NOISE SUPPRESSION IN RELATIVISTIC ELECTRON BEAMS*

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Abstract

A brief rerview is presented of the various schemes which have been proposed for removing shot noise from a beam that will be used as a driver for a free electron laser. Attention is focused on just one of these schemes; namely that proposed in Ref. [1]. We begin with a general discussion of shot noise properties and their mathematical formulation. An analysis is developed which expresses noise suppression in terms of the longitudinal impedance in the region where interaction is taking place. The impedances of a number of different interaction regions are presented with numerical examples that demonstrate their efficacy for noise suppression. Comments are made about the application of noise suppression to real systems.

INTRODUCTION

In a beam of randomly distributed particles, the longitudinal density exhibits uncorrelated fluctuations, commonly called shot noise. In free electron lasers (FEL), shot noise provides the start-up radiation for Self-Amplified Spontaneous Emission (SASE). There are situations, however, where the same density fluctuations may adversely affect FEL operation. For example, the microbunching instability incapacitates diagnostics of the beam and can lead to degradation of the FEL performance [2–7]. In seeded FELs shot noise competes with external modulations of the beam being amplified in the process of the seeding [8, 9]. In these situations suppression of the shot noise in the beam would lead to improved performance of the FEL and allow for a lower seed power in seeded machines. Suppressing shot noise could also allow controlling instabilities and increasing efficiency in cooling relativistic beams [10]. Suppression of shot noise is the subject of this paper.

Suppression of long wavelength shot noise was observed in microwave tubes as early as the 1950s, [11]. In the last few years, several groups have independently proposed suppressing shot noise at short wavelengths in relativistic electron beams [1, 12–14]. The first experimental observation of shot noise suppression at sub-micron wavelengths were recently reported in [15, 16].

We have to point out that noise suppression is not a cooling of the beam. As we will show below it involves the reactive part of the impedance associated with the particle interactions. One can say that it transfers the shot noise in the longitudinal (z) direction to the energy coordinate, where it is relatively benign for applications of interest. We will discuss how noise suppression can improve FEL performance, but also, as well, the difficulties associated with its application.

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APPROACHES TO SUPPRESS THE SHOT NOISE

One method for shot noise suppression (also chronologically first) was presented in Refs. [12, 13] where it was pointed out that the density fluctuations in a beam oscillate in time with the plasma frequency. If the beam is prepared in such initial state that there is only density fluctuation, and no energy, or velocity, fluctuation, after a quarter of the plasma period it will be fully converted to velocity fluctuation and the initial density fluctuation disappears.

This suppression method can be useful for beam energies that are not very large because the length required for a relativistic beam to execute a plasma oscillation increases with the beam energy. The beam plasma frequency, in the laboratory frame, is

$$\omega_p = c\left(\frac{4\pi}{\gamma^3 S}\frac{I}{I_A}\right)^{1/2}, \tag{1}$$

where I is the beam current, $I_A = mc^3/e \approx 17.5$ kA is the Alfven current, and S is the transverse cross section area of the beam. As an example, consider the following beam parameters: beam energy 1 GeV, $I = 1$ kA, $S = 100\ \mu$m $\times 100\ \mu$m, which give for the quarter plasma period length $\pi c/2\omega_p \approx 16$ m. Lower currents or higher beam energies make this distance longer and, hence, less atractive.

Another approach, which seems more attractive in the limit of high energies, was proposed in [1]. In this paper we will focus only on the second approach, for it has the promise of a more compact setup. In this approach a relatively short interaction region is used followed by a dispersive element, see Fig. 1. Of course it is an approximation to confine the interaction to a particular region but for conceptual purposes it is convenient to think this way. A complete calculation would be needed to quantify this approach.

Figure 1: Schematic of noise suppression system. A beam with shot noise is injected into an interaction region of length L, followed a dispersion region with a proper value of R_{56}.

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The particles are assumed frozen at their longitudinal positions throughout the interaction region. Thus we assume that plasma oscillations do not play a role and, since we neglect plasma oscillations, we assume that the length of the system is shorter than $\pi c/2\omega_p$. Particles are shifted longitudinally by passage through the dispersive element, and this shift is controlled by the (5,6) element of the transport matrix, R_{56}.

Still a third method of shot noise suppression, is outlined in Ref. [14]. It involves two wigglers, a chicane and an amplifier. Radiation from the first wiggler is amplified in an optical amplifier and then recombines with the beam in the second chicane. The interaction with this radiation, according to [14] would lead to the noise suppression in the beam. Since this approach is described, in detail, in another contribution to this Workshop, we shall not discuss it further here.

SHOT NOISE PROPERTIES

We consider fluctuations in the beam in the laboratory frame of reference. The coordinate z marks the position of a particle inside the beam (with positive z in the direction of propagation), and $\eta = \Delta E/E_0$ is the relative energy deviation with the nominal energy of the beam $E_0 = \gamma mc^2$. The 1D distribution function is $f_0(z, \eta) = n_0 F(\eta) + \delta f(z, \eta)$ where $F(\eta)$ is the averaged distribution function normalized by $\int d\eta F(\eta) = 1$, and n_0 is the averaged line density of the beam. Note that we assume on average uniform distribution over z which is a reasonable local approximation for small-scale fluctuations. The fluctuational part $\delta f(z, \eta)$ can be Fourier expanded, $\delta\hat{f}_k(\eta) = \int_{-\infty}^{\infty} dz e^{-ikz}\delta f(z, \eta)$. For shot noise, according to the statistical physics of ideal gas [17],

$$\langle \delta f(z, \eta)\delta f(z', \eta')\rangle = n_0 F(\eta)\delta(z - z')\delta(\eta - \eta'), \quad (2)$$

which after the Fourier transformation gives

$$\langle \delta\hat{f}_k(\eta)\delta\hat{f}_{k'}(\eta')\rangle = 2\pi n_0 F(\eta)\delta(k + k')\delta(\eta - \eta'). \quad (3)$$

Introducing the density fluctuation $\delta n(z)$ as $\delta n(z) = \int d\eta\, \delta f(z, \eta)$ we find by integrating (2) over η and η'

$$\langle \delta n(z)\delta n(z')\rangle = n_0\delta(z - z'). \quad (4)$$

This is the mathematical expression of the properties of the shot noise: the density fluctuations in shot noise are uncorrelated in space.

We can formally introduce the Fourier spectrum of $\delta n(z)$

$$\delta\hat{n}_k = \int_{-\infty}^{\infty} dz e^{-ikz}\delta n(z) = \int_{-\infty}^{\infty} d\eta\, \delta\hat{f}_k(\eta). \quad (5)$$

Integrating (3) over η and η' we obtain $\langle\delta\hat{n}_k\delta\hat{n}_{k'}\rangle = 2\pi n_0\delta(k + k')$, which is equivalent to (4) but expressed through the Fourier harmonics.

SUPPRESSING SHOT NOISE THROUGH INTERACTION AND DISPERSION

In the interaction region the density modulation changes particles' energy through the longitudinal wake. In this section we consider a general case of arbitrary wake function with two specific examples of the wake studied in subsequent sections of the paper. We define the wakefield function $w(z)$ so that the energy change $\Delta E(z)$ of particles at point z in the beam after passage of the interaction region is

$$\Delta E(z) = e^2 \int_{-\infty}^{\infty} w(z - z')\delta n(z'). \quad (6)$$

Associated with the wake is the longitudinal impedance $Z(k)$ defined through the Fourier transform of the wake, $Z(k) = -\hat{w}_k/c$. Using (6) it is straightforward to find the relative energy change $\Delta\eta(z)$ in terms of $Z(k)$,

$$\Delta\eta(z) = -\frac{r_e c}{2\pi\gamma}\int_{-\infty}^{\infty} dk Z(k)\delta\hat{n}_k e^{ikz}. \quad (7)$$

Changing particles' energy through interaction leads to a new distribution function at the end of the interaction section which we denote by f_1, $f_1(z, \eta) = f_0(z, \eta - \Delta\eta(z))$. Sending the beam through a chicane with the dispersive strength R_{56} changes the distribution function again, $f_1 \to f_2$, with

$$\begin{aligned} f_2(z, \eta) = f_1(z - R_{56}\eta, \eta) = n_0 F(\eta - \Delta\eta[z - R_{56}\eta]) \\ + \delta f(z - R_{56}\eta, \eta - \Delta\eta[z - R_{56}\eta]). \end{aligned} \quad (8)$$

We consider the fluctuating quantities δf, δn, and hence $\Delta\eta$, which according to (7) is proportional to δn, as small quantities and Taylor expand (8) keeping only linear terms in fluctuations. Integrating the result over η gives the density fluctuation after the chicane which we denote $\delta n_2(z) = n_2(z) - n_0$,

$$\delta n_2(z) = \int_{-\infty}^{\infty} d\eta[\delta f(\zeta, \eta) + R_{56} n_0\Delta\eta(\zeta)F'(\eta)], \quad (9)$$

where we introduced $\zeta = z - R_{56}\eta$. We now make the Fourier transformation of $\delta n_2(z)$ and compute the quantity $|\delta n_{2k}|^2$ in the relation $\langle\delta n_{2k}\delta n_{2k'}\rangle = |\delta n_{2k}|^2\delta(k + k')$. If $|\delta n_{2k}|^2$ becomes smaller than the initial $|\delta n_k|^2$ then the noise is suppressed. The quantity $|\delta n_{2k}|^2$ can be calculated using (9) and the correlators for the perturbation of the distribution function (3). The result is

$$|\delta n_{2k}|^2 = 2\pi n_0(1 - 2T\,\mathrm{Im}\, Q + |Q|^2 T), \quad (10)$$

where

$$Q = R_{56} n_0 \frac{r_e c}{\gamma} k Z(k), \quad (11)$$

and

$$T = \left|\int_{-\infty}^{\infty} d\eta e^{ikR_{56}\eta} F(\eta)\right|^2 = e^{-(kR_{56}\sigma_\eta)^2}, \quad (12)$$

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where the last expression is obtained assuming a Gaussian distribution, $F(\eta) = (2\pi)^{-1/2}\sigma_\eta^{-1} e^{-\eta^2/(2\sigma_\eta^2)}$.

Analysis of (10) shows that the full suppression of noise can be achieved in the case when Z, and hence Q, are purely imaginary with $\mathrm{Im}\, Q = T = 1$. Introducing the *noise factor* $F_\mathrm{n}(k)$ as $F_\mathrm{n}(k) = |\delta n_{2k}|^2/|\delta n_k|^2$, we find that in this case $F_\mathrm{n} = 0$. As is seen from (12) $T = 1$ only in the limiting case of a cold beam, $\sigma_\eta = 0$. Assuming a small energy spread, such that $k^2 R_{56}^2 \sigma_\eta^2 \ll 1$, and $\mathrm{Re}\, Z = 0$, it is easy to show that the minimum value of F_n is achieved at $\mathrm{Im}\, Q = 1$ and is equal to

$$\min F_\mathrm{n} \approx k^2 R_{56}^2 \sigma_\eta^2. \quad (13)$$

We see that for a beam with finite energy spread only *partial* noise suppression can be achieved.

Expressing R_{56} through other variables from equation $\mathrm{Im}\, Q = 1$ and substituting it into (13) we obtain

$$\min F_\mathrm{n} = \left(\frac{I_A}{I} \frac{\sigma_\eta \gamma}{\mathrm{Im}\, Z(k) c} \right)^2. \quad (14)$$

Note that since the required value of R_{56} from (11) is proportional to $1/n_0$ and the beam density and current vary along the bunch, the optimal suppression can be achieved only in a limited interval where the beam current is approximately constant.

INTERACTION THROUGH SPACE CHARGE FORCES

Let us assume that interaction between the particles occurs in a drift space and is due to the space charge forces in the beam, and use a 1D model to describe this interaction. In this model each electron is treated as a sheet of charge e and area S (S is associated with the beam transverse area) and variation of the field in the transverse direction is neglected. The 1D space charge model is valid if the transverse size of the beam is large, $S \gg \gamma/k$. In this case the density modulation $\delta n(z)$ in the beam creates the electric field $\mathcal{E}$ which satisfies the Poisson equation $\partial \mathcal{E}/\partial z = 4\pi e \delta n(z)/S$. Solving this equation with the help of the Fourier transformation and calculating the energy change of a particle at coordinate z gives

$$\Delta\eta(z) = \frac{e\mathcal{E}(z)L}{E_0} = -\frac{2ie^2 L}{S\gamma mc^2} \int_{-\infty}^{\infty} \frac{dk}{k} \delta\hat{n}_k e^{ikz}. \quad (15)$$

Comparing this with (7) we find the impedance of the space charge

$$Z_\mathrm{sc} = \frac{4\pi i L}{Skc}. \quad (16)$$

We see that the space charge impedance is purely imaginary and positive, and as was pointed out in the previous section, can be used to effectively suppress the beam noise.

For a numerical example we consider the following parameters: beam energy 1 GeV, $I = 1$ kA, $S = 100\ \mu$m $\times 100\ \mu$m, $\sigma_\eta = 10^{-4}$, $L \approx 5$ m (comfortably less than the plasma distance $L = \pi c/2\omega_p$). Using (14) we obtain $F_\mathrm{n} \approx 0.11$ at the wavelength of 10 nm.

In the next sections of this paper we shall develop the longitudinal impedance of three possible schemes. The original work, upon which this is based, was performed by two of the co-authors and is described – actually in more detail – in [18].

1D UNDULATOR INTERACTION

We now consider the case when an undulator in the interaction region provides a mechanism for the energy exchange between the particles. Electrons passing through an undulator interact with each other through the emitted electromagnetic field of undulator radiation. The wake and the impedance corresponding to this interaction are calculated again using the 1D model. Such a model is valid if the beam area S is much larger than L_u/k, where L_u is the undulator length.

For simplicity we consider a helical undulator which has N_u periods and the undulator parameter K. The formula for the undulator wake $w_u(z)$ was derived in Ref. [19], with the corresponding impedance given by

$$Z_u(\nu) = \frac{W}{ck_0} \left[\frac{i\nu}{1-\nu^2} + \frac{(\nu^2+1)(1-e^{-2\pi i N_u \nu})}{2\pi N_u (\nu^2-1)^2} \right],$$

where $\lambda_0 = 2\pi/k_0$ is the wavelength of the undulator radiation,

$$W = 4\pi \frac{N_u \lambda_u}{S} \frac{K^2}{1+K^2} = 8\pi \frac{N_u \lambda_0 \gamma^2}{S} \frac{K^2}{(1+K^2)^2}, \quad (17)$$

$\nu \equiv k/k_0$, λ_u is the undulator period, and N_u assumed to be an integer. It is easy to see that $Z_u(\nu)$ is purely imaginary when $N_u \nu$ is an integer. The maximal imaginary part of Z_u is obtained for $\nu = 1 \pm N_u^{-1}$ and in the limit $N_u \gg 1$ is equal to

$$\mathrm{Im}\, Z_u|_{\nu = 1 \pm N_u^{-1}} \approx \pm \frac{W N_u}{2ck_0}. \quad (18)$$

Comparing this expression with the space charge impedance (15) we find that, in the limit $K \gg 1$, the undulator impedance is $N_u/2$ times larger than Z_sc. On the other hand, while Z_sc in 1D model has a vanishing real part for all wave numbers k, the real part of Z_u vanishes only at particular values of k. This means that with undulators one can expect to achieve noise suppression in a narrow frequency bands the positions of which are determined by the undulator frequency and the width is inversely proportional to the number of periods N_u. It is interesting to point out that the product of the impedance by the frequency bandpass for the undulator, within a numerical factor, is the same as for the space charge interaction.

In order to have $\mathrm{Im}\, Z_u > 0$ we choose $\nu = 1 - N_u^{-1}$. Substituting (18) into (14) and assuming for simplicity $K = 1$ we obtain for the noise factor

$$\min F_\mathrm{n} = \left(\frac{I_A}{I} \frac{2S\sigma_\eta}{N_u^2 \lambda_0^2 \gamma} \right)^2. \quad (19)$$

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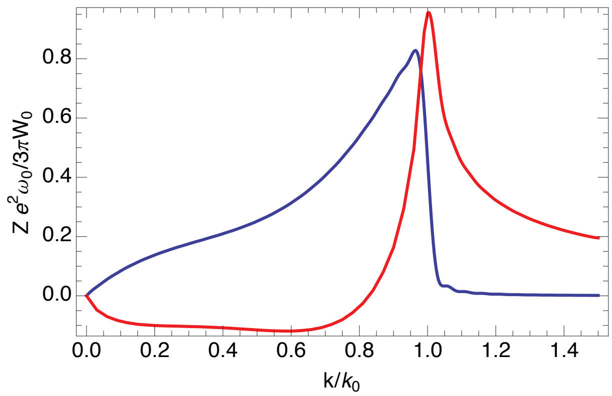

Figure 2: Imaginary (red) and real (blue) parts of the impedance in the undulator for $K \ll 1$ and $N_u = 20$.

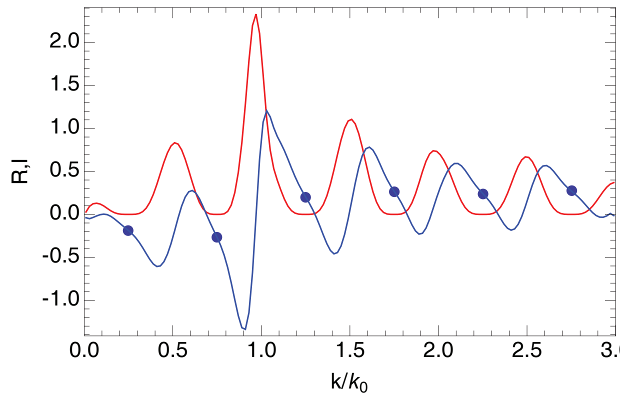

Figure 3: Functions $\mathcal{R}$ (red) and $\mathcal{I}$ (blue) for a finite length undulator. The dots on the blue curve indicates values of $\mathcal{I}$ at points where $\mathcal{R} = 0$.

As a numerical example consider the following parameters: beam energy 1 GeV, $I = 1$ kA, $S = 100\ \mu$m $\times 100\ \mu$m, $\sigma_\eta = 10^{-4}$, $\lambda = 10$ nm, $N_u = 30$. These parameters give $F_\mathrm{n} = 0.04$.

LINE-CHARGE BEAM IN UNDULATOR

As we see from (19) making the transverse size of the beam smaller increases the strength of the interaction and the wakefield. However, as was pointed out earlier, the one-dimensional model requires $S \gg L_u/k$. To consider the opposite limit, $S \ll L_u/k$, we have to treat the beam as a line charge. This is the subject of this section, in which we additionally assume a weak undulator, $K \ll 1$.

The wakefield and impedance in this case can be found using the relation between the real part of the impedance and the spectrum of radiation of a single electron, $\mathrm{Re}\, Z = (\pi/e^2) d\mathcal{W}/d\omega$, where $d\mathcal{W}/d\omega$ is the energy radiated by the electron in unit frequency interval. The imaginary part of the impedance can be found with the help of the Kramers-Cronig relation. In the limit $K \ll 1$, using standard formulas for undulator radiation [20] we found

$$Z = \frac{\pi}{e^2}\frac{3\mathcal{W}_0}{\omega_0}[\mathcal{R}(\nu, N_u) + i\mathcal{I}(\nu, N_u)] \qquad (20)$$

where

$$\mathcal{W}_0 = \frac{1}{3} r_e^2 \gamma^2 B_0^2 L_u = \frac{\pi}{3} e^2 k_0 K^2 (1 + K^2) N_u \qquad (21)$$

is the total energy radiated by the electron. The expressions for $\mathcal{R}(\nu, N_u)$ and $\mathcal{I}(\nu, N_u)$ can be found in [18].

Plots of the functions $\mathcal{R}(\nu, N_u)$ and $\mathcal{I}(\nu, N_u)$ for $N_u = 20$ are shown in Fig. 2. As one can see from Fig. 2 the real part of the impedance quickly vanishes in the region $k > k_0$. This however is not so if we drop the assumption $K \ll 1$: as is well known the radiation from an undulator, and hence $\mathrm{Re}\, Z$ extends over a wide region of frequencies without gaps. To resolve this problem we consider in the next section a setup with two undulators separated by a drift.

TWO-UNDULATORS SETUP

Consider a system in which there are two undulators separated by distance l_s. Interference between the two undulator radiation introduces a phase shift factor $\phi = k_0 l_s/2\gamma^2$ at the fundamental frequency ω_0. It adds a factor $[1 + \cos(\phi k/k_0)]^2$ in the intensity of the total radiation, which vanishes for $kl_s = \pi(n + 1/2)$, where n is an integer, $n = 0, 1, 2, \ldots$. The calculated functions $\mathcal{R}$ and $\mathcal{I}$ for the case of two undulators setup are shown in Fig. 3 for the case of $N_u = 10$ periods, $K = 1$ and $\phi = 4\pi$. For the noise suppression we now have

$$\min F_\mathrm{n} = \left(\frac{I_A}{I}\frac{\sigma_\eta \gamma}{\pi^2 K^2 (1 + K^2) N_u \mathcal{I}(\nu, N_u)}\right)^2 . \qquad (22)$$

As a numerical example consider the following parameters: beam energy 1 GeV, $I = 1$ kA, $\sigma_\eta = 10^{-4}$. We will take $\mathcal{I} = -0.26$ from Fig. 3 (corresponding to the point $\nu = 0.75$), which gives $F_\mathrm{n} = 4 \times 10^{-3}$. While this result looks impressive, unfortunately, in the range of short wavelength (of order of ten nanometers) it requires small beam radius, and hence small beam emittance. For example, taking $\lambda = 30$ nm, we find that in order for the line charge model be applicable, the beam radius should be smaller than 12 μm.

EFFECT OF ENERGY NOISE ON FEL STARTUP

In the previous sections we looked at the suppression of the density fluctuations which are characterized by the correlator $\langle \delta\hat{n}_k \delta\hat{n}_{k'} \rangle$. This quantity is proportional to the initial startup power in SASE FEL only when the beam has a negligibly small energy spread. In the case of a finite energy spread analysis shows that the FEL startup is determined by the following integral [21]:

$$J = |\mu_0|^2 \int d\eta d\eta' \frac{\langle \delta\hat{f}_k(\eta) \delta\hat{f}^*_{k'}(\eta') \rangle}{(\mu_0 - \eta)(\mu_0^* - \eta')}, \qquad (23)$$

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where μ_0 is the complex frequency of the fastest growing FEL mode normalized by $2k_u c$. Note that in the limit of a cold beam, $\sigma_\eta \to 0$, the quantity J reduces to $\langle \delta\hat{n}_k \delta\hat{n}^*_{k'} \rangle$, as expected.

In general case, making $\langle \delta\hat{n}_k \delta\hat{n}^*_{k'} \rangle$ equal to zero does not mean that J vanishes. This has been emphasized in Ref. [22] where the noise suppression with plasma oscillations was studied. Here we present the results of analysis for the case when the interaction and dispersion occur in separate regions, as shown in Fig. 1.

It turns out that suppression of the quantity J is not as effective as suppression of the density fluctuations. We now define the suppression factor $F_\mathrm{n} = J/J_0$ where J_0 is the value of J before the suppression. While for the cold beam the minimum value of F_n was given by Eq. (13), now under the same conditions, we find

$$\min F_\mathrm{n} \equiv \frac{J}{J_0} \approx k^2 R_{56}^2 \sigma_\eta^2 \cdot S\left(kR_{56}\sigma_\eta, \frac{\sigma_\eta}{\rho}\right), \qquad (24)$$

where the function S is greater than one. For illustration, Fig. 4 shows the plot of S as a function of the relative energy spread for $kR_{56}\sigma_\eta = 0.1$. One can interpret this result as an additional (to density fluctuations) energy fluctuations that are not diminished when the density fluctuations are decreased.

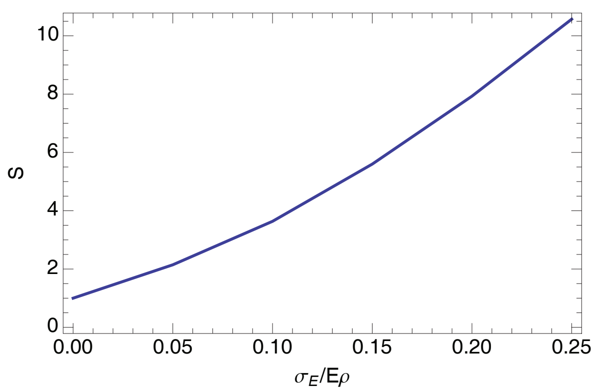

Figure 4: Plot of function S versus relative energy spread normalized by the FEL parameter ρ for $kR_{56}\sigma_\eta = 0.1$.

SUMMARY

Let us try and summarize the subject of noise suppression of beams sutable for driving an FEL. A practical beam will be Gaussian, not a flat top, and therefore the suppression methods will only work for part of the beam. Practical application requires that a beam very close to a flat top be produced. Also, we have seen that the intrinsic energy spread must be very small (even as compared to $\sigma_E/E\rho$). Again this puts a requirement on the electron gun and, most particularly, on the avoidance of instabilities in the buncher without use of a laser heater. We note that the undulator methods of noise suppression require an intense beam of very small emittance. Once again, a severe requirement on the electron gun. Alternatively, undulators of a very small wavelngth would be useful. In sum, then, the noise suppression schemes do not seem to help at the present time. On the other hand, the reduction of noise is definitely an advantage for FEL operation and the effort, here, delineates the developments necessary to make suppression useful in practical systems.

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