# Energy Recovery Linac Workshop

# Longitudinal Stability of ERL with Two Accelerating RF Structures

Getmanov Yaroslav

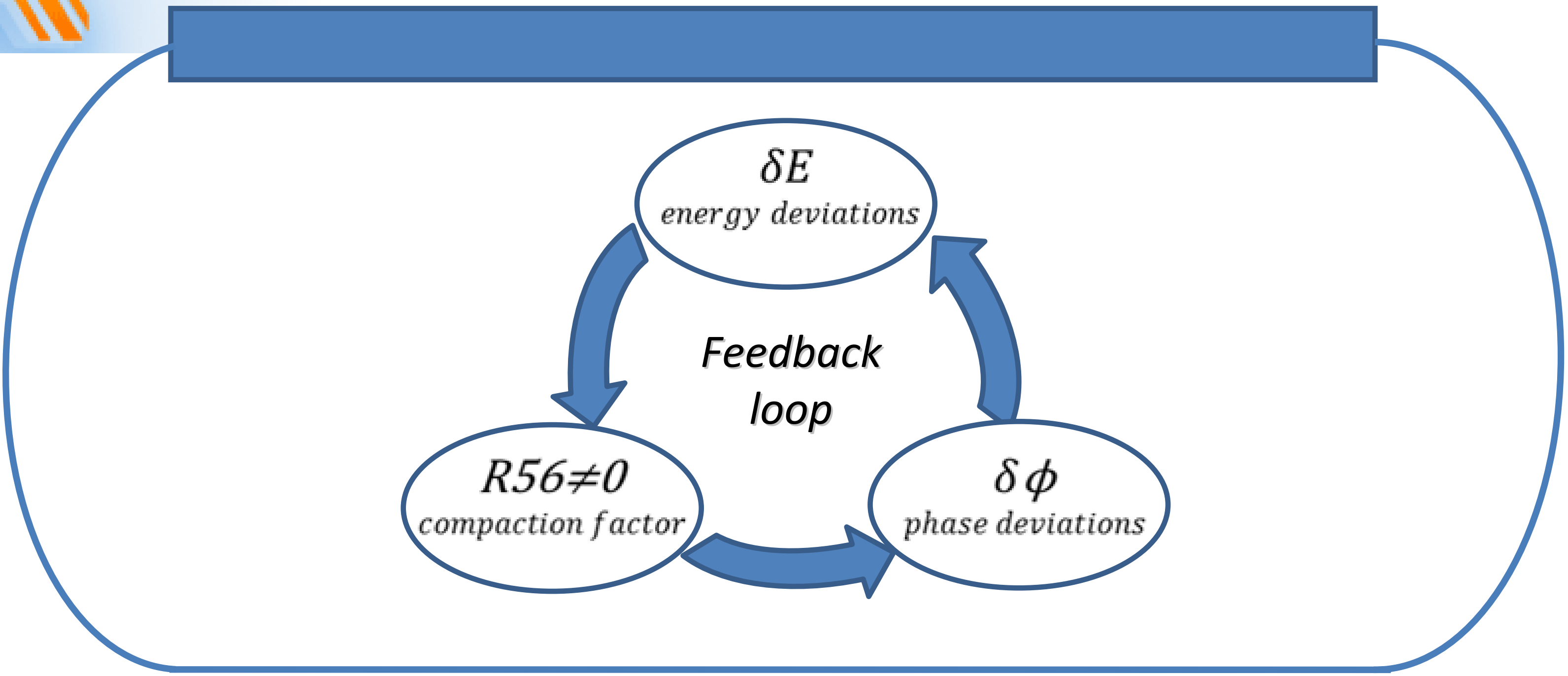
Budker INP, Russia

9-13 Sep. 2013, Novosibirsk, Russia

# OUTLINE

- Introduction
- MARS scheme
- Theory
- Simulations
- Conclusion

# Beam-cavity interaction instabilities

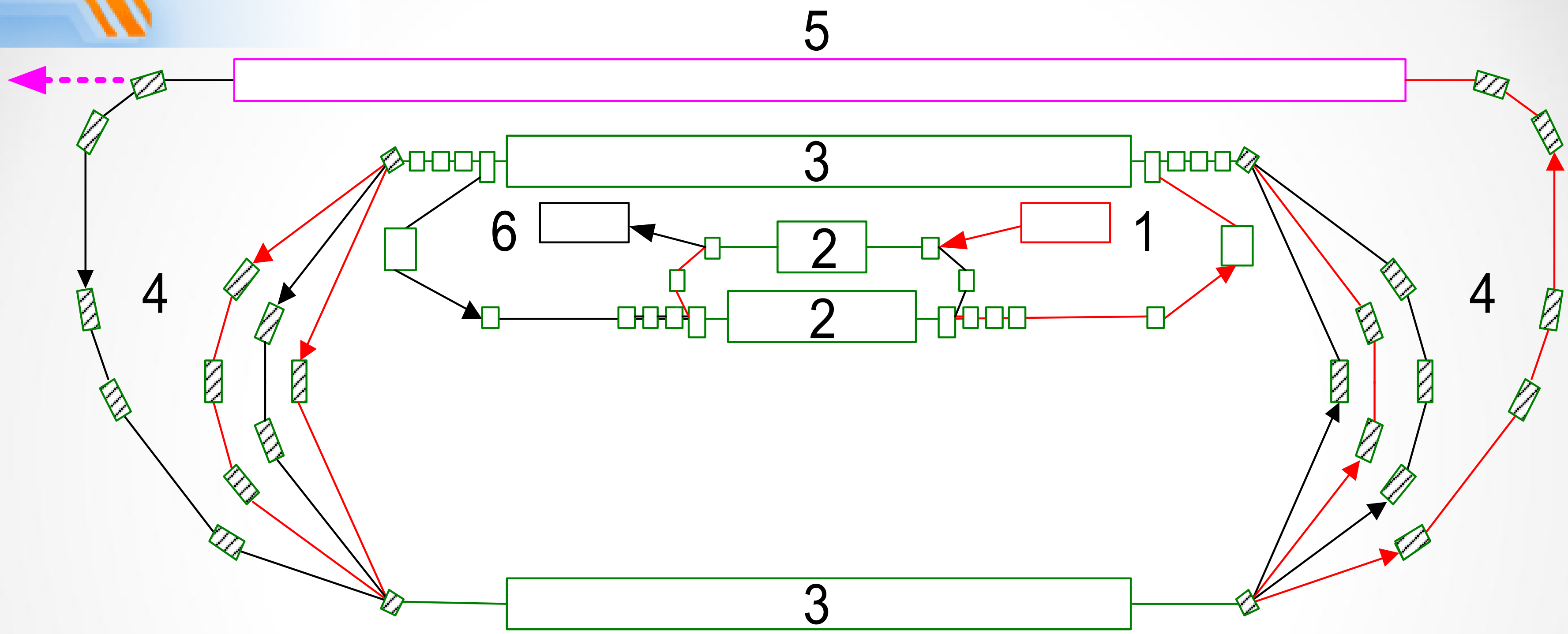



Longitudinal instability
*(Interaction with fundamental accelerating mode)*

An analytical expression for the threshold current of one-reciculation ERL applicable to all three instabilities can be derived and is given by

$$I_{th} \approx \frac{-2 p_r c}{e(R/Q)_m Q_m k_m M_{ij} \sin(\omega_m t_r + l\pi/2)}$$

# ERL with separated linacs



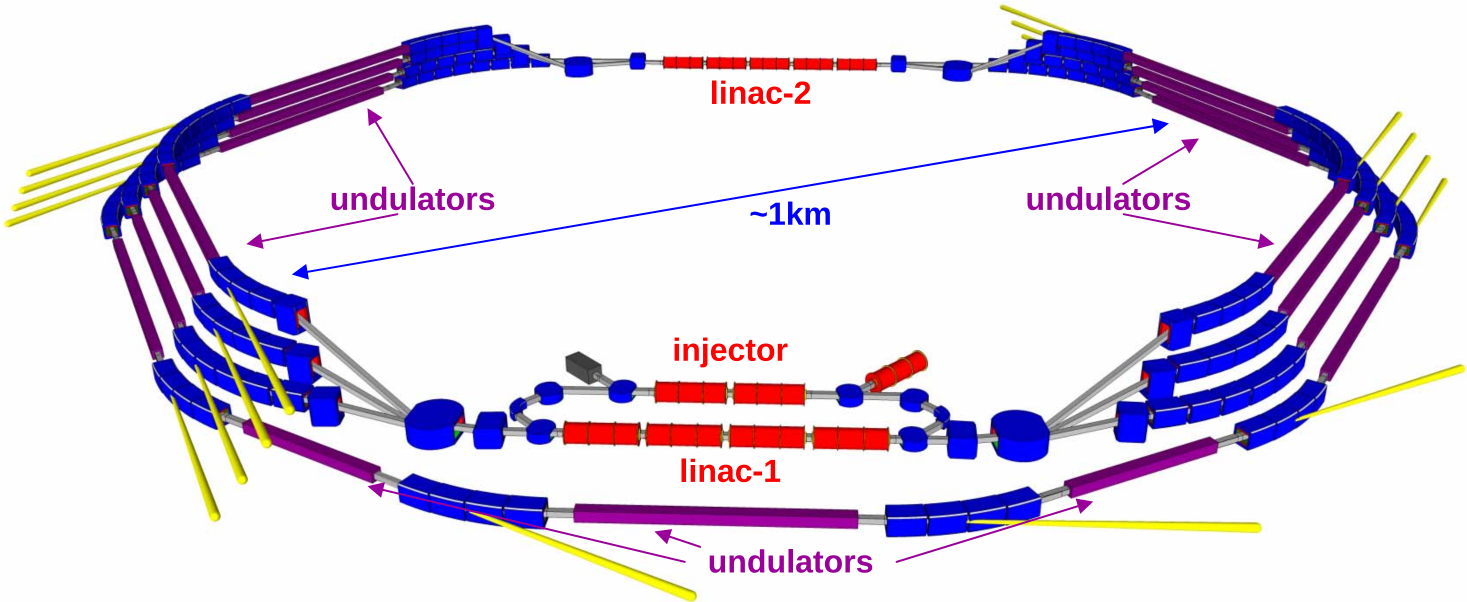

1 – injector, 2 – preliminary accelerating system, 3 – main accelerating RF structure, 4 – magnets, 5 – undulator, 6 – dump.

red arrows – accelerating bunch black arrows – used decelerating bunch

# Multipass Accelerator-Recuperator Source project (MARS)

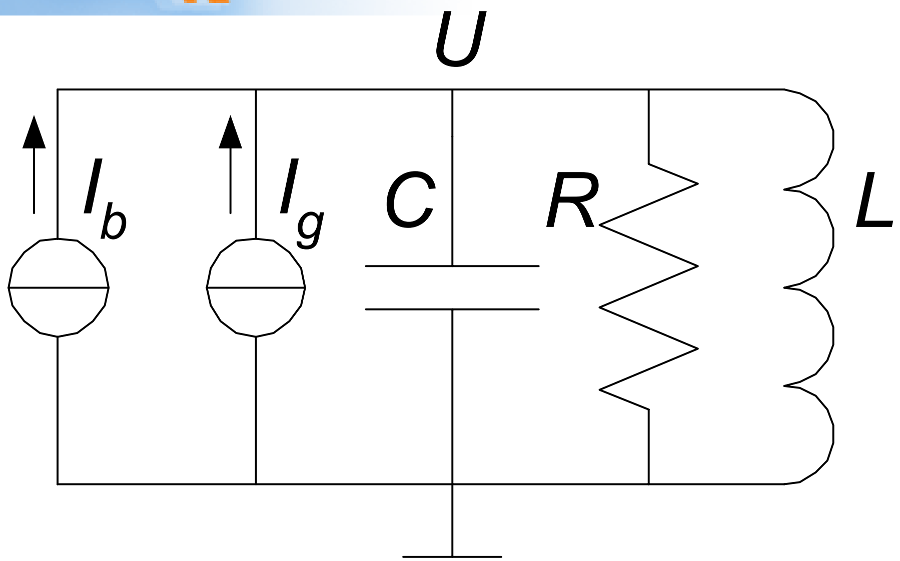
Budker INP

NRC «KI»




| | | | |
|---|---|---|---|
| ♦ Energy range | 5.6, 3.8, 3, 1.2 GeV | ♦ Bunch length | 0.1 – 1 ps |
| ♦ Average current | 10 mA | ♦ 7 undulators for | 5.6 GeV |
| ♦ Peak current | 10 A | ♦ 4 undulators for | 3.8 GeV |
| ♦ Normalized Emittance | $10^{-7}$ m·rad | ♦ 4 undulators for | 3 GeV |
| ♦ Bunch charge | $10^{-11}$ Cl | ♦ 4 undulators for | 1.2 GeV |

# Single-cavity model




Equivalent circuit of the RF cavity

Effective voltage of the linac with number *α*:

$$\mathrm{Re}(U_\alpha e^{-i\omega t})$$

where *ω* – generator frequency,

$U_\alpha$ – complex amplitude,

$I_{b\alpha}$, $I_{g\alpha}$ – beam and generator currents

$$\frac{d^2U}{dt^2} + \frac{1}{RC}\frac{dU}{dt} + \frac{1}{LC}U = \frac{1}{C}\frac{d}{dt}\left(I_b + I_g\right)$$

$$\frac{2}{\omega}\frac{dU_\alpha}{dt} = \frac{i\xi_\alpha - 1}{Q_\alpha}U_\alpha + \rho_\alpha(I_{b\alpha} + I_{g\alpha})$$

where $\omega_\alpha = \frac{1}{\sqrt{L_\alpha C_\alpha}} = (1 - \frac{\xi_\alpha}{2Q_\alpha})\omega$ resonance frequency

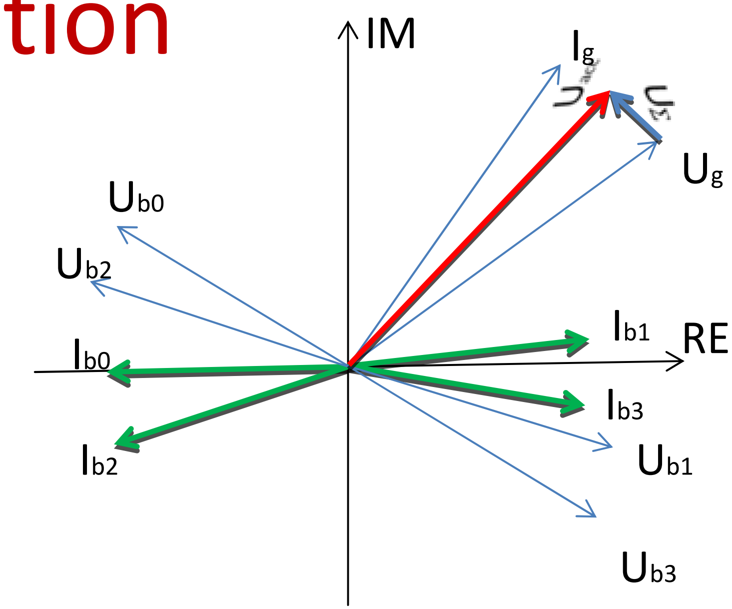
$Q_\alpha = R_\alpha \Big/ \sqrt{L_\alpha / C_\alpha} >> 1$ loaded quality of RF cavity $\rho_\alpha = R_\alpha / Q_\alpha = \sqrt{L/C}, R_\alpha$

characteristic and the loaded shunt impedances for the fundamental (*TM010*) mode

# Linearization

Linearization near the stationary solution

$$U_{0\alpha} = \frac{R_\alpha}{1 - i\xi_\alpha}\left[ I_{b\alpha}(\mathbf{U}_0) + I_{g\alpha} \right]$$




*complex diagram of linac cavity*

where $I_{b\alpha}$, $I_{g\alpha}$ complex amplitudes of beam and generator currents, gives

$$\frac{2}{\omega}\frac{d\delta U_\alpha}{dt} = \frac{i\xi_\alpha - 1}{Q_\alpha}\delta U_\alpha + \rho_\alpha \sum_\beta \left( \frac{\partial I_{b\alpha}}{\partial \operatorname{Re} U_\beta} \operatorname{Re}\delta U_\beta + \frac{\partial I_{b\alpha}}{\partial \operatorname{Im} U_\beta} \operatorname{Im}\delta U_\beta \right)$$

$$\lambda \delta \mathbf{U} = \mathbf{M} \delta \mathbf{U}$$

# Conductivity matrix of ERL with 2 linacs

Matrix of conductivity

$$\mathbf{M}=\begin{pmatrix} -\frac{1}{Q_1}+\rho_1\frac{\partial \operatorname{Re} I_{b1}}{\partial \operatorname{Re} U_1} & -\frac{\xi_1}{Q_1}+\rho_1\frac{\partial \operatorname{Re} I_{b1}}{\partial \operatorname{Im} U_1} & \rho_1\frac{\partial \operatorname{Re} I_{b1}}{\partial \operatorname{Re} U_2} & \rho_1\frac{\partial \operatorname{Re} I_{b1}}{\partial \operatorname{Im} U_2} \\ \frac{\xi_1}{Q_1}+\rho_1\frac{\partial \operatorname{Im} I_{b1}}{\partial \operatorname{Re} U_1} & -\frac{1}{Q_1}+\rho_1\frac{\partial \operatorname{Im} I_{b1}}{\partial \operatorname{Im} U_1} & \rho_1\frac{\partial \operatorname{Im} I_{b1}}{\partial \operatorname{Re} U_2} & \rho_1\frac{\partial \operatorname{Im} I_{b1}}{\partial \operatorname{Im} U_2} \\ \rho_2\frac{\partial \operatorname{Re} I_{b2}}{\partial \operatorname{Re} U_1} & \rho_2\frac{\partial \operatorname{Re} I_{b2}}{\partial \operatorname{Im} U_1} & -\frac{1}{Q_2}+\rho_2\frac{\partial \operatorname{Re} I_{b2}}{\partial \operatorname{Re} U_2} & -\frac{\xi_2}{Q_2}+\rho_2\frac{\partial \operatorname{Re} I_{b2}}{\partial \operatorname{Im} U_2} \\ \rho_2\frac{\partial \operatorname{Im} I_{b2}}{\partial \operatorname{Re} U_1} & \rho_2\frac{\partial \operatorname{Im} I_{b2}}{\partial \operatorname{Im} U_1} & \frac{\xi_2}{Q_2}+\rho_2\frac{\partial \operatorname{Im} I_{b2}}{\partial \operatorname{Re} U_2} & -\frac{1}{Q_2}+\rho_2\frac{\partial \operatorname{Im} I_{b2}}{\partial \operatorname{Im} U_2} \end{pmatrix}$$

the characteristic equation

$$\lambda^4 - S_1\lambda^3 + S_2\lambda^2 - S_3\lambda + S_4 = 0$$

Stable conditions are given by Lienard-Chipard criterion (4 equations)

# Threshold current

Additional constraints

Energy recuperation

$$\begin{cases} \mathrm{Re}\left[U_{01}\sum_{n=0}^{N-1}\left(e^{-i\varphi_{2n}}+e^{-i\varphi_{4N-2n-1}}\right)\right]=0 \\ \mathrm{Re}\left[U_{02}\sum_{n=0}^{N-1}\left(e^{-i\varphi_{2n+1}}+e^{-i\varphi_{4N-2n-2}}\right)\right]=0 \end{cases}$$

Longitudinal focusing

$$e\,\mathrm{Im}\left[U_{0\alpha(n)}e^{-\varphi_n}\right]<0$$

Assume that equilibrium phases are equal during acceleration. In this case

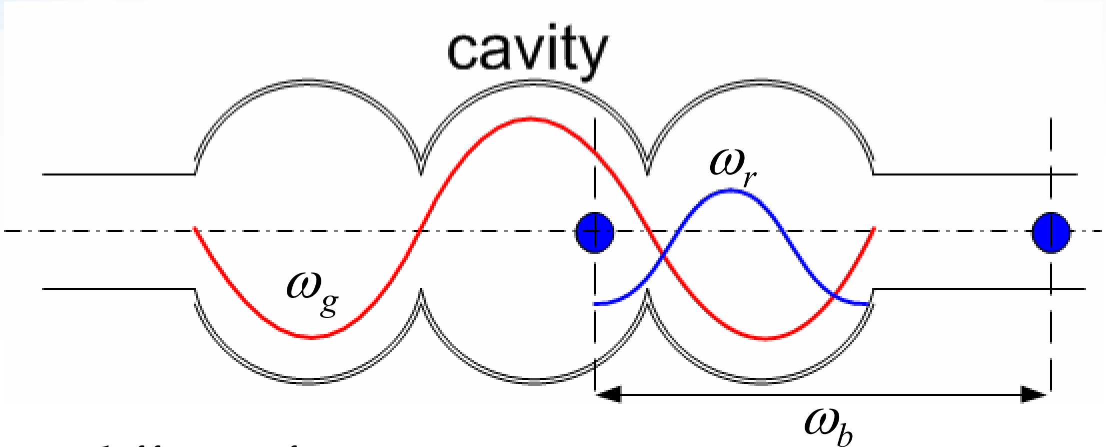
$$\varphi_{2n}-\arg\left(eU_{01}\right)=\Phi_1,\ \varphi_{2n+1}-\arg\left(eU_{02}\right)=\Phi_2$$

If the Liénard-Chipart criterion is satisfied, the threshold current is

$$I_{th}=\frac{1/Q_1+1/Q_2}{e\rho_1\sin\left(2\Phi_1\right)\sum_{n=0}^{N-1}\sum_{k=0}^{N-1}S_{4N-2n-1,2k}+e\rho_2\sin\left(2\Phi_2\right)\sum_{n=0}^{N-1}\sum_{k=0}^{N-1}S_{4N-2n-2,2k+1}}$$

ERL 2013

# Simulations: wakes




There are three different frequencies in system: generator, gun and cavity resonance

Additional energy gain by electron $\Delta E = -eV_{\|}(t)$ where induced voltage $V$(t)

for bunch distribution $\lambda$(t) is $V_{\|}(\tau) = \int_0^{+\infty} G_{\|}(t)\lambda(t-\tau)dt$

*G(t)* is the wake function

$$G_{\|}(t) \cong \frac{\omega_r R_s}{Q}\cos(\omega_r t)\exp(-\frac{\omega_r}{2Q}t)H(t)$$

For point bunches with charge q, the induced voltage is

$$V_{\|}(t) = q\frac{\omega_r R_s}{Q}\cos(\omega_r t)\exp(-\frac{\omega_r}{2Q}t)H(t)$$

# Phases of the induced fields

Phase of induced filed by bunch 2 at the pass bunch 1.

$$\phi = \frac{\omega_r}{\omega_b}\left[\left(\phi_{1ref} - \phi_{2ref}\right) + \left(\delta\phi_{1N_1} - \delta\phi_{2N_2}\right)\right]$$

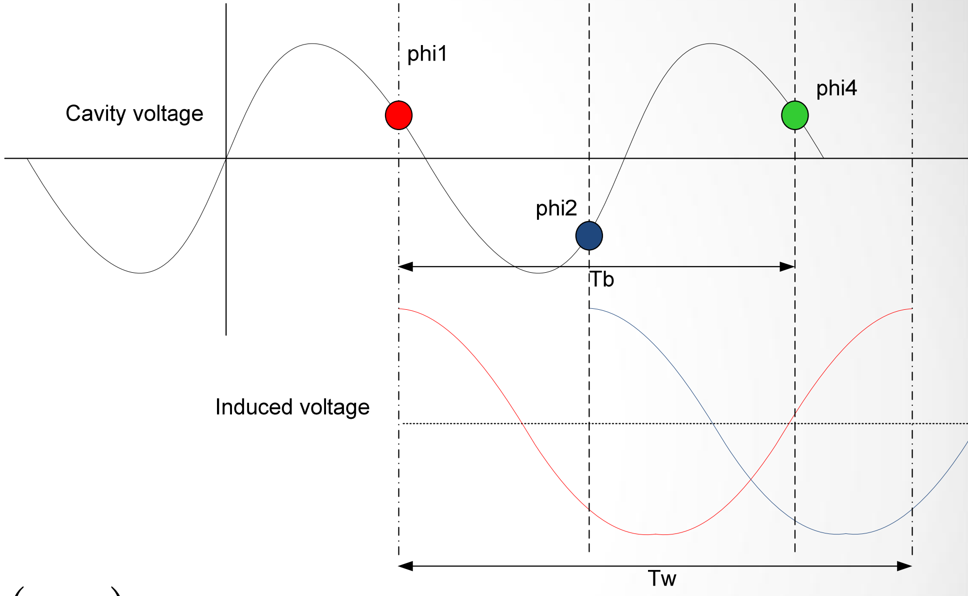
Additional energy gain by electron

$$\Delta E_{n+1} = -W_n(t_{n+1}) i^*(t_{n+1}) T =$$

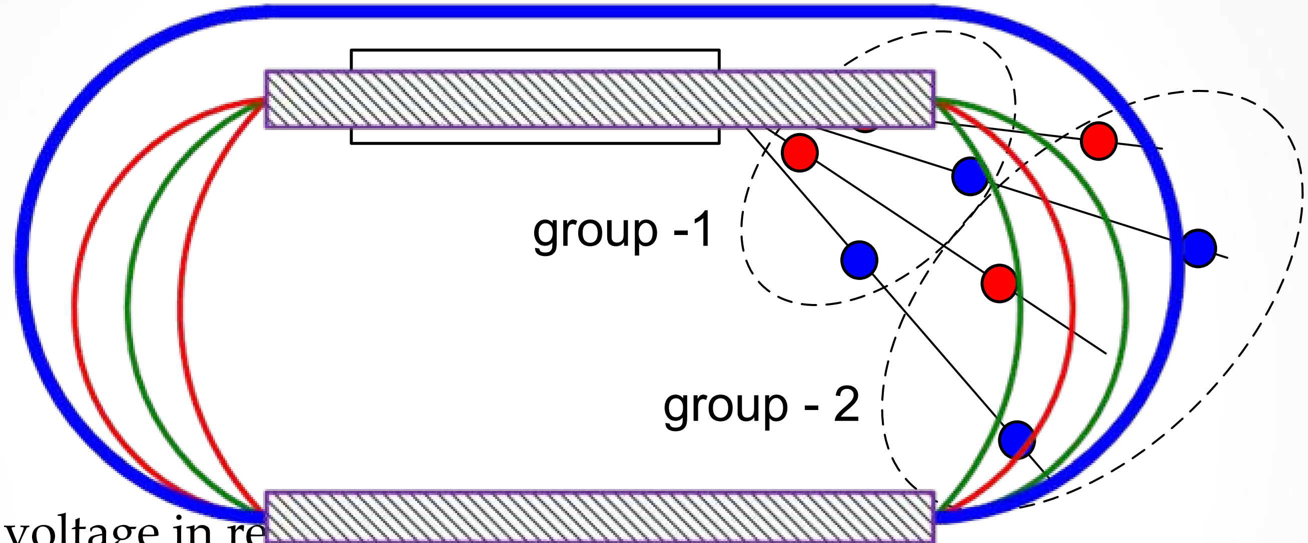
$$-W_n(t_n)\exp\left(-\frac{\omega_r}{2Qc}\Delta L\right)\exp\left(i\frac{\omega_r}{\omega_b}2\pi\right)\exp(-i\phi)$$

where

$$W_n(\phi_1) = q\frac{\omega_r R_s}{Q}\exp\left(-i\frac{\omega_r}{\omega_b}\left(\phi_{2ref} + \delta\phi_2\right)\right) \qquad i(\phi_1) = \frac{e}{T}\exp\left(-i\frac{\omega_r}{\omega_b}\left(\phi_{1ref} + \delta\phi_1\right)\right)$$



# simulations




Induced voltage in recurrence form

$$W_n = W_{n-1}\exp\left(-\frac{\omega_r}{2Qc}\Delta L\right)\exp\left(i\frac{\omega_r}{\omega_b}2\pi\right) + q\frac{\omega_r R_s}{Q}\sum_{m=0}^{Nb-1}\exp\left(-i\frac{\omega_r}{\omega_b}\left(\phi_{mk}+\delta\phi_{mk}\right)\right)$$

energy gain

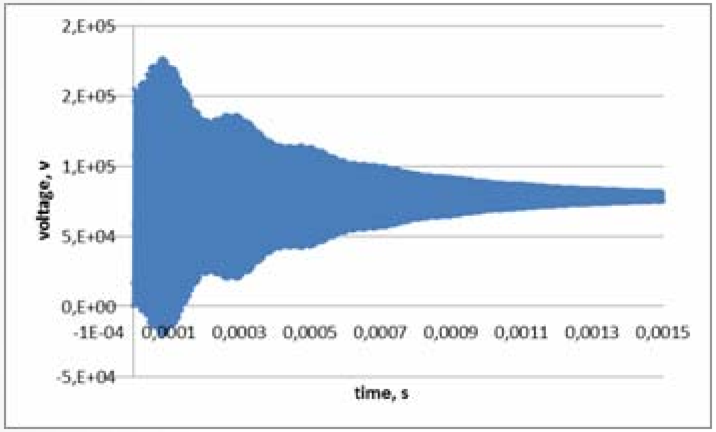
$$\Delta E_{n+1} = -W_n\exp\left(-\frac{\omega_r}{2Qc}\Delta L\right)\exp\left(i\frac{\omega_r}{\omega_b}2\pi\right)i^*T$$

ERL 2013

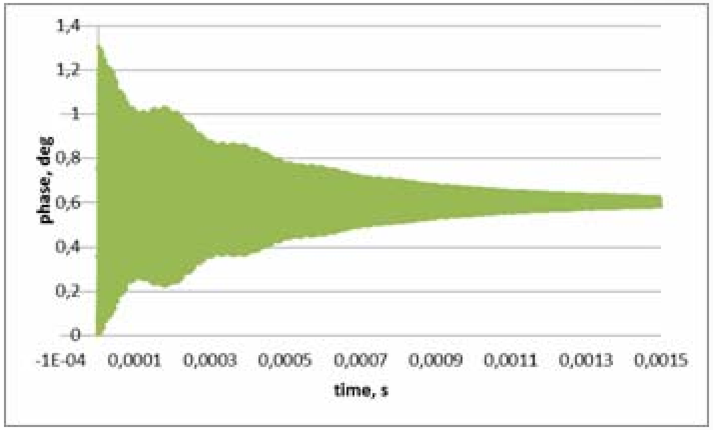
# Examples

## stable operation

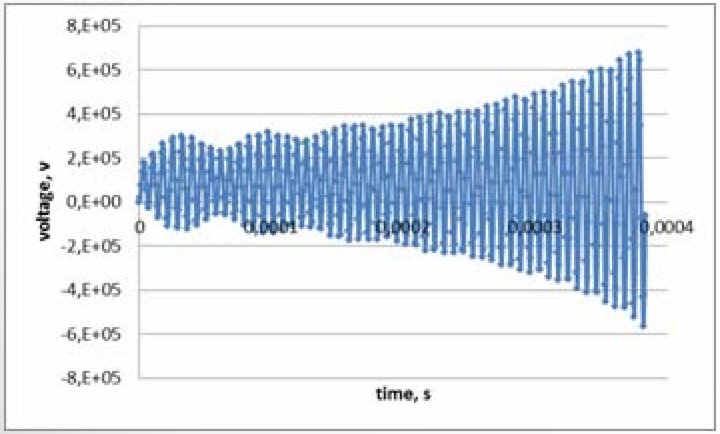
Voltage to time on 2-d linac

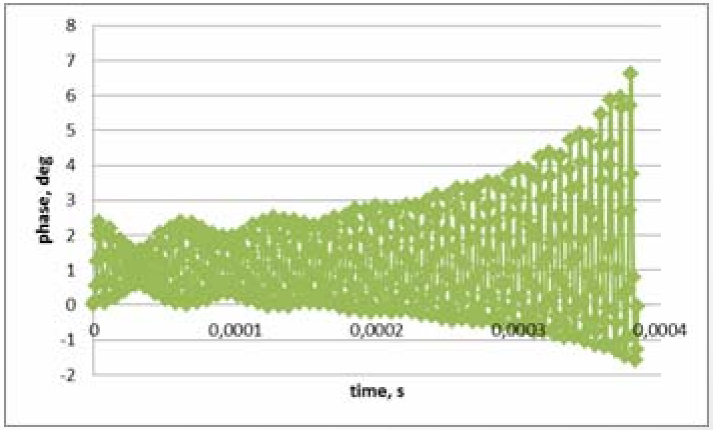



Phase to time on 2-st linac

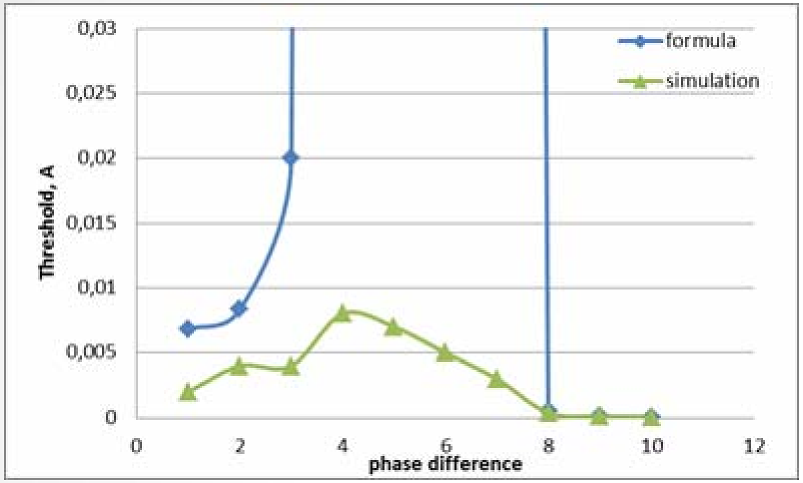


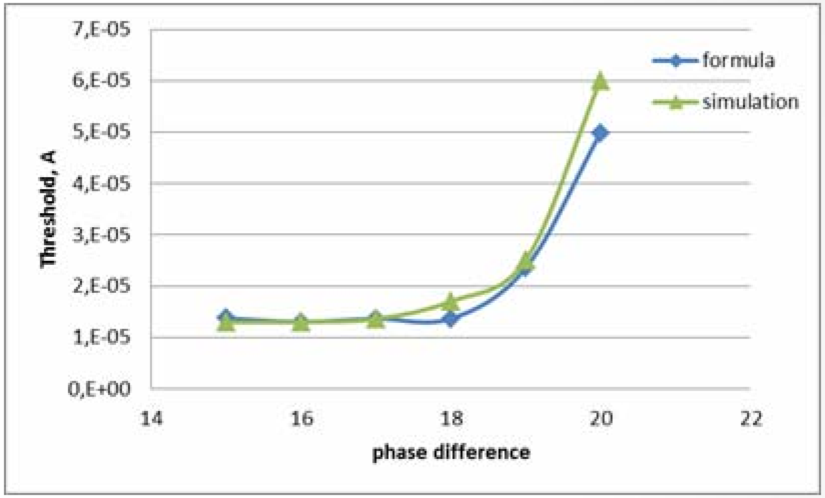

## unstable operation



# Threshold current



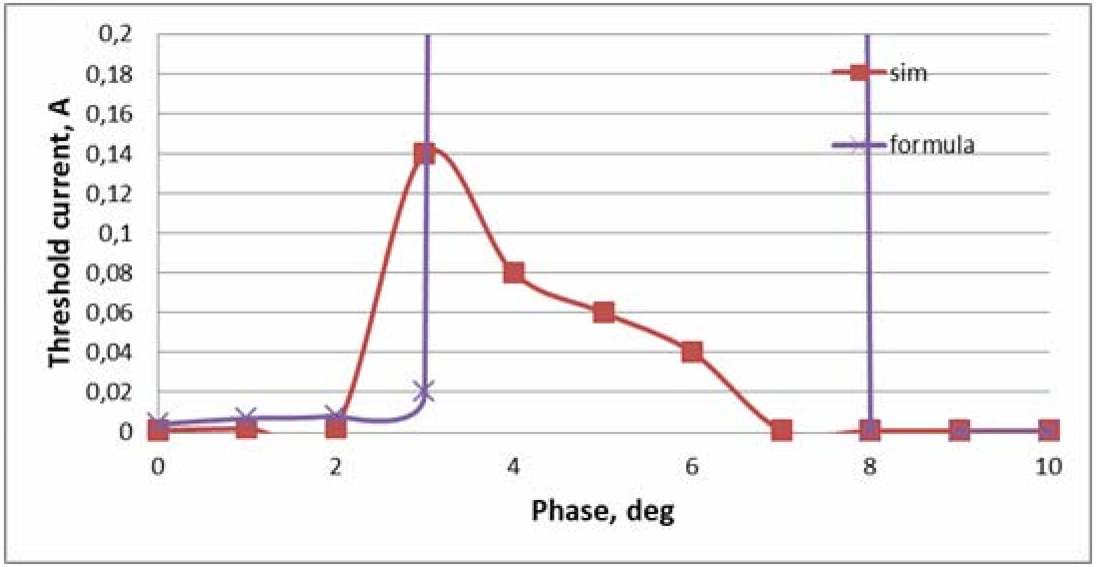




The threshold current dependence on phases with equal detunings of cavities $\xi_{1,2} = 100$

Accelerating phases are equal:

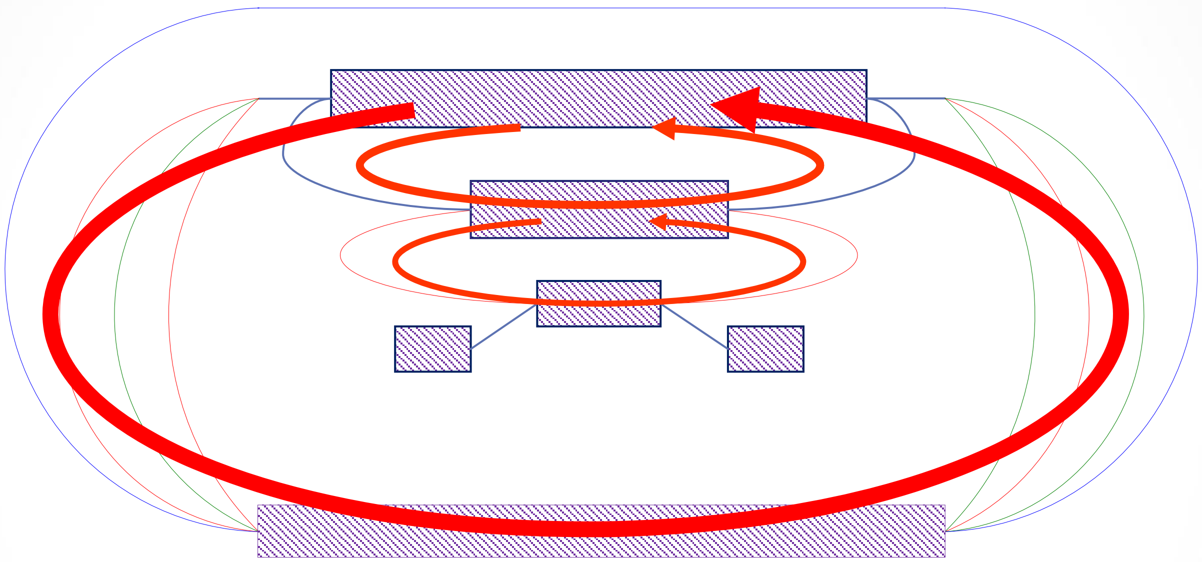
$$\varphi_{2n} - \arg(eU_{01}) = \Phi_1 = \varphi_{2n+1} - \arg(eU_{02}) = \Phi_2$$

# Threshold without preinjection




The threshold current dependence on phases with equal detunings of cavities $\xi_{1,2} = 1000$

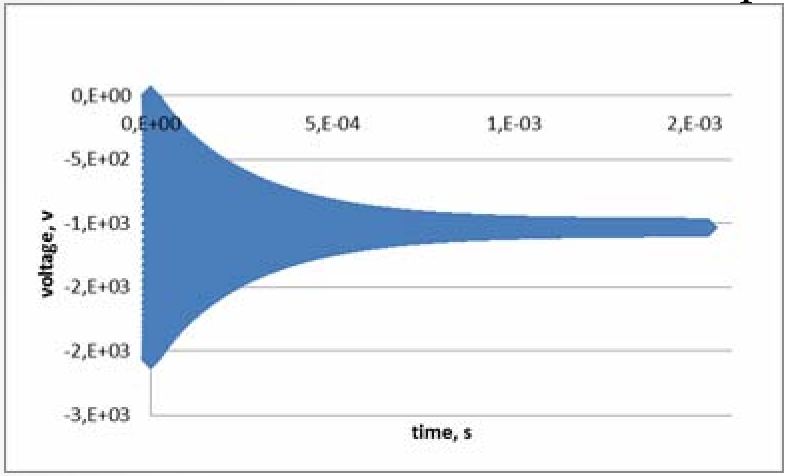
# Preinjection

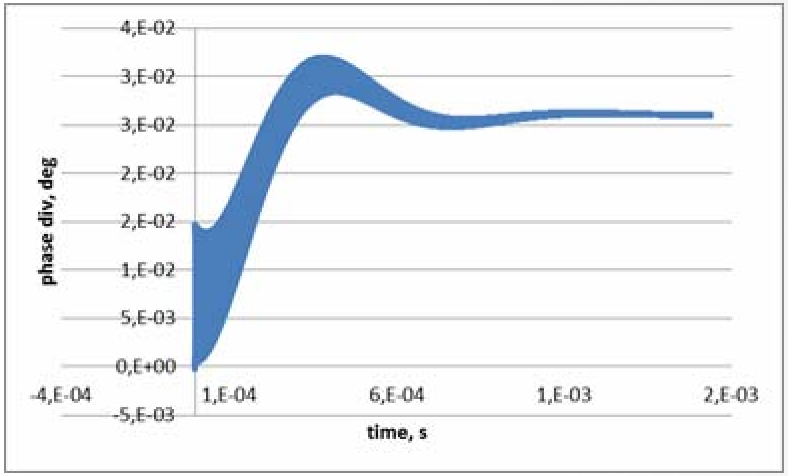


There are two additional feedback loops at the facility with preinjection.

# Examples

Stable operation

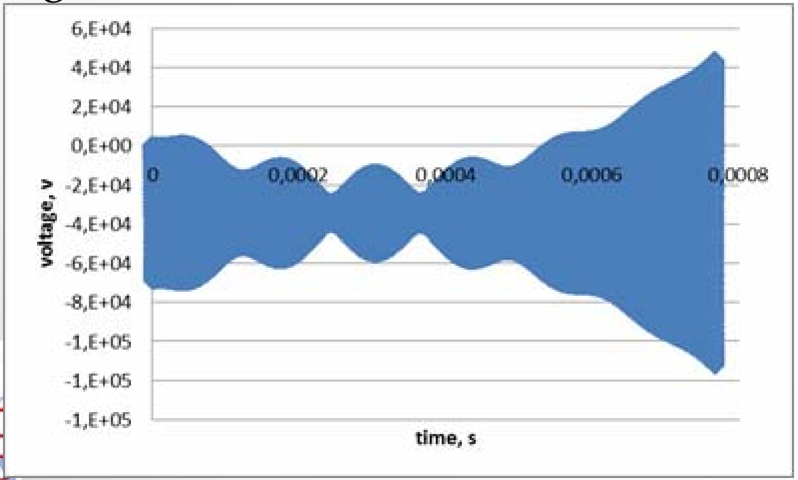


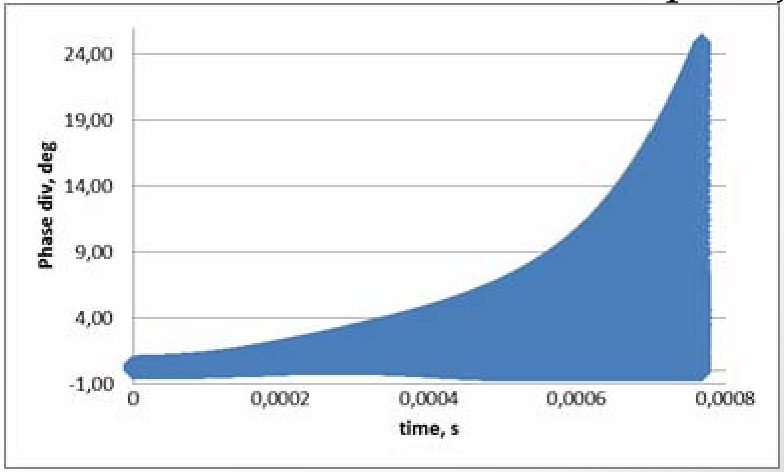




Unstable operation

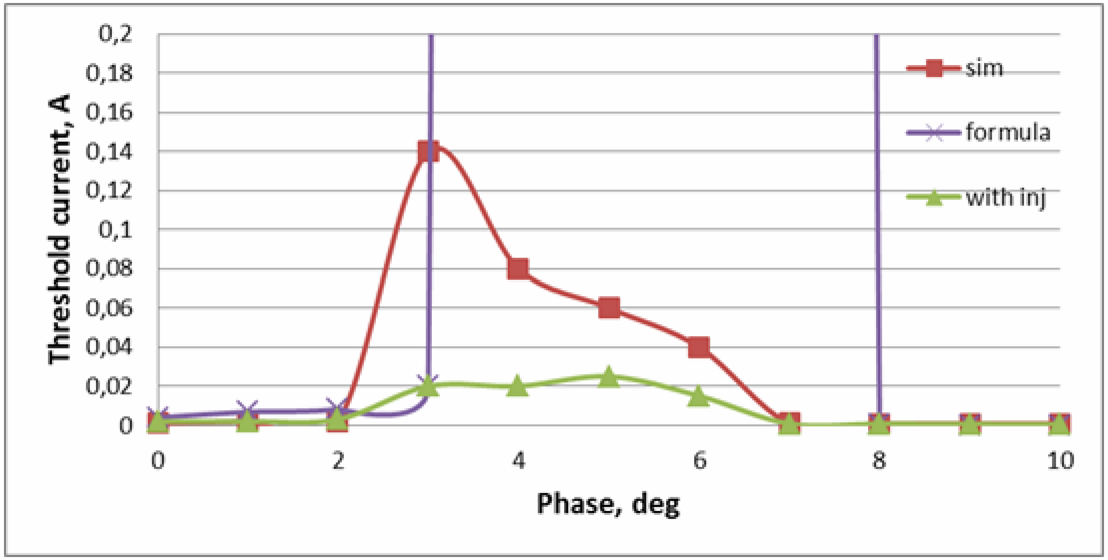
Voltage to time at the main linac

Phase deviation to time at the preinj



# Threshold current with injection




The threshold current dependence on phases with equal detunings of cavities $\xi_{1,2} = 1000$

# Conclusion

- The criterion of the longitudinal stability for the ERL with two accelerating structures is derived
- Numerical simulations were made for the project of the light source based on multiturn ERLs.
- The simulated threshold current is lower than the theoretical lower limit of the threshold current.
- To increase the threshold current, it necessary to develop a proper feedback system.

# Thank you for your attention!

# Liénard-Chipart criterion

The characteristic equation

$$\lambda^4 - S_1\lambda^3 + S_2\lambda^2 - S_3\lambda + S_4 = 0$$

where

$$S_1 = \sum_{1\le k\le 4} A\begin{pmatrix} k \\ k \end{pmatrix} = \sum_{1\le k\le 4} M_{kk} = Tr(\mathbf{M}) \quad S_2 = \sum_{1\le k<l\le 4} A\begin{pmatrix} k & l \\ k & l \end{pmatrix} \quad S_3 = \sum_{1\le k<l<m\le 4} A\begin{pmatrix} k & l & m \\ k & l & m \end{pmatrix}$$

$$S_4 = A\begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix} = |\mathbf{M}|$$

are the sums of main minors of the matrix **M**.

---

For polynomial with real coefficients $f(z) = a_0 z^n + a_1 z^{n-1} + \ldots + a_n, a_0 > 0$

Positivity of roots requires one of the following conditions:

1) $: \; a_n > 0, a_{n-2} > 0 \ldots; \Delta_1 > 0, \Delta_3 > 0, \ldots$

2) $: \; a_n > 0, a_{n-2} > 0 \ldots; \Delta_2 > 0, \Delta_4 > 0, \ldots$

3) $: \; a_n > 0, a_{n-1} > 0, a_{n-3} > 0 \ldots; \Delta_1 > 0, \Delta_3 > 0, \ldots$

4) $: \; a_n > 0, a_{n-1} > 0, a_{n-3} > 0 \ldots; \Delta_2 > 0, \Delta_4 > 0, \ldots$

Hurwitz determinant:

$$\Delta_i = \begin{Vmatrix} a_1 & a_3 & a_5 & \ldots & \ldots \\ a_0 & a_2 & a_4 & . & . \\ 0 & a_1 & a_3 & . & . \\ 0 & a_0 & a_2 & a_4 & \ldots \\ \ldots. & . & . & \ldots & a_i \end{Vmatrix}$$

ERL 2013

F.R. Gantmacher, The theory of matrices