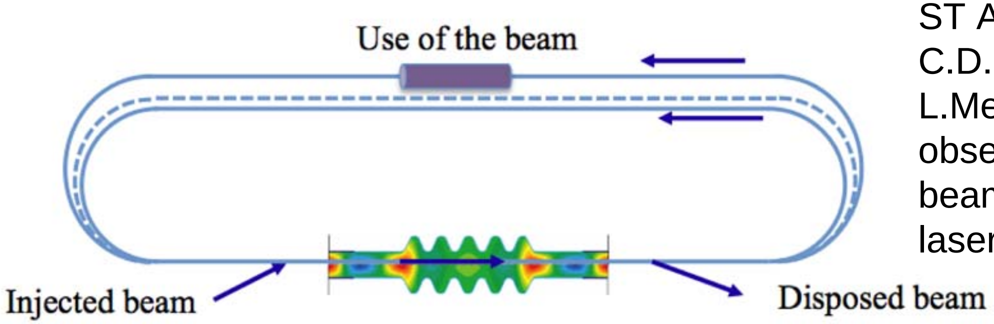
Chromaticity and TBBU

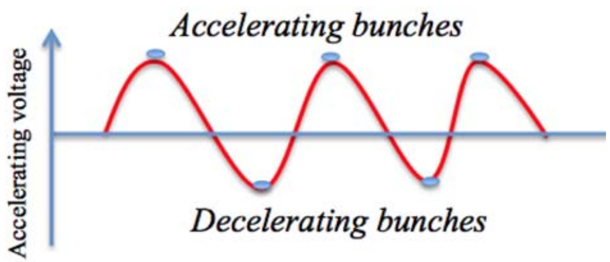
Vladimir N. Litvinenko
Brookhaven National Laboratory, Upton, NY, USA
Stony Brook University, Stony Brook, NY, USA
Center for Accelerator Science and Education

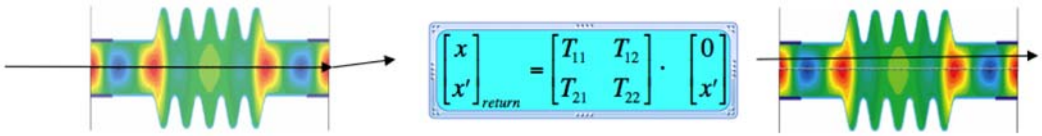
TBBU!
A killer of effective ERLs

It is believed that for a given Q*R/Q and spread of the HOM, the TBBU threshold is inverse proportional to number of ERL passes squared

G.H. Hoffstaetter and I.V. Bazarov, “Beam-breakup instability theory for energy recovery linacs”, Phys. Rev. ST AB 7, 054401 (2004)
C.D. Tennant, K.B. Beard, D.R. Kouglas, K.C. Jordan, L.Merminga, E.G. Pozdeyev, T.I. Smith “First observations and suppression of multipass, multibunch beam breakup in the Jefferson Laboratory free electron laser upgrade”, Phys. Rev. ST AB 8, 074403 (2005)







G.H. Hoffstaetter and I.V. Bazarov, “Beam-breakup instability theory for energy recovery linacs”, Phys. Rev. ST AB 7, 054401 (2004)

$$I_{th} = \frac{2c^2}{e R_g \cdot Q \cdot \omega} \cdot \frac{1}{\sum_{J=1}^{2N} \sum_{I=J+1}^{2N} T^{IJ} \sin\omega\left(t_I - t_J\right)} ,$$

where $T^{IJ} \equiv T_{12}(s_J|s_I)$ is the element of transport matrix between J^{th} and I^{th} pass through the linac. Hoffstaetter and Bazarov come to a natural conjecture that the TBBU threshold scales as following:

$$I_{th} \propto \left(R_g \cdot Q \cdot N^2 \left\langle \left|T^{IJ}\right| \right\rangle\right)^{-1} .$$

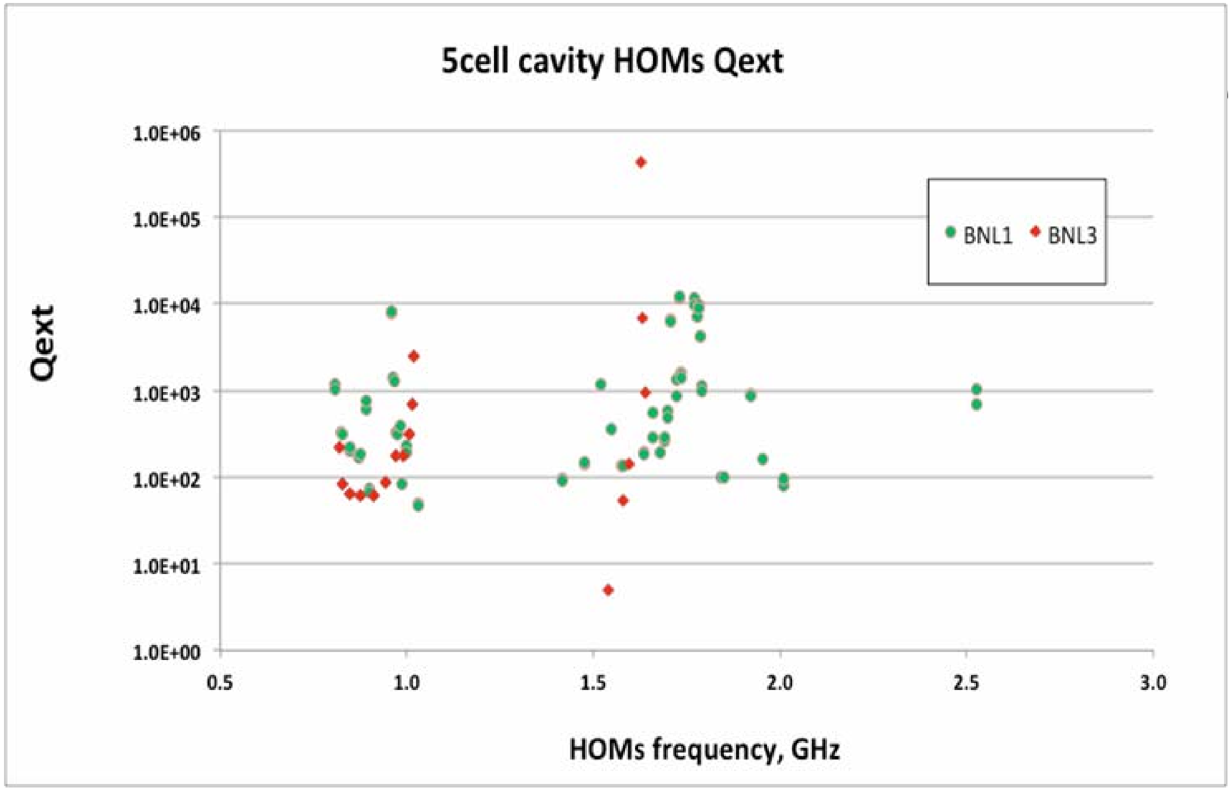
One can argue that the linac HOM impedance scales with it length and therefore, for a fixed top energy of ERL, the geometrical impedance scales as $R_g \sim E_{top} / N$, and

$$I_{th} \propto \left(\tilde{R}_g \cdot Q \cdot N \left\langle \left|T^{IJ}\right| \right\rangle\right)^{-1} .$$

where $\tilde{R}_g$ is HOM impedance for a unit length of the linac. On the other hand, typical values of β-functions, and therefore the typical values of T^{IJ}, in the linac are proportional to its length

This observation can bring back N^{-2} dependence of the TBBU threshold.

HOMs used for BBU



Comparison of BNL1 and BNL3 dipole HOM's

BNL1

F (GHz)	R/Q (Ω)	Q	(R/Q)Q
0.8892	57.2	600	3.4e4
0.8916	57.2	750	4.3e4
1.7773	3.4	7084	2.4e4
1.7774	3.4	7167	2.4e4
1.7827	1.7	9899	1.7e4
1.7828	1.7	8967	1.5e4
1.7847	5.1	4200	2.1e4
1.7848	5.1	4200	2.1e4

BNL3

F (GHz)	R/Q (Ω)	Q	(R/Q)Q
1.01E+09	30.6	313.0	9562.7
1.01E+09	30.5	313.0	9551.2
1.63E+09	1.0	6730.0	7030.9
1.02E+09	7.7	693.0	5328.8
1.02E+09	7.6	693.0	5301.0
9.11E+08	67.2	61.1	4108.1
9.11E+08	67.1	61.1	4101.6
9.90E+08	22.7	176.0	3991.7

Chromatic ERL Arcs

- ✓ The driver of the TBBU is the displacement of the beam in a RF cavity caused by a kick in another cavity, i.e. $T_{12}(s_1/s_2)$.
- ✓ Strong focusing ERL arcs (such as eRHIC) have very large natural chromaticity ~ 100
- ✓ It means that in combination with reasonable energy spread, there is exponential suppression of whole beam response

$$f(\delta) = \frac{1}{\sqrt{2\pi}\sigma_\delta} \exp\left(-\frac{\delta^2}{2\sigma_\delta^2}\right) \qquad \phi = 2\pi C$$

$$\langle T_{12} \rangle = \frac{\langle x(s) \rangle}{x'} = \exp\left(-\frac{(\phi\sigma_\delta)^2}{2}\right) w_{io} w_o \left(\cos(\psi_o - \pi/2) - \frac{\upsilon\phi\sigma_\delta^2}{w_o} \sin(\psi_o - \pi/2)\right)$$

V.N. Litvinenko, Chromaticity and beam stability in energy recovery linacs, in press

BROOKHAVEN NATIONAL LABORATORY

Office of Nuclear Physics — Exploring Nuclear Matter - Quarks to Stars

Simple analytical formulae

$$x = a\mathrm{w}(s)\cos\left(\psi(s)+\varphi\right);\ \psi' = 1/\mathrm{w}^2;$$

$$x' = a\mathrm{w}'(s)\cos\left(\psi(s)+\varphi\right) - \frac{a}{\mathrm{w}(s)}\sin\left(\psi(s)+\varphi\right),$$

$$\mathrm{w}(s,\delta) \cong \mathrm{w}_o(s)\left(1+\delta\cdot\upsilon(s)\right);\ \psi(s,\delta) \cong \psi_o(s) + \delta\cdot\phi(s)$$

$$\phi(s) = -\frac{\delta}{2}\int_0^s K(z)\mathrm{w}_o^2(z)\left(1-\cos 2\left(\psi_o(s)-\psi_o(z)\right)\right)dz;$$

$$\upsilon(s) = \mathrm{w}_o(s)\int_0^s \sin 2\left(\psi_o(s)-\psi_o(z)\right)K(z)\mathrm{w}_o^2(z)dz$$

Do not use sextupoles in ERL and enjoy extra stability and multi-pass economy

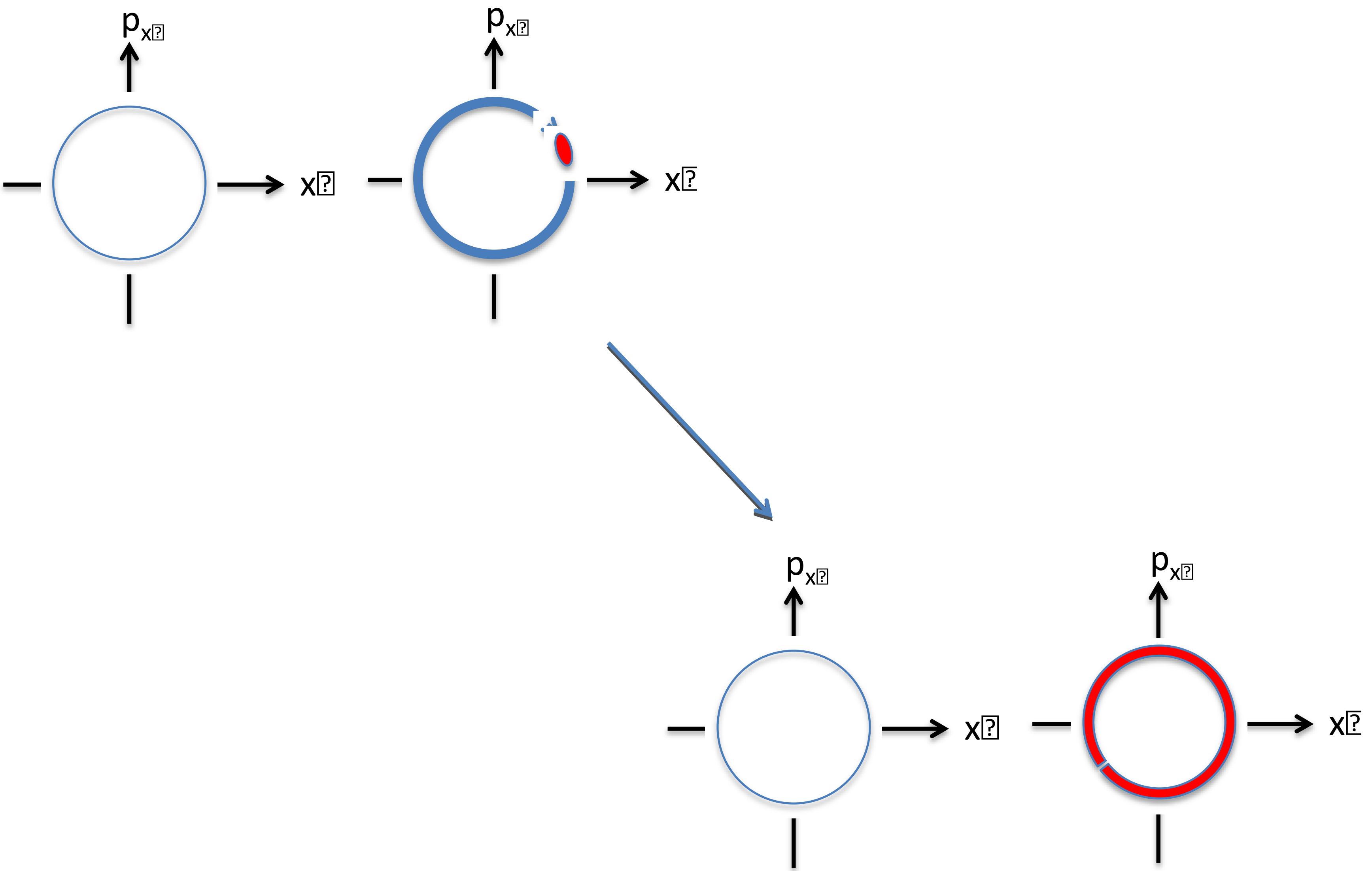
$$\langle T_{12} \rangle \propto \exp\left[-\frac{(\phi\sigma_\delta)^2}{2} \right] T_{12}(\max)$$

$$I_{th}(chromatic) \propto \exp\left[\frac{(\phi\sigma_\delta)^2}{2} \right] I_{th}(achromatic)$$

Assuming a strong focusing lattice for return loops, similar to that designed for eRHIC electron-hadron colliders the loop's chromaticity can be $c(s) \sim$ □300 and □$(s) \sim 2\#10^3$. Then for a beam with RMS energy spread of 0.2% the response $\langle T_{12} \rangle$ will be suppressed 3,000 fold, and according to formula (2) the threshold for TBBU instability will increase about 3,000 fold.

BROOKHAVEN NATIONAL LABORATORY

Office of Nuclear Physics — Exploring Nuclear Matter - Quarks to Stars


p_x
x
p_x
x
p_x
x
p_x
x

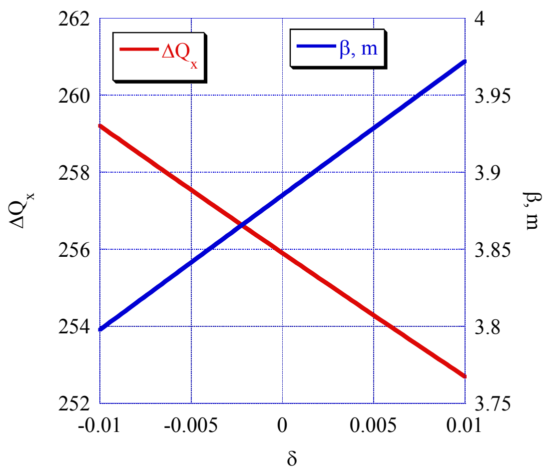
Other distributions

$f(\delta)$	Suppression factor $\langle T_{12} \rangle / \mathrm{w}_{io}\mathrm{w}_{o}$	Name
$\frac{1}{\sqrt{2\pi}\sigma_\delta}\exp\left(-\frac{\delta^2}{2\sigma_\delta^2}\right)$	$e^{-X^2/2}\cdot(\sin\psi + X\cdot Y\cos\psi)$	Gaussian
$\frac{1}{\pi\sigma_\delta}\left(1+\frac{\delta^2}{\sigma_\delta^2}\right)^{-1}$	$e^{-\lvert X\rvert}\cdot(\sin\psi + Y\cdot sign(X)\cos\psi)$	Lorentzian
$\frac{2}{\pi\sigma_\delta}\left(1+\frac{\delta^2}{\sigma_\delta^2}\right)^{-2}$	$e^{-\lvert X\rvert}\cdot((1+\lvert X\rvert)\sin\psi + X\cdot Y\cos\psi)$	$\kappa - 2$
$\frac{1}{2\sigma_\delta}\begin{pmatrix}\theta(\delta-\sigma_\delta)- \\ \theta(\delta+\sigma_\delta)\end{pmatrix}$	$\frac{\sin X}{X}\sin\psi + Y\frac{\sin X - X\cos X}{X^2}\cos\psi$	Rectangular
$\frac{\lvert 1-\delta/\sigma_\delta\rvert}{\sigma_\delta}, \lvert\delta\rvert \le \sigma_\delta$	$2\left(\frac{\cos X\ -1}{X^2}\sin\psi + Y\cdot\frac{2(\cos X - 1) + X\sin X}{X^3}\cos\psi\right)$	Triangular

Exact Numerical Example: FODO

$$T_{total}(\delta) = \left[T_{FODO}(\delta)\right]^{1024}$$

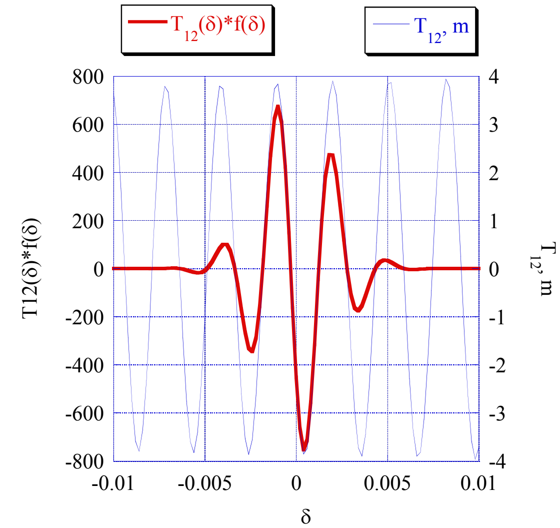
$$T_{FODO}(\delta) = \begin{bmatrix} 1 & L \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\varphi & \kappa^{-1}\sin\varphi \\ -\kappa\sin\varphi & \cos\varphi \end{bmatrix} \begin{bmatrix} 1 & L \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \cosh\varphi & \kappa^{-1}\sinh\varphi \\ \kappa\sinh\varphi & \cosh\varphi \end{bmatrix};$$



$$\kappa(\delta) = \sqrt{\frac{K_1}{1+\delta}}; \quad \varphi(\delta) = \kappa(\delta) l.$$

$$\sigma_\delta = 2 \cdot 10^{-3}$$

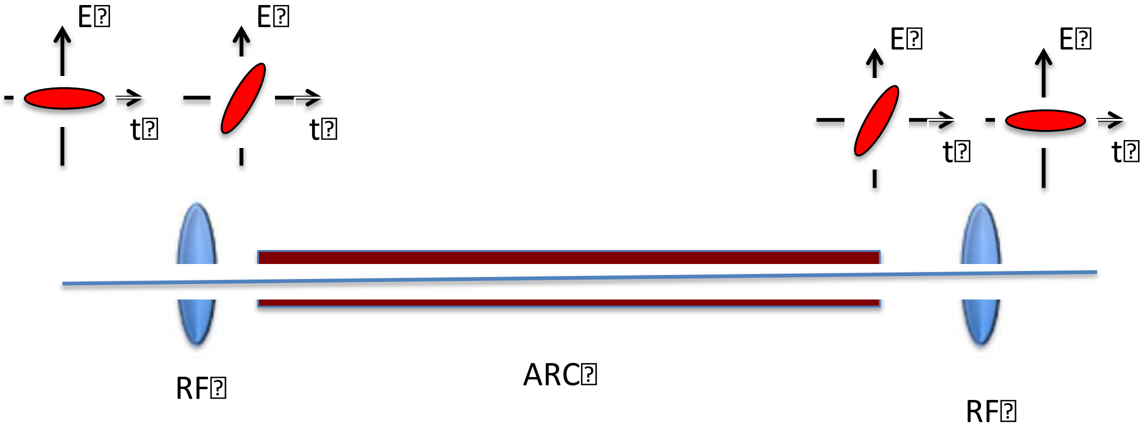
$$C = -328.51$$



Tune advance per cell 0.2499

$\langle T_{12}(s|s+C) \rangle$ suppression factor to $5.93 \cdot 10^3$

Using SRF linacs



electron beam generated from photoinjectors frequently have Gaussian longitudinal distribution, and a linear chirp in the RF cavity will introduce Gaussian energy-spread in the beam

Examples

eRHIC's low emittance ERL lattice has natural chromaticity of $C_{x\_s} = -28.571;\ C_{y\_s} = -20.242$ per sextant.

Single turn

$$C_{x\_t} \cong -175;\ C_{y\_t} \cong -125$$

$$\phi_{x\_t} \approx 1100;\ \phi_{y\_t} \approx 800$$

$\sigma_\delta = 5\square 10^{-3}$ $\langle T_{12}(s|s+C\rangle$ Suppression $2.9\square 10^{3}$

Examples

eRHIC's low emittance ERL lattice has natural chromaticity of $C_{x\_s} = -28.571;\ C_{y\_s} = -20.242$ per sextant.

Full path ERL

$$C_{x\_ERL} \sim -2000;\ C_{y\_ERL} \sim -1400$$

$$\phi_{x\_ERL} \approx 1.25 \times 10^{4};\ \phi_{y\_ERL} \approx 8.5 \times 10^{3}$$

$$\sigma_{\delta} = 5 \times 10^{-3}$$

$$\langle T_{12}(s|s+C) \rangle$$

Formally suppression is astronomical

$$3.7 \times 10^{6}$$

In reality other effects will limit the beam current!

Conclusions

- Using natural chromaticity in the arcs may significantly increase tolerance to HOM impedances and Qs and reduce complexity and cost of SRF linacs
- Low emittance eRHIC lattice would be perfect for such suppression to occur naturally
- Running linacs off-crest with alternating energy chirp would provide sufficient suppression in present eRHIC lattice
- This suppression mechanism provides for violating predicted $I_{th} \sim N^{-2}$ behavior for multi-pass ERLs

BROOKHAVEN NATIONAL LABORATORY

The work was supported by Brookhaven Science Associates under Contract No. DE-AC02-98CH10886 with the U.S. DOE

Office of Nuclear Physics
Exploring Nuclear Matter - Quarks to Stars

Emittance preservation

$$\mathbf{H} = \frac{p_x^2}{2} + \frac{K(s)}{1+\delta}\frac{x^2}{2} = \mathbf{H}_o - \nu K(s)\frac{x^2}{2}$$

$$x = a \cdot \mathbf{w}_o(s) \cdot \cos\left(\psi_o(s) + \varphi\right);\ \beta_o \equiv \mathbf{w}_o^2;\ \psi_o' = 1/\beta_o$$

$$h = \mathbf{H} - \mathbf{H}_o = -\nu I \cdot K(s)\mathbf{w}_o^2(s) \cdot \cos^2\left(\psi_o(s) + \varphi\right)$$

$$\varphi = \frac{\partial h}{\partial I} = -\nu \cdot K(s)\mathbf{w}_o^2(s) \cdot \cos^2\left(\psi_o(s) + \varphi\right);$$

$$I' = -\frac{\partial h}{\partial \varphi} = -\nu \cdot I \cdot K(s)\mathbf{w}_o^2(s) \cdot \sin\left(2\left(\psi_o(s) + \varphi\right)\right).$$

$$\varphi(s) = \varphi_o - \delta \cdot \int_o^s K(z)\mathbf{w}_o^2(z) \cdot \frac{1 + \cos\left(2\left(\psi_o(z) + \varphi_o\right)\right)}{2} dz + O(\delta^2);$$

$$I = I_o \left[1 - \delta \cdot \int_o^s K(z)\mathbf{w}_o^2(z) \cdot \sin\left(2\left(\psi_o(z) + \varphi_o\right)\right) dz\right] + O(\delta^2)$$

$$x \cong a_0 \cdot \mathbf{w}_\varepsilon(s) \cdot \cos\left(\psi_o(s) + \varphi(s)\right);$$

$$K(s) = K(s+C);\ \mathrm{w}_o(s+C) = \mathrm{w}_o(s)$$

$$\mathbf{w}_\varepsilon(s) = \mathbf{w}_o(s)\left[1 - \frac{\delta}{2} \cdot \int_o^s K(z)\mathbf{w}_o^2(z) \cdot \sin\left(2\left(\psi_o(z) + \varphi_o\right)\right) dz\right] + O(\delta^2).$$

$$\langle I \rangle = \langle I_o \rangle + O(\delta^2)$$

$$\varepsilon = \langle I \rangle$$

BROOKHAVEN NATIONAL LABORATORY

Office of Nuclear Physics — Exploring Nuclear Matter - Quarks to Stars

The later is a well know experimental fact that chromaticity of the betatron oscillations does not result in emittance growth in storage ring, where particles propagate for millions of turns and accumulate astronomic value of the phase spread caused by chromaticity.

Matching into the arcs it the key!

BBU simulation results

For simulation:

- 28 dipole HOMs are used for BNL3 and 70 HOMs for BNL1
- HOM Frequency spread 0-0.01
- Two different set of phase advances per each arc.

Challenges Exist both for eRHIC and LHeC ERLs