# GROWTH RATES AND COHERENCE PROPERTIES OF FODO-LATTICE BASED X-RAY FREE ELECTRON LASERS

S. Reiche and E. Prat *
Paul Scherrer Institute, Villigen, Switzerland

*Abstract*

Most hard X-ray Free Electron Lasers are designed with a super-imposed FODO lattice to focus the electron beam for optimum performance of the FEL. Theory predicts an optimum value of the beta-function, where the induced axial velocity spread starts to counteract the increased Pierce-parameter due to higher electron density. However in a FODO lattice the electron beam envelope varies significantly and disrupts the coupling of the electron beam to the radiation field. This is particularly relevant for hard X-ray FELs, where the radiation mode is smaller than the electron beam size. In this presentation we study the impact of the FODO cell length and the beta-function variation on the FEL gain length and growth of the coherence properties for SASE FELs.

## INTRODUCTION

Free-electron lasers in the soft and hard X-ray regime require a sufficiently high electron density for the shortest gain length and an overall reasonable requirement for the undulator length. This cannot be provided by the natural focusing of the undulator field [1], which vanishes with higher beam energies. Therefore all operating and planned X-ray FELs [2] utilize quadrupoles in a FODO lattice to focus the beam. Because the beam orbit is much more sensitive to quadrupole misalignment than undulator misalignment, the quadrupoles are placed in drift sections between undulators. It is easier to align the quadrupoles alone than entire undulator modules with integrated quadrupoles.

The variation in the beam size along the undulator violates the assumption of a rigid beam in most 3D FEL theories [3]. It is impossible to obtain a fully analytical solution because the length of the FODO cell can be comparable or shorter than the characteristic length of the FEL amplification: the gain length. However simulations, which derive their results on a more basic set of equations with less assumptions on the beam transport, can give an insight on the effect of the explicit focusing on the FEL performance.

We ran simulations with Genesis 1.3 [4] to study this effect. While similar studies have been done for the FEL output power [5, 6], none of these publications include the aspect of coherence growth for SASE FELs. We studied the impact of the FEL power and the coherence for operation at 1 and 10 Ångstrom, based on SwissFEL facility layout[7]. The latter case exhibits stronger diffraction and any fine structure in the transverse electron beam distribution will be washed out stronger, implying less impact from the FODO lattice than for the former case.

* sven.reiche@psi.ch

## 3D FEL THEORY AND FEL MODES

In the 3D theory the FEL equations are scaled differently, resulting in a definition of the FEL parameter [3], which deviates from the standard 1D FEL parameter $\rho$ [8]. In addition, a new independent parameter is introduced, expressing the ratio between the characteristic scaling length and the length of diffraction (Rayleigh length). It is called the Diffraction parameter [3], where a large value indicates negligible effects from diffraction.

The FEL equation is transformed into a form very similar to that of a 2D field equation in quantum mechanics, except that the system is not hermite. The transverse current profile plays the role of the quantum mechanical potential and for a well-defined solution the FEL wavefront has to drop faster than $\propto r^{-2}$ to avoid unphysical infinite radiation power.

The different solutions (modes) can be grouped by their number of azimuthal and radial nodes, each yielding their own dispersion equation to determine the growth rate. In a system with strong diffraction the growth rate of the fundamental mode is the strongest and the radiation mode is larger than the electron beam. With vanishing diffraction the growth rates and gain curves of all modes collapse into the one of the 1D FEL model and even a distorted wavefront is preserved during amplification. Low diffraction causes problems for SASE FELs reducing the overall transverse coherence of the FEL pulse.

In all 3D models the beam is treated radially symmetric and rigid and the effect of the betatron oscillation is expressed by any effective shift in the beam electron energy [9] . However this is not the case if a FODO based focusing structure is used to keep the beam size small along the undulator. Typically, the beam exhibits a sawtooth like variation in its beam size with its variation proportional to the spacing of the quadrupoles. Fig. 1 shows the variation in the beam size as a function for different FODO cell lengths, based on the hard X-ray FEL Aramis of the SwissFEL facility. Because the FEL eigenmode is also imprinted into the bunching profile of the electron beam any variation in the electron distribution reduces the overlap of radiation field and bunching profile. We expect a reduction in the growth rate of the mode, however it is very difficult to calculate it analytically. For this we present the numerical calculation with Genesis 1.3

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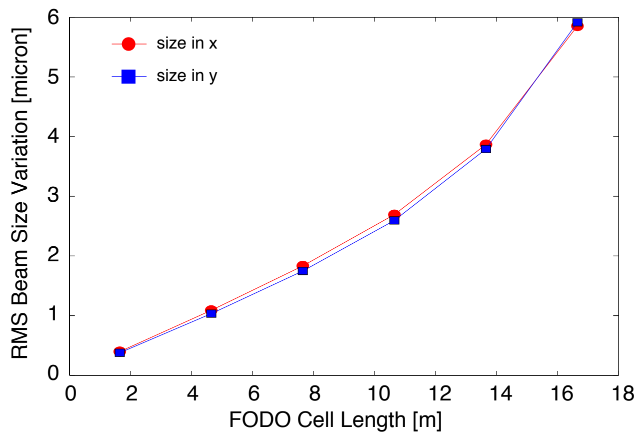


Figure 1: Beam size variation for different cell lengths of the FODO lattice, based on the SwissFEL hard X-ray FEL parameters.

## COHERENCE

A Self-Amplifying Spontaneous Emission (SASE) FEL can be regarded as the amplification of a narrow frequency band in a broad band signal of the electron shot noise. In the case of a bunch length much longer than the resonant wavelength the positions of the individual electrons are truly random and the shot noise signal is described by a "white noise" signal. During the FEL process coherence in both longitudinal and transverse direction is induced by means of slippage and diffraction.

The definition of coherence is the correlation between two field amplitudes in time and space, defining the mutual coherence function [10]:

$$\Gamma_{12} = < E(\vec{r}_1, t+\tau) E^*(\vec{r}_2, t) >, \qquad (1)$$

where the average is taken over many shots at the same observation time $t$. Because the SASE signal for long pulses is a stationary process the shot average is identical to the time-average over the entire bunch. This simplifies the computational time because only one single pulse with constant beam parameters needs to be calculated and analyzed as long as it exhibits sufficient number of SASE spikes.

To evaluate the transverse coherence we set $\tau$ to zero and normalize by the mean field amplitude with $\mu_{12} = \Gamma_{12}(0)/\sqrt{\Gamma_{11}(0)\Gamma_{22}(0)}$. This complex coherence function defines the coherence between two spatial points and has a value between zero (no correlation between field amplitudes) and unity (full coherence). For an expression of the coherence of the whole bunch the mutual coherence function is weighted by the local intensities and integrated over $r_1$ and $r_2$:

$$\zeta = \frac{\int |\mu_{12}(\vec{r}_1, \vec{r}_2)|^2 I(\vec{r}_1) I(\vec{r}_2) d\vec{r}_1 d\vec{r}_2}{\left[\int I(\vec{r}_1) d\vec{r}_1\right]^2} \quad . \qquad (2)$$

A value of unity means full coherence over the entire transverse profile.

The evaluation of $\zeta$ is very computational and memory intensive and rather impractical for systematic coherence studies. However for the "white-noise" character of the spontaneous emission, seeding the FEL process, there is an alternative method to calculate $\zeta$ [11] based on the fluctuation in the power level along the pulse with

$$\zeta = \frac{< (P - < P >)^2 >}{< P >^2} \qquad (3)$$

Simulations have shown that both expressions yield the same results in the linear regime of the FEL [12]. The physical meaning of $\zeta = 1/(M_L \cdot M_T)$ is the inverse product of the number of longitudinal and transverse modes $M_L$ and $M_T$, respectively. The instantaneous power has only one longitudinal mode in the linear regime but increases at saturation [13], which results in an artificially lower degree of coherence. Nevertheless the evaluation of the fluctuation of the radiation power is a fast and effective method to study the coherence, presented in the following sections.

## HARD X-RAY FEL PERFORMANCE

The following discussion is based on the SwissFEL hard X-ray FEL beamline Aramis, operating at one Ångstrom. The driving electron beam has a flat current profile of 2.7 kA, an energy spread of 350 keV, a normalized emittance of 0.43 mm·mrad, and a beam energy of 5.8 GeV. The undulator consists of 12 undulator modules of 4 m length, a period length of 15 mm and an undulator parameter of $K = 1.2$. The undulator breaks, which hold the diagnostic and the focusing quadrupoles, is 75 cm long. At nominal operation 12 modules are sufficient to achieve saturation.

We model an electron bunch with constant beam parameters sufficiently long enough to obtain enough SASE spikes for an evaluation of the coherence factor by Eq. 3. In reality the electron pulse will be shorter and thus prone to stronger fluctuation.

Using the Ming Xie model [14] to estimate the FEL performance the shortest gain length occurs at a beta function of 12 m, which is the optimum between induced axial velocity spread at lower values for the beta-function and reduced electron density for larger values. For lower emittance values or longer wavelength this value is decreased beyond the limit of $\beta = 6$ m, which is imposed by an unstable beam transport when the focal length of the quadrupoles is shorter than one fourth of the FODO cell length.

For comparison we scan over different focal strengths of the FODO lattice with the code Genesis 1.3. The results are shown in Fig. 2. There is a shift towards weaker focusing (average beta function value of 15 m) for the best performance of the FEL in comparison to the Ming Xie model, while operation at an average value of 10 m has a stronger penalty than expected. With respect to achievable saturation power an even weaker focusing shows a better performance although the saturation length is slightly increased. This behavior is similar to detuning an FEL ampli-



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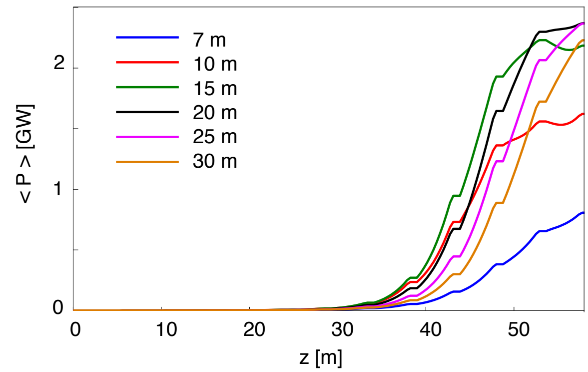


Figure 2: FEL performance of the SwissFEL 1 Å X-ray FEL for various focussing strengths of the FODO lattice, expressed by its average beta function.

fier, where a slightly off-resonance operation yields larger FEL power at longer saturation lengths [15]. The origin for the observed increase in saturation power is that weaker focusing couples the axial velocity of the betatron motion less to the longitudinal velocity and therefore the electrons stay longer at emission phase before they are shifted out of resonance due to the energy loss by the FEL process.

We evaluated the coherence according to Eq. 3 and reached a maximum for the case of 10 m beta-function. Larger values decrease the degree of coherence and is not simultaneously achieved with the optimization of the saturation power. In addition this seems to be in contrast to the FEL theory [16], which predicts better coherence for longer saturation lengths because the radiation field has more "time" to spread out by diffraction and to impose a single phase over the entire wavefront. However the theory assumes a rigid beam, excluding the impact of the electron motion. For strong focusing the variation of the beam size is significant and should have a stronger impact on higher modes because the overlap in the phase front of the bunching and radiation field is disrupted. A more systematic study of this effect is discussed in the next section.

## *Variable FODO Lattices*

Scanning over the focusing strength has the disadvantage that the average electron density is changed and therefore the FEL performance is altered, which makes it harder to investigate the impact of the FODO lattice. Therefore we conducted simulations with different lengths of the FODO cells but keeping the average beta-function constant for all runs at a value of 15 m. Also, we modeled the undulator as one long continuous device, eliminating the drift sections to hold the quadrupoles. Otherwise a shorter FODO cell length would have the penalty of requiring more drift sections and therefore artificially increase the saturation length.

Fig. 3 shows the FEL performance and coherence for different FODO cell lengths. The point of saturation is similar in all cases but a shorter FODO cell length provides more power with less beam size variations (see Fig. 1). Only for extreme variations (not shown here) it has also an impact on the saturation length, which is significantly increased.

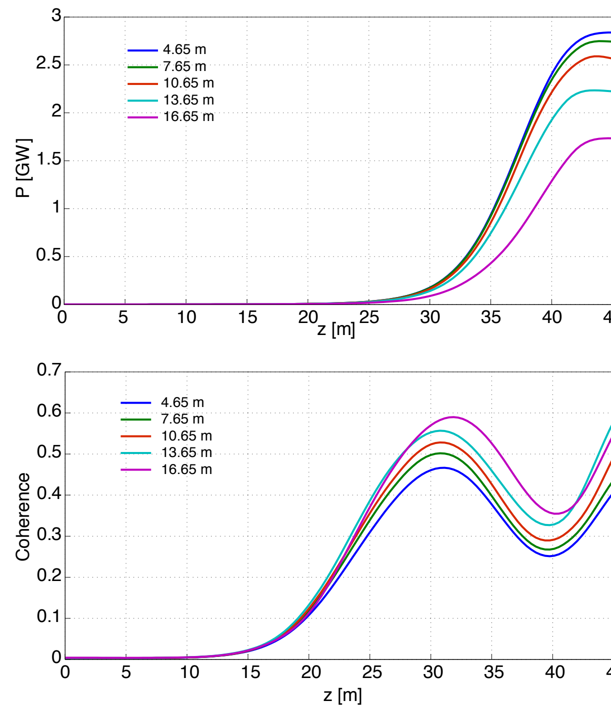


Figure 3: FEL power and coherence of an 1 Å X-ray FEL for various cell lengths of the FODO lattice (upper and lower plot, respectively).

For hard X-ray FELs the radiation mode size is smaller than the electron beam size (for the case of the SwissFEL 18 microns for the radiation size and 24 microns for the electron beam) and the alternating squeezing and stretching of the electron beam by the FODO quadrupoles results in even less electrons staying within the radiation field for uninterrupted FEL amplification. This effectively reduces the beam current and therefore the achievable saturation power.

In contrast to the saturation power the coherence is improved with a larger beam size variation. Similar to the results of the beta-function scan, maximum power and best coherence are exclusive for hard X-ray FELs. It is caused because higher modes are more sensitive to a missing overlap between the radiation field and the transverse bunching profile. It is hard to extract the growth rates for higher modes except for a $TEM_{01}$ or $TEM_{10}$ Gauss-Hermite mode [17], which has a double lobe distribution with opposite phases and excludes coupling to the fundamental FEL mode. The growth rate is not constant but varies with the periodicity of the FODO cell. For larger periodicity the average growth rate is smaller and the separation in amplitude of the fundamental and higher mode due to the FEL amplification becomes apparent earlier within the undulator.

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## SOFT X-RAY FEL PERFORMANCE

We studied the impact of the FODO lattice at a longer wavelength. As basis for the undulator we used the soft X-ray FEL beamline ATHOS at SwissFEL, tuned to 1 nm. The period length of the undulator is 4 cm with a K-value of 1.2. The beam energy is 3 GeV and the beam current is 1.5 kA. The saturation length is 45 m and thus comparable to the hard X-ray case.

The growth of the radiation power and the coherence for the soft X-ray FEL is shown in Fig. 4. The dependence of the saturation power on the FODO cell is rather weak with a slightly better performance for larger beam size variations. The lattice was matched in a way that the mean of the minimum and maximum values of the beta-function yields 15 m. However at larger spacing of the quadrupoles and smaller beta-values at the points of minimum beam sizes the quadratic term of the beta-function for a drift between the quadrupoles becomes more pronounced, resulting in an overall smaller beam size for larger cell lengths. This is the dominant effect, seen in Fig. 4, on the FEL performance. The impact of the varying beam size can be regarded as negligible, in particular, unlike the hard X-ray case, the FEL eigenmode is larger in size (37 microns) than the electron beam size of 32 microns.

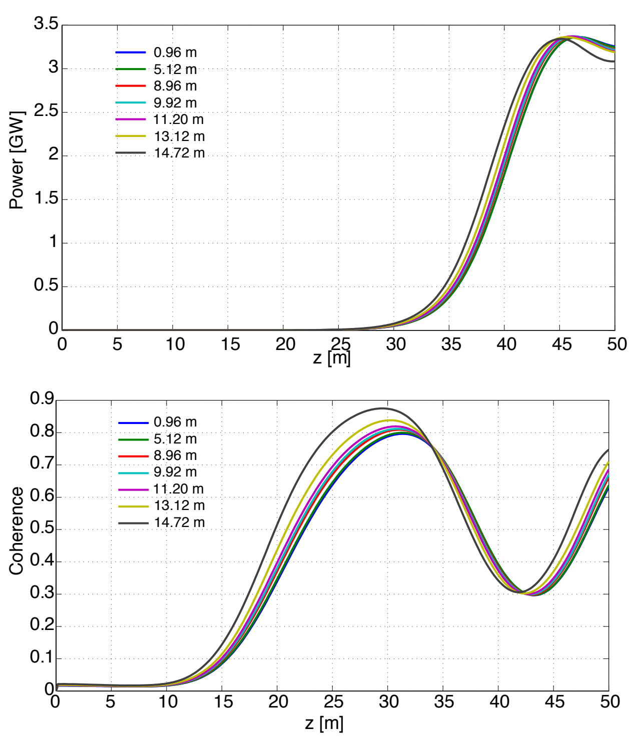


Figure 4: FEL power and coherence of a 1 nm X-ray FEL for various cell lengths of the FODO lattice (upper and lower plot, respectively).

The coherence of the soft X-ray FEL pulse is overall better because the emittance condition $\epsilon_N/\gamma < \lambda/4\pi$ is well fulfilled. The higher modes have a reduced growth rate due to higher diffraction and the contrast between fundamental mode and all higher modes is built up at a much faster rate. Nevertheless an improvement in the coherence is seen for a larger FODO cell, similar to the results from the hard X-ray FEL.

## CONCLUSION

For hard X-ray Free-electron Lasers the optical mode lies within the electron beam and is sensitive to any variation in beam size, e.g. by a superimposed FODO lattice. The stronger the distortion is, the less efficient the FEL operates. Therefore it is recommended to operate with reduced focusing when the optimum beta-function would push the FODO lattice to the unstable limit. Relaxed focusing improves the FEL performance with respect to power. However strong beam size variations have the benefit of improving the coherence of the FEL when operated in SASE mode, relaxing the emittance constraint of a hard X-ray FEL somewhat.

This effect is less pronounced at longer wavelengths and the subtle change in the average beam size has a stronger effect on the FEL performance than the variation in the beam size due to the focusing. Still the growth in the coherence can be enhanced with a stronger focusing. Normally the optimum beta-function value lies well below the capability of the focussing lattice, suggesting that the soft X-ray FEL is operated at the limit of the FODO lattice without any penalty in the FEL performance.

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