LONGITUDINAL STABILITY OF MULTITURN ERL WITH SPLIT ACCELERATING STRUCTURE

Ya. V. Getmanov#, O. A. Shevchenko, Budker INP, Novosibirsk, Russia
N. A. Vinokurov, Budker INP, Novosibirsk, Russia and KAERI, Daejon, Korea

Abstract

Some modern projects of the new generation light sources use the conception of multipass energy recovery linac with split (CEBAF-like) accelerating structures [1 - 5]. One of the advantages of these light sources is the possibility to obtain a small bunch length. To help reduce it, the longitudinal dispersion should be non-zero in some arcs of the accelerator. However small deviations in voltages of the accelerating structures can be enhanced by induced fields from circulating bunches due to the dependence of the flight time on the energy deviation and the high quality factor of the superconducting radio-frequency cavities. Therefore, instabilities caused by interaction of electron bunches and fundamental modes of the cavities can take place. The corresponding stability conditions are discussed in this paper. Numerical simulations were performed for project MARS [4].

INTRODUCTION

The scheme of an ERL with two accelerating structures is shown in Fig. 1.

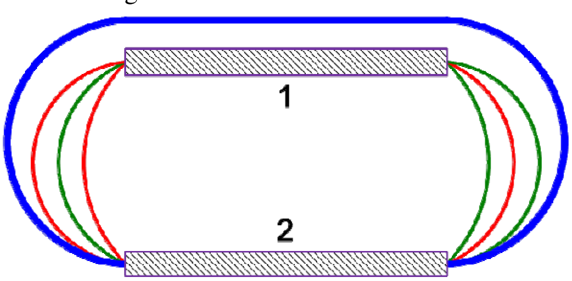

Figure 1: Scheme of ERL with two linacs.

Electrons are injected to the linac 1. After two passes through linac 1 and linac 2 they are used, for example, in undulators. After that electrons are decelerated.

There are four electron beams in each linac simultaneously. Each beam induced large voltage in the linac, but the sum is not so large. If the phases of the beams vary, the sum voltage also varies, and initially small phase deviation may increase due to the dependence of flight times through arcs on the particle energy. This longitudinal instability is considered in our paper.

THEORY

The Voltage Equations

To simplify the picture, consider each linac as one RF cavity. Its equivalent circuit is shown in Fig. 2.

The gap voltage expression $U = L\,d\left(I_b + I_g - C\,dU/dt - U/R\right)/dt$, I_b and I_g are the currents of the beam and of the RF generator, leads to the standard equation

$$\frac{d^2U}{dt^2} + \frac{1}{RC}\frac{dU}{dt} + \frac{1}{LC}U = \frac{1}{C}\frac{d}{dt}\left(I_b + I_g\right) \quad (1)$$

Taking the effective voltage on the linac with number α in the form $\mathrm{Re}(U_\alpha e^{-i\omega t})$ (ω is the frequency of the RF generator), one obtains:

$$\frac{2}{\omega}\frac{dU_\alpha}{dt} = \frac{i\xi_\alpha - 1}{Q_\alpha}U_\alpha + \rho_\alpha(I_{b\alpha} + I_{g\alpha}), \quad (2)$$

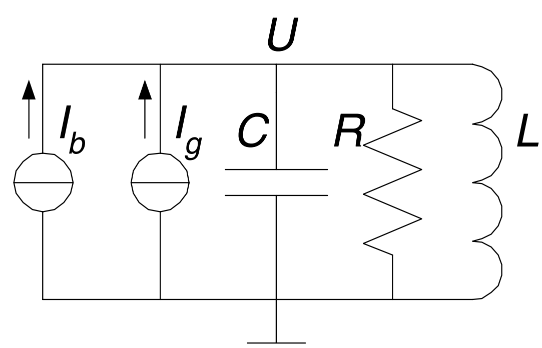

Figure 2: Equivalent circuit of the RF cavity.

where $\omega_\alpha = 1/\sqrt{L_\alpha C_\alpha} = (1 - \xi_\alpha / 2Q_\alpha)\omega$ is the resonant frequency, $Q_\alpha = R_\alpha \big/ \sqrt{L_\alpha / C_\alpha} >> 1$ is the loaded quality of the cavity, $\rho_\alpha = R_\alpha / Q_\alpha = \sqrt{L/C}$ and R_α are the characteristic and the loaded shunt impedances for the fundamental (TM_{010}) mode, and $I_{b\alpha}$ and $I_{g\alpha}$ are the complex amplitudes of the beam and (reduced to the gap) generator currents correspondingly. We are interested in the case of constant $I_{g\alpha}$. The beam currents $I_{b\alpha}$ depend on all U_α due to phase motion. Linearization of Eq. (2) near the stationary solution

$$U_{0\alpha} = \frac{R_\alpha}{1 - i\xi_\alpha}\left[I_{b\alpha}(\mathbf{U}_0) + I_{g\alpha}\right] \quad (3)$$

gives:

$$\frac{2}{\omega}\frac{d\delta U_\alpha}{dt} = \frac{i\xi_\alpha - 1}{Q_\alpha}\delta U_\alpha + \quad (4)$$

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$$+\rho_\alpha \sum_\beta \left(\frac{\partial I_{b\alpha}}{\partial \operatorname{Re} U_\beta} \operatorname{Re} \delta U_\beta + \frac{\partial I_{b\alpha}}{\partial \operatorname{Im} U_\beta} \operatorname{Im} \delta U_\beta \right)$$

Strictly speaking, I_b depends on the values of U at previous moments of time, so Eq. (4) is valid only if the damping times Q_α/ω are much longer than the time of flight through the ERL.

The Stability Conditions

Considering the exponential solutions $\exp(\omega\lambda t/2)$ of system of linear differential equations Eq. (4), one can find the stability conditions. Indeed, the system Eq. (4) corresponds to the system of the linear homogeneous equations $\lambda\delta\mathbf{U} = \mathbf{M}\delta\mathbf{U}$ with the consistency condition $|\mathbf{M} - \lambda\mathbf{E}| = 0$. Re($\lambda$) < 0 for all roots of this equation (i. e., eigenvalues of the matrix **M**) is the stability condition.

The stability condition for ERL with one linac was derived in paper [2]. In this case

$$\mathbf{M} = \begin{pmatrix} -\frac{1}{Q} + \rho\frac{\partial \operatorname{Re} I_b}{\partial \operatorname{Re} U} & -\frac{\xi}{Q} + \rho\frac{\partial \operatorname{Re} I_b}{\partial \operatorname{Im} U} \\ \frac{\xi}{Q} + \rho\frac{\partial \operatorname{Im} I_b}{\partial \operatorname{Re} U} & -\frac{1}{Q} + \rho\frac{\partial \operatorname{Im} I_b}{\partial \operatorname{Im} U} \end{pmatrix} \quad (5)$$

and the characteristic equation is

$$\lambda^2 - \lambda Tr(\mathbf{M}) + |\mathbf{M}| = 0 \quad (6)$$

According to Eq. (5) the stability condition is

$$Tr(\mathbf{M}) = \rho\left(\frac{\partial \operatorname{Re} I_b}{\partial \operatorname{Re} U} + \frac{\partial \operatorname{Im} I_b}{\partial \operatorname{Im} U}\right) - \frac{2}{Q} < 0 . \quad (7)$$

One can say, that the beam "active conductivity" $(\partial \operatorname{Re} I_b/\partial \operatorname{Re} U + \partial \operatorname{Im} I_b/\partial \operatorname{Im} U)/2$ has not to exceed the linac active conductivity $(\rho Q)^{-1}$.

For the ERL with two linacs

$$\mathbf{M} = \begin{pmatrix} \rho_1\frac{\partial \operatorname{Re} I_{b1}}{\partial \operatorname{Re} U_1} - \frac{1}{Q_1} & \rho_1\frac{\partial \operatorname{Re} I_{b1}}{\partial \operatorname{Im} U_1} - \frac{\xi_1}{Q_1} & \rho_1\frac{\partial \operatorname{Re} I_{b1}}{\partial \operatorname{Re} U_2} & \rho_1\frac{\partial \operatorname{Re} I_{b1}}{\partial \operatorname{Im} U_2} \\ \rho_1\frac{\partial \operatorname{Im} I_{b1}}{\partial \operatorname{Re} U_1} + \frac{\xi_1}{Q_1} & \rho_1\frac{\partial \operatorname{Im} I_{b1}}{\partial \operatorname{Im} U_1} - \frac{1}{Q_1} & \rho_1\frac{\partial \operatorname{Im} I_{b1}}{\partial \operatorname{Re} U_2} & \rho_1\frac{\partial \operatorname{Im} I_{b1}}{\partial \operatorname{Im} U_2} \\ \rho_2\frac{\partial \operatorname{Re} I_{b2}}{\partial \operatorname{Re} U_1} & \rho_2\frac{\partial \operatorname{Re} I_{b2}}{\partial \operatorname{Im} U_1} & \rho_2\frac{\partial \operatorname{Re} I_{b2}}{\partial \operatorname{Re} U_2} - \frac{1}{Q_2} & \rho_2\frac{\partial \operatorname{Re} I_{b2}}{\partial \operatorname{Im} U_2} - \frac{\xi_2}{Q_2} \\ \rho_2\frac{\partial \operatorname{Im} I_{b2}}{\partial \operatorname{Re} U_1} & \rho_2\frac{\partial \operatorname{Im} I_{b2}}{\partial \operatorname{Im} U_1} & \rho_2\frac{\partial \operatorname{Im} I_{b2}}{\partial \operatorname{Re} U_2} + \frac{\xi_2}{Q_2} & \rho_2\frac{\partial \operatorname{Im} I_{b2}}{\partial \operatorname{Im} U_2} - \frac{1}{Q_2} \end{pmatrix} \quad (8)$$

and the characteristic equation is (see, e. g., [6])

$$\lambda^4 - S_1\lambda^3 + S_2\lambda^2 - S_3\lambda + S_4 = 0 , \quad (9)$$

where $S_1 = \sum_{1\le k\le 4} A\begin{pmatrix} k \\ k \end{pmatrix} = \sum_{1\le k\le 4} M_{kk} = Tr(\mathbf{M})$,

$$S_2 = \sum_{1\le k<l\le 4} A\begin{pmatrix} k & l \\ k & l \end{pmatrix}, S_3 = \sum_{1\le k<l<m\le 4} A\begin{pmatrix} k & l & m \\ k & l & m \end{pmatrix},$$

and $S_4 = A\begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix} = |\mathbf{M}|$ are the sums of main minors of the matrix **M**. The necessary conditions for stability (Re(λ) < 0 for all four roots of Eq. (9)) is positivity of all the coefficients of the polynomial Eq. (9). In particular, the only independent on detunings ξ_1 and ξ_2 condition $S_1 < 0$ gives

$$\rho_1\left(\frac{\partial \operatorname{Re} I_{b1}}{\partial \operatorname{Re} U_1} + \frac{\partial \operatorname{Im} I_{b1}}{\partial \operatorname{Im} U_1}\right) + \rho_2\left(\frac{\partial \operatorname{Re} I_{b2}}{\partial \operatorname{Re} U_2} + \frac{\partial \operatorname{Im} I_{b2}}{\partial \operatorname{Im} U_2}\right) < \frac{2}{Q_1} + \frac{2}{Q_2} \quad (10)$$

The sufficient conditions are given by the Liénard-Chipart criterion [6]. It requires the positivity of of all the coefficients of the polynomial Eq. (9) and the third Hurwitz minor

$$S_1 < 0, S_2 > 0, S_4 > 0, \Delta_3 = S_1(S_2S_3 - S_1S_4) - S_3^2 > 0 \quad (11)$$

In the simplest case of the isochronous ERL arcs the conductivity matrix is zero. Then it is easy to proof, that all stability conditions are satisfied.

As the qualities of the superconducting cavities are very large, it is interesting to consider the opposite limiting case, neglecting small terms $1/Q_{1,2}$ in the matrix Eq. (8). Then all stability conditions do not depend on the beam current. They depend only on the ratio ρ_1/ρ_2 and the beam conductivity matrix, which is fully defined by the ERL magnetic system.

The Conductivity Matrix

To proceed further, we have to specify the elements of the beam conductivity matrix in the stability conditions. The complex amplitude of the beam current I_b may be written in the form

$$I_{b\alpha} = -2I\sum_{n=0}^{N-1}\left(e^{i\varphi_{2n+\alpha-1} + i\psi_{2n+\alpha-1}} + e^{i\varphi_{4N-2n-\alpha} + i\psi_{4N-2n-\alpha}}\right) \approx$$
$$\approx I_{b\alpha}(\mathbf{U}_0) - 2iI\sum_{n=0}^{N-1}\left(e^{i\varphi_{2n+\alpha-1}}\psi_{2n+\alpha-1} + e^{i\varphi_{4N-2n-\alpha}}\psi_{4N-2n-\alpha}\right) \quad (12)$$

where I is the average beam current, $\varphi_{2n+\alpha-1}$ is the equilibrium phase for the n-th pass through the resonator with the number α (α = 1, 2), and N is the number of orbits for acceleration. The small energy and phase deviations ε_n and ψ_n obey the linear equations:

$$\varepsilon_{n+1} = \varepsilon_n + e\operatorname{Im}\left[U_{0\alpha(n)}e^{-i\varphi_n}\right]\psi_n + e\operatorname{Re}\left[\delta U_{\alpha(n)}e^{-i\varphi_n}\right], \quad (13)$$

$$\psi_{n+1} = \psi_n + \omega\left(\frac{dt}{dE}\right)_{n+1}\varepsilon_{n+1} , \quad (14)$$

where

$\alpha(2n) = 1, \alpha(2n+1) = 2 \; for \; 0 \le n \le N-1$ and

$\alpha(2n) = 2, \alpha(2n+1) = 1 \; for \; N \le n \le 2N-1$.

$(dt/dE)_n$ is the longitudinal dispersion of the n-th 180-degree bend. The initial conditions for the system of Eqs. (13) and (14) are, certainly, ε_0=0 and ψ_0=0, if we have no special devices to control them for the sake of the beam

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stabilization, or other purposes. The solution of Eq. (13) and Eq. (14) may be written using the longitudinal sine-like trajectory S_{nk} and its "derivative" S'_{nk} (elements 56 and 66 of the transport matrix). These functions are the solutions of the homogenous system

$$S'_{n+1,k} = S'_{n,k} + e\,\mathrm{Im}\left[U_{0\alpha(n)}e^{-i\varphi_n}\right]S_{n,k}\,, \qquad (15)$$

$$S_{n+1,k} = S_{n,k} + \omega\left(\frac{dt}{dE}\right)_{n+1} S'_{n+1,k}\,, \qquad (16)$$

with the initial conditions $S_{k,k} = 0$, $S'_{k,k} = 1$. Then

$$\psi_n = e\sum_{k=0}^{n-1} S_{nk}\,\mathrm{Re}\left[\delta U_{\alpha(k)}e^{-i\varphi_k}\right], \qquad (17)$$

$$\varepsilon_n = e\sum_{k=0}^{n-1} S'_{nk}\,\mathrm{Re}\left[\delta U_{\alpha(k)}e^{-i\varphi_k}\right]. \qquad (18)$$

Substitution of Eq. (20) to Eq. (15) gives

$$\delta I_{b\alpha} = -2ieI\sum_{n=0}^{N-1}\{e^{i\varphi_{2n+\alpha-1}}\sum_{k=0}^{2n+\alpha-2} S_{2n+\alpha-1,k}\,\mathrm{Re}\left[\delta U_{\alpha(k)}e^{-i\varphi_k}\right] + e^{i\varphi_{4N-2n-\alpha}}\sum_{k=0}^{4N-2n-\alpha-1} S_{4N-2n-\alpha,k}\,\mathrm{Re}\left[\delta U_{\alpha(k)}e^{-i\varphi_k}\right]\} \qquad (19)$$

For an ERL it needs to satisfy (at least approximately) the recuperation condition

$$\mathrm{Re}\left[U_{01}\sum_{n=0}^{N-1}\left(e^{-i\varphi_{2n}} + e^{-i\varphi_{4N-2n-1}}\right)\right] = 0$$
$$\mathrm{Re}\left[U_{02}\sum_{n=0}^{N-1}\left(e^{-i\varphi_{2n+1}} + e^{-i\varphi_{4N-2n-2}}\right)\right] = 0 \qquad (20)$$

For the longitudinal stability it also needs to have longitudinal focusing for most of passes through the linac (see Eq. (12, 13)):

$$e\,\mathrm{Im}\left[U_{0\alpha(n)}e^{-\varphi_n}\right] < 0 \qquad (21)$$

if all $(dt/dE)_n > 0$). Conditions Eq. (20) and Eq. (21) may be satisfied simultaneously, if ($0 \le n \le 2N-1$)

$$\arg(eU_{0\alpha(n)}e^{-i\varphi_n}) + \arg(eU_{0\alpha(4N-n-1)}e^{-i\varphi_{4N-n-1}}) = -\pi, \qquad (22)$$

which leads to

$$\varphi_{4N-n-1} = \pi - \varphi_n + 2\arg\left(eU_{0\alpha(n)}\right) \qquad (23)$$

Conditions Eq. (23) affords equality of beam energies after n-th and $(4N-n)$-th passes through a linac.

The Threshold Current

To make the stability condition Eq. (10) more explicit, consider a simple example. Assume that equilibrium phases are equal during acceleration. In this simplest case φ_{2n} - $\arg\left(eU_{01}\right) = \Phi_1$, φ_{2n+1} - $\arg\left(eU_{02}\right) = \Phi_2$ for $0 \le n \le N-1$. Eq. (20) defines the equilibrium phases for deceleration. Then Eq. (19) gives

$$e\rho_1 I\sin(2\Phi_1)\sum_{n=0}^{N-1}\sum_{k=0}^{N-1} S_{4N-2n-1,2k} + e\rho_2 I\sin(2\Phi_2)\sum_{n=0}^{N-1}\sum_{k=0}^{N-1} S_{4N-2n-2,2k+1} < \frac{1}{Q_1} + \frac{1}{Q_2} \qquad (24)$$

SIMULATIONS

The Induced Voltage

To simulate the instability evolution one can use the turn-by-turn calculations of the voltage induced by each bunch. Passing through the cavity the electron excites the voltage oscillations on the cavity resonance frequency ω_r. Considering the bunch as the point with charge q one can calculate it induced voltage [9]

$$V_{\parallel}(t) = q\frac{\omega_r R_s}{Q}\cos(\omega_r t)\exp\left(-\frac{\omega_r}{2Q}t\right)H(t) \qquad (25)$$

where R_s is the shunt impedance, Q is the loaded quality factor, $H(t)$ is Heaviside's step-function. As the recirculating frequency of the generator is close to resonance, but not equal, electrons will see the phase of induced voltage by previous bunch slightly changed. Let's consider two electron bunches pass the cavity from the same magnet arc. The distance difference between them is cT_g (considering the equal frequencies of the cavity and gun), the reference phases are the same and its deviations are $\delta\phi_1$, $\delta\phi_2$ respectively:

$$\omega_g t_{1,2} = 2\pi n + \phi_{ref} + \delta\phi_{1,2}, n \in N$$

Therefore the 2-nd bunch will see the induced voltage by 1-st bunch with phase

$$\omega_r(t_1 - t_2) = \frac{\omega_r}{\omega_g}\omega_g(t_1 - t_2) = \frac{\omega_r}{\omega_g}\left[2\pi + (\delta\phi_1 - \delta\phi_2)\right] \qquad (26)$$

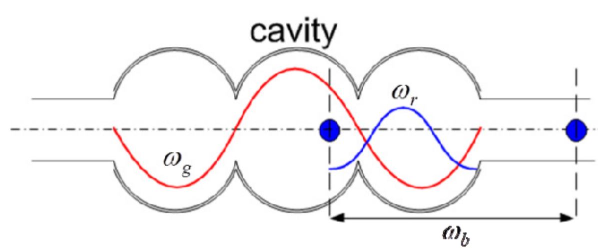

Figure 3. Frequencies in the bunch-cavity system

In case of bunches from different arcs, i.e. with different reference phases, the voltage phase is

$$\phi = \frac{\omega_r}{\omega_b}\left[2\pi(N_1 - N_2) + \left(\phi_{1ref} - \phi_{2ref}\right) + \left(\delta\phi_{1N_1} - \delta\phi_{2N_2}\right)\right]$$

where N_1, N_2 – the numbers of RF-buckets.

Induced voltage by Nb bunches at the times $(t_1,\ldots,t_{Nb})$ in complex form $V_{\parallel}(t) = \mathrm{Re}\,W_{\parallel}(t)$ is

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$$V_{\|}(t) = \operatorname{Re} W_{\|}(t - t_n) =$$
$$\operatorname{Re} \exp(i\omega_r t) q \frac{\omega_r R_s}{2Q} \exp(-i\omega_r t_n) \exp\left(-\frac{\omega_r}{2Q}(t - t_n) \right)$$

This formula can be transformed in phasors representation as the production bunch complex current $i(t)$ and complex voltage, induced by previous bunches with saturation and phase shift

$$\Delta W_{n+1} = -\operatorname{Re} W(t_{n+1}) i^*(t_{n+1}) T =$$
$$-\operatorname{Re} i^*(t_{n+1}) W(t_n) \exp\left(-\frac{\omega_r}{2Qc} \Delta L \right) \exp\left(i \frac{\omega_r}{\omega_b} 2\pi \right) T_g \quad (27)$$

where $\Delta L = cT_g$ is the distance between bunches.

The time dependence of current and voltage transforms to phases as

$$i(\phi_1) = \frac{e}{T_g} \exp\left(-i \frac{\omega_r}{\omega_b} \left(\phi_{1ref} + \delta\phi_1 \right) \right),$$
$$W(\phi_2) = q \frac{\omega_r R_s}{Q} \exp\left(-i \frac{\omega_r}{\omega_b} \left(\phi_{2ref} + \delta\phi_2 \right) \right).$$

Since the time of the saturation of the excited oscillations is much longer than gun period bunches can be united in groups by recuperation parameter (Fig. 4), where the total gain and induced voltages are close to zero

$$W_g(t_n) = \sum_{n=1}^{2Na} W_n(t_n) = q \frac{\omega_r R_s}{Q} \sum_{n=1}^{2Na} \exp(-i \frac{\omega_r}{\omega_g} \phi_n) \quad (28)$$

Accordingly the bunch will see the total voltage induced by previous N groups as

$$W(t_{N+1}) = W_g(t_N) + W(t_{N-1}) \exp(-\frac{\omega_r}{2cQ} \Delta L) \exp(i \frac{\omega_r}{\omega_g} \phi_{N+1}) \quad (29)$$

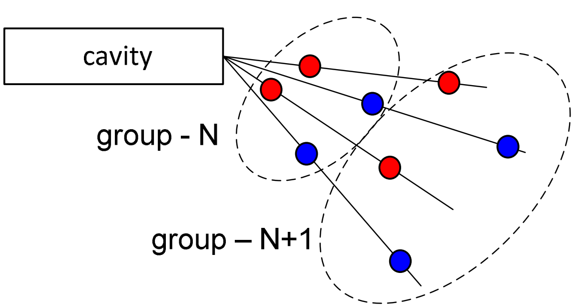

Figure 4. Recuperation groups of bunches: red – accelerating, blue - decelerating

For each point-like bunch the energy phase system is

$$\Delta E_n = \Delta E_{n-1} + e\Delta U(\delta\phi_n) + \Delta W_{n-1}(\delta\phi_n)$$
$$\delta\phi_n = \delta\phi_{n-1} + \frac{\omega_g}{c} (R_{56})_n \frac{\Delta E_n}{E_n} \quad (30)$$

where $\Delta U(\delta\phi_n)$ is additional energy received by electron with phase deviation $\delta\phi_n$, $\Delta W_{n-1}(\delta\phi_n)$ - additional energy due to the induced voltage by previous *n-1* turns (30).

The MARS Structure

Numerical calculations were made for proposed structure of MARS [7,8] (Multipass Accelerator-Recuperator Source) (the scheme is shown in Fig. 5).

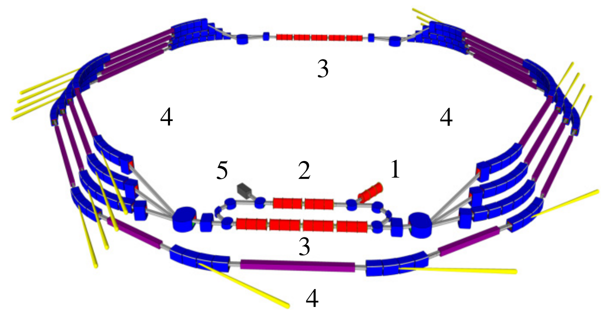

Figure 5: Scheme of MARS - ERL with two linacs: 1 – injector, 2 – preinjection linac, 3 – main linacs, 4 – bending arcs with undulators, 5 – dump.

Parameters of the main accelerating structures are: $Q_1 = Q_2 = 10^6$, $\rho_1 = 40K\Omega$, $\rho_2 = 90K\Omega$, $\omega = 2\pi \cdot 1.3 \cdot 10^9$ Hz, $U_1 = 0.8$ GV, $U_2 = 1.8$ GV. The transport matrix elements $R_{56} \sim 1$m at all arcs. The dependence of the threshold currents calculated by stability condition (24) and by numerical simulation on accelerating phases $\Phi_1 = \Phi_2$.

Simulations: without Preinjection Accelerating Structure

To compare the theoretical limit of the beam current given by (24) and by wakes simulations it's necessary to calculate system without preinjection.

Simulations start with filling the accelerator trajectory by array of bunches without initial deviation. Bunches with appropriate numbers interact with cavities. After that, the unperturbed bunch is injected in the facility and the last one goes to the dump. The examples of the phase to time dependence are shown in Fig. 6 and Fig. 7. The deviation of the bunch phase is decreasing and increasing exponentially defining the stable and unstable operations.

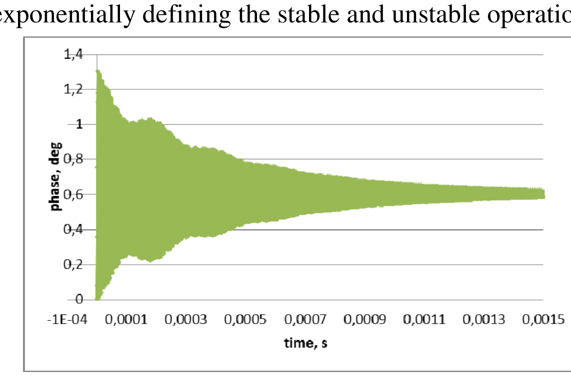

Figure 6: Example of the relaxation of the bunch's phase after the last deceleration to the equilibrium value.

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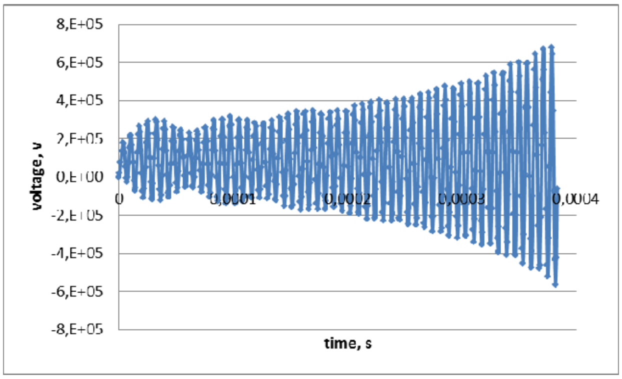

Figure 7: Example of the unstable operation.

Theoretical formulas (24) define the area of beam phases with extremely high threshold current. The comparison between simulated and theoretically calculated threshold currents is shown on Fig. 8. The numerical calculated current is much lower than that given by theory, but the areas of the maximum threshold parameters are correlated. The difference in the current values can be explained by second and high order terms: theory formulas use the linearization of the voltage-phase dependence (see (12), (13)), and near the zero reference phase the influence of the second order perturbations can be significant.

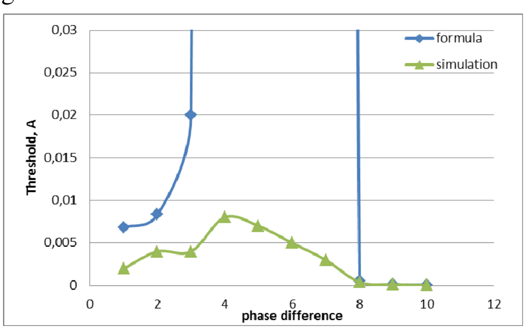

Figure 8: Threshold current: blue – formula (24), green – wakefields simulations ($\xi_{1,2} = 100$).

The Fig. 9 shows the comparison between the theory and simulations with high values of the reference phases.

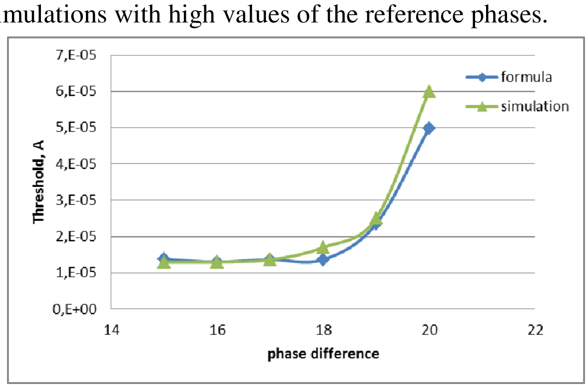

Figure 9: Threshold current: blue – formula (24), green – wakefields simulations ($\xi_{1,2} = 100$).

The maximum values of the threshold currents should be at the highest detunings of the cavities (all conditions of the Lienard-Chipard criterion (9, 10) are satisfied, excepts the last one (24), which does not depends on detunings). Fig. 10 shows the thresholds currents at the highest detunings.

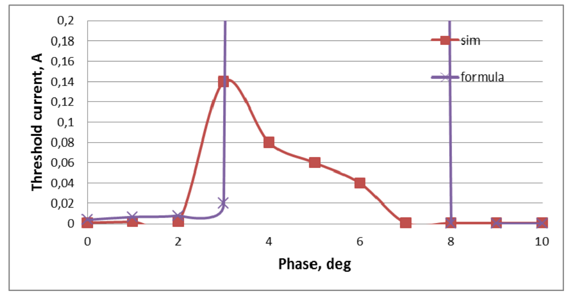

Figure 10: Threshold current: violet – formula (24), red – wakefields simulations ($\xi_{1,2} = 1000$).

Simulations: with Preinjection Accelerating Structure

The proposed scheme of the multiturn ERL (Fig. 5) has also the preliminary acceleration/deceleration system to reduce beam induced radiation and RF power consumption. Simplified scheme with one undulator is shown on Fig.11.

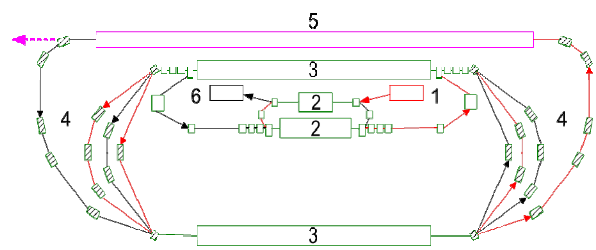

Figure 11. Recuperation groups of bunches: red – accelerating, blue – decelerating. 1-injector, 2 – two preinjection linacs, 3 – main linacs, 4 – bending arcs, 5 – undulator, 6 – beam dump.

Preliminary accelerating system consist of two linacs with energy gain 350 MeV and 40 MeV. Parameters of the linacs are: $Q_{in1} = Q_{in2} = 10^6$, $\rho_{in1} \cong 60\Omega$, $\rho_{in2} \cong 500\Omega$. On the Fig. 13 is shown the comparison of threshold currents calculated in the three cases: by stability condition (24), by numerical simulation without preliminary acceleration and with preliminary accelerating structure.

The examples of the stable and unstable operations are shown on the Fig. 12 and Fig. 13. The threshold current in the case of the maximum cavities detunings is shown on Fig. 14. The current if lower than it for two linacs system, but however satisfy the necessary condition for the accelerator (higher than 10 mA).

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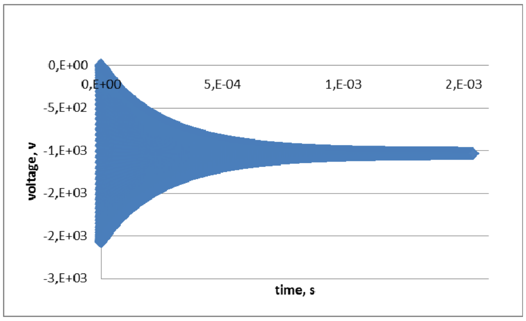

Figure 12. Example of the stable operation: the voltage deviations at the main linac

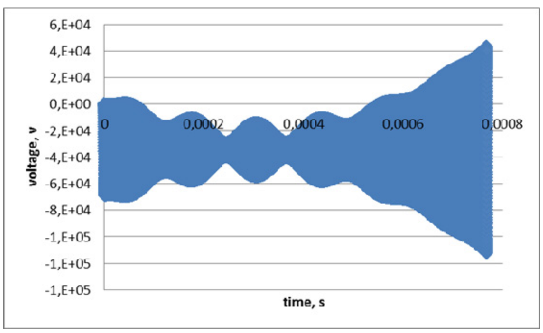

Figure 13. Example of the unstable operation: the voltage deviations at the preinjection linac

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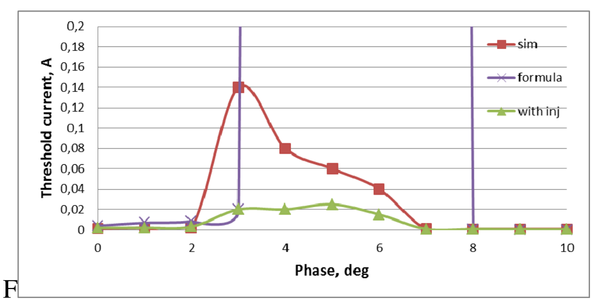

Figure 14: Threshold current: violet – formula (24), red – wakefields simulations, green – with preinjection linacs ($\xi_{1,2} = 1000$).

CONCLUSION

In this paper we derived the criterion of the longitudinal stability for the ERL with two accelerating structures. Numerical calculations specify stability phase region with high threshold current for the accelerating cavities of accelerator with two linacs.

Numerical simulations were made light source projects based on multiturn ERLs. The simulated threshold current is lower than the theoretical lower limit in the areas of the maximum current. To increase the threshold current, it is necessary to develop a proper feedback system.

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