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ON-AXIS BEAM ACCUMULATION BASED ON A TRIPLE-FREQUENCY RF SYSTEM

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Abstract

Considering the incompatible off-axis injection scheme on the newly constructed light sources, we have proposed a new on-axis accumulation scheme based on the so-called triple-frequency RF system. By means of additional second harmonic cavities, the original static longitudinal acceptance will be lengthened, which will provide the sufficient time to raise a full-strength kicker pulse. Through imposing the specific restriction on the RF parameters, the final bunch length can also be stretched to satisfy the functions of the conventional bunch lengthening system. In this paper, we will move on to explain how to build this complex triple-frequency RF system, and present the relevant simulation works.

INTRODUCTION

One of the intrinsic characteristics of advanced light sources, is their small dynamic aperture, which brings new challenges for the design of corresponding injection scheme [1]. Due to this specific characteristic, conventional off-axis injection schemes might be incompatible. Several new injection schemes are being designed, in which swap-out injection is a mature case and it will be utilized in the latest light sources [2, 3]. Enlightened by few on-axis injection schemes, we also proposed a new on-axis accumulation scheme, which is based on a triple-frequency RF system, consisting of fundamental, second harmonic and third harmonic cavities [4]. Compared to the conventional bunch lengthening system, normally indicating a double-frequency RF system, the extra harmonic cavities will help lengthen the original static longitudinal acceptance. The local extreme point in the potential curve, corresponding to the fixed point in the longitudinal acceptance, is away from the synchrotron phase. As well as the time interval between the circular bunch and the outermost injection point, we expect this value, employing general lattice parameters of the fourth-generation light sources, could be larger if the "golf club" effect is taken into consideration [5]. Based on the current design, the time interval between this two points, can be lengthened to nearly 2 ns, and this value in the single frequency RF system or the double-frequency RF system is less than 1.5 ns. Furthermore if considering the energy loss per turn is related to the energy spread, this time interval is able to be increased about 10% to 15%. So long as the design of the kicker is compatible with the above time limit, there will not be any disturbance to the circular bunch in the whole injection process. Also the injection can be done in multiturn, when the last injected bunch merged to the synchrotron phase, the next kicker pulse can be raised again. This injection scheme also relieve the design for the booster, if a high charge bunch is needed in the storage ring, and all the RF parameters can remain unchanged during the injection. In this paper, we will explain how to build such triple-frequency RF system, and present the relevant simulation results, in allusion to a typical fourth-generation light source.

THE CONSTRUCTION OF THE TRIPLE-FREQUENCY RF SYSTEM

Without radiation damping, the single-particle motion while considering the triple-frequency RF system,

$$H(\phi,\delta;t) = \frac{h_f\omega_0\eta}{2}\delta^2 + \frac{\omega_0 e}{2\pi E_0\beta^2}\Big[\sum_{i=1}^{N_f} V_f^i\cos(\phi+\phi_f^i) + \frac{h_f}{h_1}\sum_{j=1}^{N_{h_1}} V_{h_1}^j\cos(\frac{h_1}{h_f}\phi+\phi_{h_1}^j) + \frac{h_f}{h_2}\sum_{k=1}^{N_{h_2}} V_{h_2}^k\cos(\frac{h_2}{h_f}\phi+\phi_{h_2}^k) + \phi U_0\Big], \tag{1}$$

where ϕ and δ are a pair of canonical variables with respect to the time variable t, $\omega_0 = 2\pi c/C$ is the angular revolution frequency of the synchrotron particle, c is the speed of light, C is the circumference of the storage ring. $\eta = \alpha_c - 1/\gamma^2$, $\beta = \sqrt{1-\gamma^2}$, where α_c is the momentum compaction factor of the storage ring, γ is the relativistic factor. Suppose there are N_f fundamental cavities with a harmonic number h_f, N_{h_1} harmonic cavities with a harmonic number h_1, and N_{h_2} harmonic cavities with a harmonic number h_2. V_f^i, $V_{h_1}^j$ and $V_{h_2}^k$ are the voltages of the i-th fundamental cavity, the j-th 2nd harmonic cavity and the k-th 3rd harmonic cavity respectively. ϕ_f^i, $\phi_{h_1}^j$ and $\phi_{h_2}^k$ are the phases of the synchrotron particle relative to the above cavities.

According to the natural mathematical features of the potential curve, and lengthening the bunch longitudinally, we have several restrictions,

$$P(\phi_b) = P_{max}, P'(\phi_b) = 0, P''(\phi_s) = 0. \tag{2}$$

In which function $P(\phi)$ is the beam potential, ϕ_b is the fixed point in the longitudinal acceptance, corresponding to the

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local extreme point in $P(\phi)$, and ϕ_s is the synchrotron phase. For easier writing, denoting $\phi_1 = \phi_f$, $\phi_2 = \phi_{h_1}$, $\phi_3 = \phi_{h_2}$, $V_1 = V_f, V_2 = V_{h_1}, V_3 = V_{h_2}$ in a triple-frequency RF system. Combining the above Eq. 1 and Eq. 2, finally we have a unified system of nonlinear equations,

$$\begin{cases} -V_1\cos(\phi_1+\phi_s) - 2V_2\cos(2\phi_s+\phi_2) - 3V_3\cos(3\phi_s+\phi_3) = 0, \\ U_0 - V_1\sin(\phi_1+\phi_s) - V_2\sin(2\phi_s+\phi_2) - V_3\sin(3\phi_s+\phi_3) = 0, \\ U_0 - V_1\sin(\phi_b+\phi_1) - V_2\sin(2\phi_b+\phi_2) - V_3\sin(3\phi_b+\phi_3) = 0, \\ U_0(\phi_b-\phi_s) + V_1(\cos(\phi_b+\phi_1) - \cos(\phi_s+\phi_1)) + \\ \frac{V_2}{2}(\cos(2\phi_b+\phi_2) - \cos(2\phi_s+\phi_2)) + \frac{V_3}{3}(\cos(3\phi_b+\phi_3) - \cos(3\phi_s+\phi_3)) = P_{max}. \end{cases} \tag{3}$$

All the RF parameters are able to be solved from Eq. 3, corresponding to three different frequencies respectively. In the Appendix of [1], we have written down their verbose expressions for reference. Considering the optimal bunch lengthening condition, in which the first and second derivatives of the total voltage are nearly zero, the Taylor expansion to it can be simplified while omitting low order terms. Here we introduce a series of auxiliary quantities χ_i, $i = 0, 1, 2, 3, \cdots, n$, where n is the above harmonic number, $\chi_i = V_{n_i}\cos\phi_{n_i}/(V_1\cos\phi_1)$, and $\chi_0 = V_1\cos\phi_1/(V_1\cos\phi_1) = 1$. How to pick up and combine n is based on the RF system, in which $\{0,1\}$ represents a conventional bunch lengthening system, also the double-frequency RF system. Specifically $\{0,1,2\}$ is our proposed triple-frequency RF system. For a dualistic combination $\{0,n\}$, χ_n has a concise form $\chi_n = -1/n$ with a positive integer n. But for the other multivariate cases, χ_n is unable to be expressed so brief. By means of these auxiliary quantities, we can write down a unified V_z up to third order term, $V_z = (-1 - n_1^3\chi_1^3 - n_2^3\chi_2^3 - n_3^3\chi_3^3 - \cdots)V_1\cos\phi_1/6$. Here for a triple-frequency RF system, the first three terms are kept. Thus the Hamiltonian of single-particle motion with the quartic potential $\mathcal{H} = \alpha c\delta^2/2 + \alpha cqz^4/4$,

$$q = \frac{(-1 - n_1^3\chi_1 - n_2^3\chi_2^3)}{6}\frac{eV_1k_1^3}{\alpha cE_0T_0}\cos\phi_1, \tag{4}$$

and k_1 is the wave number, χ_1, χ_2 are defined before.

Due to the additional degree of freedom in the triple-frequency RF system $\{0,\chi_1,\chi_2\}$, there are indeed lots of combinations. Such as the simplest one, the fundamental, second and third harmonic cavities, denoting it as $\{0,1,2\}$. By that analogy if we increase χ_2, there will be $\{0,1,3\}$, $\{0,2,3\}$, a larger χ_2 will make this problem more complicated. Till now we just consider χ_2 up to 5, for each combination, the above steps are forced to repeat to solve all RF parameters. Figure 1 presents the results for the different combinations, in which x-axis is the time interval between ϕ_b and ϕ_s, and y-axis is the cavity voltage in the fundamental cavity.

From the figure we could find that the other combinations of harmonic number are still applicative, while the remaining combinations, which are not presented in the figure, may not be solvable through our methods. Other combinations compared to $\{0,1,2\}$, their cavity voltages are located in different areas, and obviously the numbers of the solutions

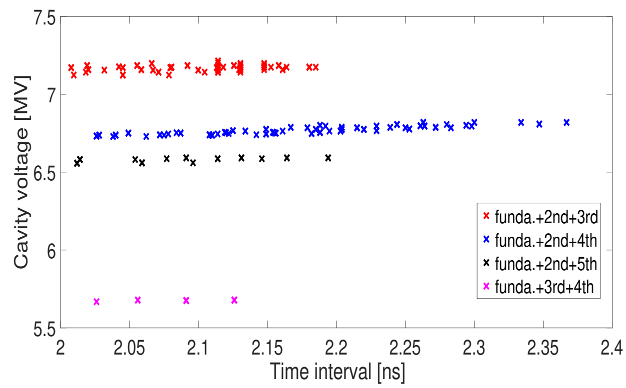

Figure 1: Cavity voltage and time interval between ϕ_b and ϕ_s for the different combinations of harmonic number.

for the first two combinations $\{0,1,2\}$, $\{0,1,3\}$ are much more.

RELEVANT SIMULATION STUDIES

We consider the random noises on the cavity voltages and phases, evaluating the impact to the original injection process. Here we utilize the lattice of High Energy Photon Source (HEPS), the random noises are part of input in the macro-particle simulation, major parameters of HEPS are listed in Table 1 [6].

Figure 2 presents the trajectories of the bunch centroid whether including the random noises in the triple-frequency RF system. Different injection points in the left and right picture, corresponding to dt=0, dp=0 and dt=-2.2 ns, dp=0.03. From the figure the original trajectories are not affected, indicating that no obvious impact to the injection process. In fact, the fundamental cavities are the main power contributor, whether in a double-frequency or a triple-frequency RF system. Energy loss per turn U_0 is larger than 4.3 MeV in HEPS, and a bucket height 3.5% requires more than 7 MV in the fundamental cavities, which may need 5-6 superconducting cavities. For the other harmonic cavites, the designed voltages are only half or less as much, and lesser power exchanges with the beam. Obviously the fundamental cavities are more easily affected and sensitive to the noises.

Figure 3 presents the result whether removing the noises from the fundamental cavities. The right picture only includes the noises on the second and third harmonic cavities, the bunch merges to the synchrotron phase after nearly 10000

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Table 1: Major parameters of HEPS, noted that energy loss per turn and relevant parameters induced by 14 insertion devices are included, the values are derived from ELEGANT simulation.

Parameter	Value	Unit
Circumference	1360.4	m
Beam energy	6	GeV
Beam current	200	mA
Natural emittance	26.14	pm
Betatron tunes	114.19/106.18	
Momentum compaction	1.28e-5	
Natural energy spread	1.14e-3	
Energy loss per turn	4.38	MeV
Damping time	7.4/12.4/9.5	ms
Harmonic number	756	
Main RF frequency	166.6	MHz
Main RF cavity voltage	7.16	MV
Second harm. cavity voltage	3.59	MV
Third harm. cavity voltage	0.90	MV
Main RF cavity phase	2.43	rad
Second harm. cavity phase	0.11	rad
Third harm. cavity phase	4.03	rad
Bunch length (no HCs)	2.6	mm
Bunch length with HCs	32.2	mm
Linear synchr. tune (no HCs)	1.2e-3	
Avg. synchr. tune with HCs	7.85e-5	
ID length	84	m
Vacuum chamber radius	3	mm

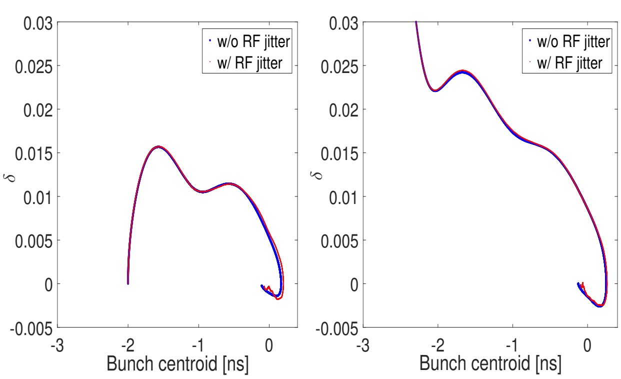

Figure 2: Longitudinal motion of bunch centroid at the presence of noises, injecting at dt=-2 ns, dp=0 (left), and dt=-2.2 ns, dp=0.03 (right), red and blue lines represent the result with and without the noises respectively.

turns and in the next 90000 turns there are only very weak oscillations, less than 0.02 ns from the warm color area. But once adding the noises on the fundamental cavities, in the left picture, the bunch centroid starts irregular oscillations in a larger longitudinal scale, the maximal offset is nearly 0.2 ns.

As for the beam collective instabilities, we investigate the transverse mode-coupling instability (TMCI) using the

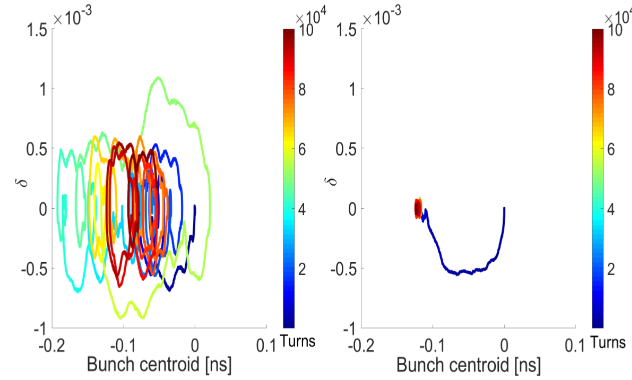

Figure 3: Comparison of the motion of bunch centroid whether removing the noises in the fundamental cavities, in which the picture on the right is after removing.

above triple-frequency RF system. A same numerical method which can refer to [7], numerical analysis results to the TMCI at the presence of the triple-frequency RF system are in Figure 4. Unstable motions will emerge at the

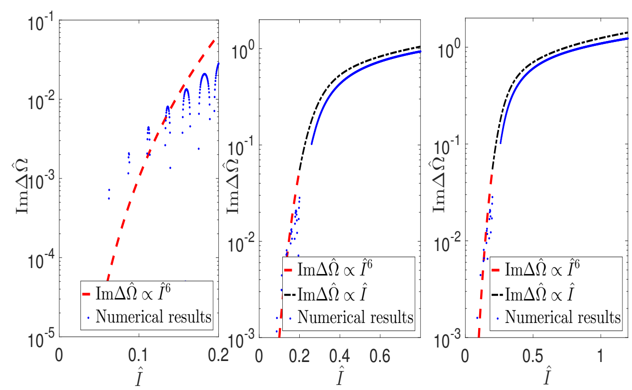

Figure 4: Numerical results and estimated values by scaling law to $\mathrm{Im}\Delta\hat{\Omega}$, representing by blue discrete dots and imaginary lines respectively. Which obeys $\propto \hat{I}^6$ for $\hat{I} < 0.2$ and $\propto \hat{I}$ for greater than 0.2.

convergence of the transverse mode $m = 0$ and $m = 1$, and give the single bunch current threshold I_{th}. The red imaginary line stands for their different asymptotic properties $\mathrm{Im}\Delta\hat{\Omega} \propto \hat{I}^6$ for $\hat{I} < 0.2$, and $\mathrm{Im}\Delta\hat{\Omega} \propto \hat{I}$ for the other side, which is already proposed in M. Venturini's results for the double-frequency RF system. Here we consider this segmented scaling law, and the framework, are still applicative for the triple-frequency RF system.

Simulation results are given by ELEGANT [8] . ILMATRIX gives a single-turn beam transport, the triple-frequency RF system is built through RFCA, and the RW impedance is given in ZTRANVERSE. By means of tuning the single bunch charges, the ever-increasing oscillation of the bunch centroid exactly indicates unstable motion. Thereby the single bunch current threshold, and the growth rates by fitting the growth trajectory of the bunch centroid could be derived. Figure 5 presents the comparison between the estimated values by the scaling law in [7], and the simulation results. The point

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of intersection with the radiation damping rate, representing by the blue imaginary line, indicates the the single bunch current threshold.

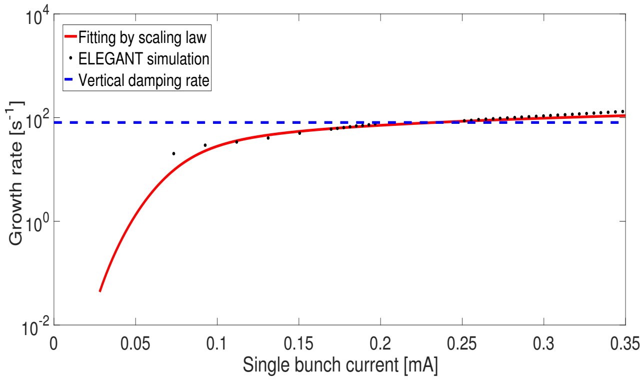

Figure 5: Comparison between ELEGANT simulation and estimated value by fitting scaling law in black dots and red lines, while blue imagnary line indicates vertical damping rate. And the point of intersection nearly 0.23 mA gives single bunch current threshold.

CONCLUSION

The on-axis beam accumulation scheme are given based on a triple-frequency RF system, and we briefly introduce how to build it. In allusion to the noises on the RF system, which may not be influential to the injection. Relevant beam dynamic issues are being studied, the similar analytical method is still applicative compared to the double-frequency RF system. Due to their similar Hamiltonian with the quartic potential in the optimal bunch lengthening condition.

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