

# COMPARATIVE STUDIES OF RECONSTRUCTION METHODS TO ACHIEVE MULTI-DIMENSIONAL PHASE SPACE INFORMATION

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## Abstract

High Intensity Proton Accelerators like SNS, PSI or future machines like ESS or Isis upgrade cannot tolerate high losses due to activation. Standard beam diagnostics may not provide enough information about potential loss sources like beam filamentation or halo. Moreover, the application of interceptive methods like slits or pepperpot can be seriously discouraged by either high power deposition or explicit requirements for non-destructive methods like on-line diagnostics near superconducting cavities. Reconstruction of the beam distribution with a tomography method based on Maximum Entropy could help to overcome those problems and is easily to integrate in already existing facilities because the algorithm does not depend on the experimental profile measurement technique. Furthermore beam tomography can be employed on both spatial and phase-space reconstruction. The paper compares results from two different software packages from PSI (Maximum Entropy Tomography MENT) with the code used at RAL (MemSys5).

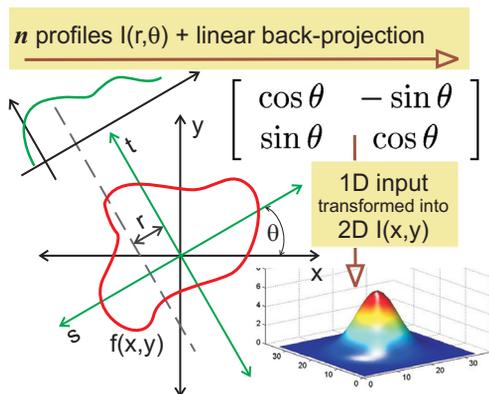


Figure 1: Sketch of a simple tomographic method. Red shows an 2D object  $F(x, y)$  to probe. Several profiles  $I(r, \theta)$  are acquired varying the parameter  $\theta$ . If the correct transformation is applied (*here*: rotation matrix) the 2D distribution  $F$  may be reconstructed.

## INTRODUCTION

Image reconstruction methods can be beneficially to diagnose ion beams and some early work has been carried out at Los Alamos [1, 2] can be found. Different mathematical approaches may be employed, for this paper only maximum entropy, closely related to Bayesian probability the-

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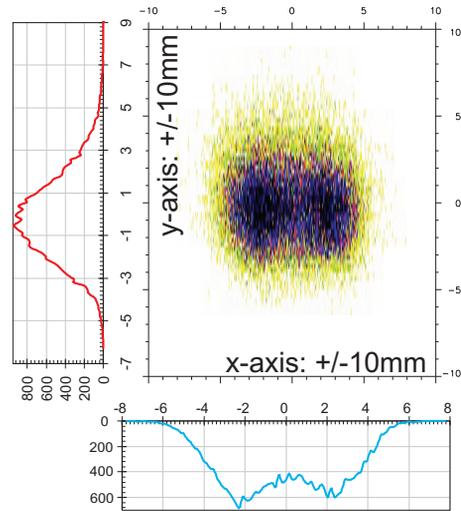


Figure 2: Particle distribution, typical for the FETS ion source downstream the post-acceleration, shown in pseudo-colours as an intensity map. As a unique feature,  $I(x)$  has two maxima according to the intensity peaks,  $I(y)$  shows some halo. From the multi-particle distribution are extracted all profiles.

ory (scientific inference), is considered [3]. The main advantage is no need of a high number of “input data points” and excellent dealing with noisy or otherwise incomplete data.

Nevertheless this technique is not often utilized in beam instrumentation but more recently, there is growing interest at laboratories like DESY [4], SNS/ PSI and RAL. It is worth to consider that image reconstruction is independent of the way how beam data are acquired, i.e. it could be used in combination with (existing) wire scanners [5] or applied to non-destructive diagnostics [6]. In either way, it means to supply data in  $(N - 1)$  dimensions to achieve  $N$  D (phase)-space information.

Additional information are needed in the from of knowledge how to transform the existing data onto the missing distribution. This is usually a  $2 \times 2$  matrix (see Fig.1) and the reconstruction may take place either in phase space or real space, the former usually called emittance reconstruction and the latter tomography. The terminology borrowed from Bayesian calculus describes the tomographic process as

$$\underbrace{\text{Posterior Probability}}_{\text{Reconstructed distribution}} \propto \underbrace{\text{Prior Probability}}_{\text{Unconditional probability}} \times \underbrace{\text{Likelihood}}_{\text{input, profiles}}$$

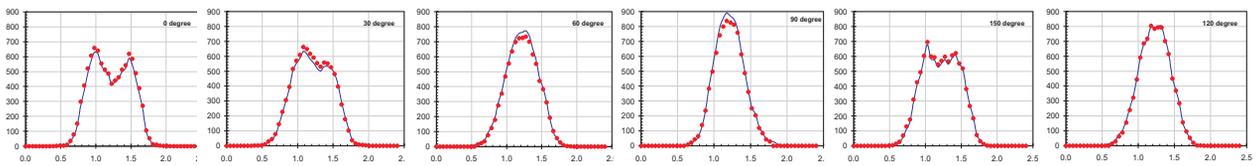


Figure 3: Shown are all 6 profiles used for the MENT computation. The blue graphs represent the input distribution, the red dots are the calculated data points. Especially  $0^\circ$  and  $120^\circ$  represent correct the two peaks of the original distribution.

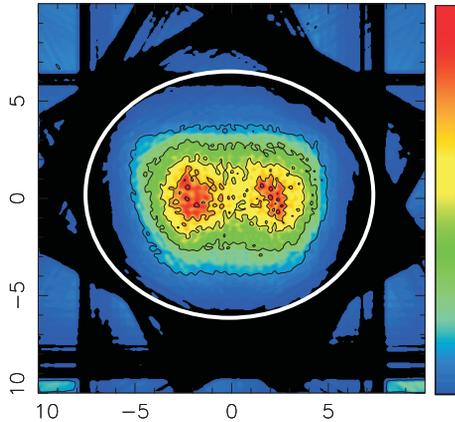


Figure 4: MemSys5 posterior distribution with 6 profiles as input. The prior spreads over the  $10 \times 10$  mm space homogeneously hence there are left-over areas which are separated from the actual distribution and should not be considered in further data mining.

Maximizing the entropy (Maxent principle) and considering all other constraints (Input profiles but also knowledge of previous iterations) leads eventually to the reconstruction. Maxent helps to interpolate missing information (e.g. limited number of profiles) but not falsify the distribution by incorrect noise.

This paper compares two different software codes used at PSI (MENT) and RAL (MemSys5), both have demonstrated their use in emittance reconstruction under different preconditions: MENT bases on the Los Alamos code [1] and was improved by W. Joho and U. Rohrer [5]. Recently time has been put in to reemployed MENT to the existing PSI beamline [7] and to improve the output (graphics) “capabilities” [8]. MemSys5 used in emittance reconstruction was first published in [9] with the intention to be used at the front end test stand (FETS) beamline [10] for photo-detachment emittance measurements. In co-operation with D.S. Sivia MemSys5<sup>1</sup> was adopted to the demands of emittance reconstruction.

## APPLIED METHOD

So far, both codes were used to reconstruct emittance in phase space but not tomography in real space. The math is

<sup>1</sup>see also <http://www.maxent.co.uk>

the same but you may have different characteristic patterns which could effect the outcome. The underlying principle is to use a know input distribution (Fig.2) as a starting point to extract profiles as well as reference to compare the simulations. It was thought that 6 profiles at angles  $\theta = 0, 30, 60, 90, 120, 150^\circ$  is a reasonable number and the coverage should be in favor of the tomography.

A problem which needs to be considered arises due to different file formats and generation/ transformation between multi-particle distributions and intensity maps with isolines (contour map). To minimize this effect the number of particles was kept sensible high, i.e.  $> 30,000$  in order to avoid unnecessarily quantization errors. Another issue occurs if the profiles are generated by spreading all particles to a given bin array due to the finite number of particles and may lead to Poisson noise. Contrariwise this procedure is similar to beam instrumentation when e.g. the wire collects a finite number of particles measuring a certain charge. The bin-arrays to acquire the are different, MemSys5 has no limitations but usually 100 bins produce reasonable results. In contrast, MENT needs a fixed abscissa and is limited to 51 bins. The most broad profile defines the x-axis, all other profiles subsequently cover less bins.

The quantitative study of the results relies on the moments of a distribution  $f(x_1, \dots, x_k)$ . Any distribution can be characterized by a number of features (such as mean, variance, skewness ...), and the moments  $\langle x_i^n \rangle$  of a function describe the nature of its distribution and more general you may note

$$\langle x_i^n \rangle = \int \dots \int x_i^n \cdot f \, dx_1 \dots dx_k \quad .$$

Most important information for particle distributions contain the 1<sup>st</sup> and 2<sup>nd</sup> momentum whereas higher order momentums refer to the central momentum  $\langle (x_i - \langle x_i \rangle)^n \rangle$ . The use of the momentum is usually employed to the rms-emittance as it was introduced by Sacherer [11] describing an area  $F = \pi \cdot \varepsilon_{\text{rms}}$  and one may follow a similar route to define an “rms-area”  $xy_{\text{rms}}$  in  $[\pi \text{ mm}^2]$

$$xy_{\text{rms}} = \sqrt{\langle x^2 \rangle \langle y^2 \rangle - \langle xy \rangle^2} \quad .$$

The MemSys5 output is shown in Fig. 4, the shown circle identifies what is considered as beam distribution. The reason is the initial flat and homogeneous prior and if no information available this remains constant. The MENT result is presented in Fig. 3 comparing the input profiles and the

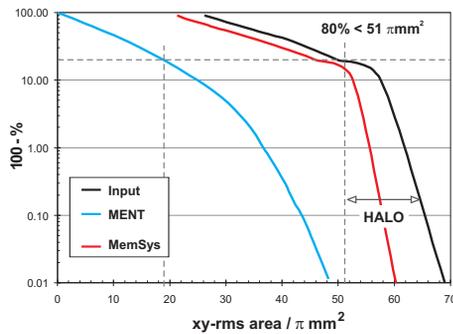


Figure 5: Comparison of the rms–area as described in the text of input and calculated distributions. The graphs show what lies outside a certain intensity level, e.g. the input has at  $y = 80$  only 20% of  $xy_{rms}$ –area bigger that  $52 \pi\text{mm}^2$ . Such a presentation is sometimes called halo–plot.

simulated ones. One of the key features of the input distribution, namely the two maxima is also represented by the profiles.

The rms–area is given in Fig. 5 where input and MemSys5 have a similar characteristic but MENT shows some different behaviour. The curve indicates a more smooth density variation. At first sight, this might be a contradiction to the good agreement between input profiles and tomography:  $x$  and  $y$  dimensions are correct as well as the two intensity maxima. One possibility to falsify the  $xy_{rms}$  might be the profiles, they are more coarse than the MemSys5 ones (Fig. 6). On the other hand, a MemSys5 test with the same profiles used for MENT show at least a similar good agreement with the input distribution but the result is far more sensitive to Poisson noise and certainly plays a more important role due to fewer data points available (the likelihood is more sparse populated). Another reason to explain the differences between MemSys5 and MENT is that the latter assumes another prior qualified more for Gaussian–like posterior probability.

## SUMMARY & OUTLOOK

PSI and RAL began to work together on the field of image reconstruction. The paper presents first results of a comparative study to judge differences in the software. Both MENT and MemSys5 have demonstrated their potential on emittance reconstruction so the study concentrated on tomography. The input distribution was compared with the simulations by the momentums of the distributions. The codes show difficulties to represent correct the halo of original  $F(x, y)$ . Since MENT is more optimized for high energy beam parameters (smooth variation of the particle density and Gaussian shaped beam) difficulties occur to reflect all the features of the input distribution. The design idea of MemSys5 follows a more general strategy assuming a flat prior distribution. This results in areas need to cut away manually to calculate the correct beam distribution.

It is hoped that the two laboratories intensify their cooperation to clarify remaining questions like influence of the

noise or which experimental constraints could lead to more appropriate prior distribution.

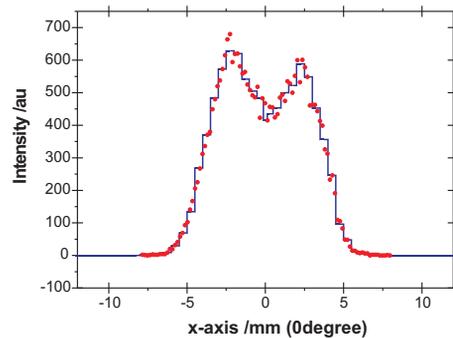


Figure 6: The profiles used by MENT are more coarse than for MemSys5, in fact there is no limit how fine the bin–size can be. Usually 100 data points are enough.

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