HARMONIC LASING OF THIN ELECTRON BEAM

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Abstract

For a typical operating range of hard X-ray FELs the condition for ratio of the electron beam emittance to radiation wavelength $2\pi\epsilon/\lambda \simeq 1$ is usually a design goal for the shortest wavelength. In the case of the simultaneous lasing the fundamental mode has shorter gain length than harmonics. If the same electron beam is used to drive an FEL in a soft X-ray beamline, the regime with $2\pi\epsilon/\lambda \ll 1$ is realized which corresponds to the case of a small value of diffraction parameter. Here we present a detailed study of this regime. We discover that in a part of the parameter space, corresponding to the operating range of soft X-ray beamlines of X-ray FEL facilities (like SASE3 beamline of the European XFEL), harmonics can grow faster than the fundamental wavelength. This feature can be used in some experiments, but might also be an unwanted phenomenon, and we discuss possible measures to diminish it.

INTRODUCTION

Once pronounced harmonic of the beam current density exists in the electron beam, it allows to produce powerful coherent radiation. Two mechanisms are usually considered providing electron beam modulation at higher harmonics. Bunching at higher harmonics always takes place when FEL apmplification process driven by the fundamental frequency reaches saturation regime. This mechanism is referred to as nonlinear harmonic generation [1–7]. In the case of planar undulator there always exists also an amplification of odd harmonics due to FEL instability. This mechanism is usually referred as linear harmonic generation [1,3,8–14]. It is generally accepted that self-consistent amplification of the radiation at higher harmonics is weaker than that of the fundamental. This is true in the framework of one-dimensional approximation (or, in a wide beam limit) as it has been show in early studies. However, situation changes qualitatively for diffraction limited electron beams with small value of diffraction parameter. This parameter range refers to as thin beam limit [21]. Our studies have shown that there exists range of parameters when gain at higher harmonics exceeds the gain at the fundamental. This range of parameters is not of pure academical interest, but can be experimentally realized for long wavelength FELS driven by high energy electron beams. Free electron laser at SASE3 beam line of the European XFEL falls in this parameter range.

FEL gain length is calculated from solution of an eigenvalue equation. Eigenvalue equation for harmonic lasing was derived in the framework of one-dimensional (1D) model in [1, 13], and a thorough 1D analysis can be found in [14]. An eigenvalue equation for three-dimensional case has been derived in [3]. However, this eigenvalue equation is rather complicated and can be solved only numerically. One can correctly calculate the gain length for a specific set of parameters, but it is very difficult to trace general dependencies and perform analysis of the parameter space. In paper [23] we performed a parametrization of the solution of the eigenvalue equation for lasing at odd harmonics [3], and presented explicit expressions for FEL gain length, optimal beta-function, and saturation length taking into account emittance, betatron motion, diffraction of radiation, energy spread and its growth along the undulator length due to quantum fluctuations of the undulator radiation. Considering 3rd harmonic lasing as a practical example, we come to the conclusion that it is much more robust than usually thought, and can be widely used at the present level of accelerator and FEL technology. We surprisingly find out that in many cases the 3D model of harmonic lasing gives more optimistic results than the 1D model. For instance, one of the results of our studies is that in a part of the parameter space, corresponding to the operating range of soft X-ray beamlines of X-ray FEL facilities, harmonics can grow faster than the fundamental mode.

SIMULTANEOUS LASING IN THE CASE OF A THIN ELECTRON BEAM

For a typical operating range of hard X-ray FELs the condition $2\pi\epsilon/\lambda \simeq 1$ is usually a design goal for the shortest wavelength. In the case of the simultaneous lasing the fundamental mode has shorter gain length than harmonics, as it was shown above in paper [23]. However, if the same electron beam is supposed to drive an FEL in a soft X-ray beamline, the regime with $2\pi\epsilon/\lambda \ll 1$ can be realized. Here we present a detailed study of this regime. In this Section we assume that beta-function is much longer than FEL field gain length, $\beta \gg L_g^{(h)}$. Here subscript h denotes harmonic number. In this case we can use the model of parallel beam (no betatron oscillations), and can also neglect an influence of longitudinal velocity spread due to emittance on FEL process. If in addition the energy spread is negligibly small, then the normalized FEL growth rate at the fundamental frequency is described by the only dimensionless parameter, namely the diffraction parameter B [21]. The generalized diffraction parameter $\tilde{B}$, that can be used for harmonics, is written as follows [23]:

$$\tilde{B} = 2\epsilon\beta\tilde{\Gamma}\omega_h/c \, . \qquad (1)$$

Here $\omega_h = 2\pi c/\lambda_h$, ω_h (λ_h) is frequency (wavelength) of the hth harmonic, c is velocity of light, and $\tilde{\Gamma}$ is the gain factor that also depends on harmonic number:

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$$\tilde{\Gamma} = \left(\frac{A_{\mathrm{JJh}}^2 I \omega_h^2 K^2 (1+K^2)}{I_{\mathrm{A}} c^2 \gamma^5} \right)^{1/2} \qquad (2)$$

The gain length of a harmonic is defined by the universal function of $\tilde{B}$:

$$L_g^{(h)} = [\tilde{\Gamma} f_1(\tilde{B})]^{-1} \qquad (3)$$

The function $f_1(\tilde{B})$ can be calculated from the general eigenvalue equation [23]. However, within the parallel beam model, accepted in this Section, the eigenvalue equation can be significantly simplified. We use here the solution of the equation presented in [21, 22] for the Gaussian transverse distribution of current density (see Fig. 4.52 of Ref. [21]). In the parameter range, that is the most interesting for our purpose, we can approximate the function $f_1(\tilde{B})$ as follows:

$$f_1(\tilde{B}) \simeq 0.66 - 0.37 \log_{10}(\tilde{B}) \qquad for \;\; \tilde{B} < 3\,. \qquad (4)$$

Using the superscript (h) to indicate the harmonic number for the diffraction parameter and the gain factor, we can see that

$$\frac{\tilde{B}^{(h)}}{\tilde{B}^{(1)}} = \frac{h\tilde{\Gamma}^{(h)}}{\tilde{\Gamma}^{(1)}} = \frac{h^2 A_{JJh}}{A_{JJ1}}\,. \qquad (5)$$

According to (3) and (2), the ratio of gain lengths can be presented as follows:

$$\frac{L_g^{(1)}}{L_g^{(h)}} = \frac{h A_{JJh}}{A_{JJ1}} \frac{f_1(\tilde{B}^{(h)})}{f_1(\tilde{B}^{(1)})} \qquad (6)$$

One can easily observe from (5) and (6) that for a given value of diffraction parameter for the fundamental frequency, $B = \tilde{B}^{(1)}$, this ratio depends only on the parameter K for a considered harmonic. If K is sufficiently large (see Fig. 1), one can obtain a universal dependence which is presented in Fig. 2 for the case of the third harmonic. For large values of the diffraction parameter (wide electron beam limit) one can use an asymptotic expression for the growth rate [21], so that the function f_1 is proportional to $(\tilde{B}^{(h)})^{-1/3}$. In this case one obtains the result of 1D theory [14]:

$$\frac{L_g^{(1)}}{L_g^{(h)}} \simeq \left(\frac{h A_{JJh}^2}{A_{JJ1}^2} \right)^{1/3}\,.$$

In the case of the third harmonic and large K this ratio is equal to 0.87. One can see that the curve in Fig. 2 slowly approaches this value when B is large. So, in the limit of wide electron beam, corresponding to 1D model, the fundamental frequency has shorter gain length than harmonics.

In the limit of small diffraction parameter (thin electron beam) we wave the opposite situation, as one can see from Fig. 2. When diffraction parameter is smaller than 0.4, the gain length of the fundamental frequency is larger than that

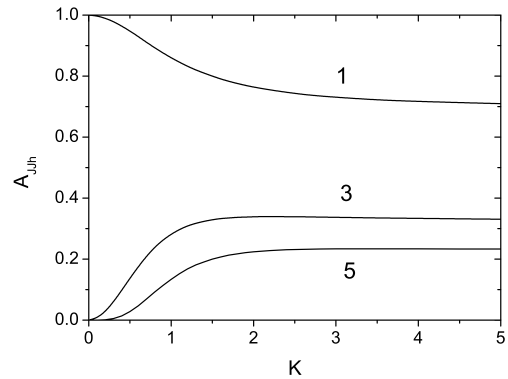

Figure 1: Coupling factors for the 1st, 3rd, and 5th harmonics (denoted with 1, 3, and 5, correspondingly) versus rms undulator parameter.

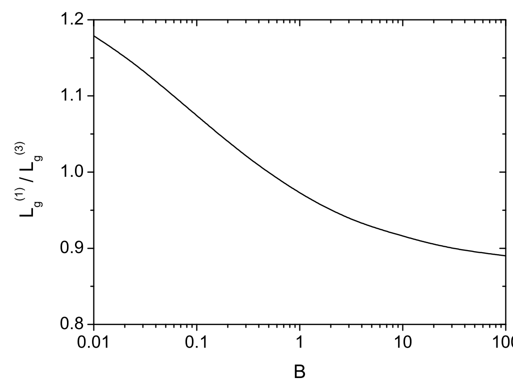

Figure 2: Ratio of gain lengths for lasing at the fundamental wavelength and at the third harmonic versus diffraction parameter of the fundamental wavelength for large values of the undulator parameter K.

of the third harmonic for large values of K. A similar dependence can be calculated for the fifth harmonic, in this case the gain length of the fundamental harmonic is larger than that of the fifth harmonic (for a sufficiently large K) when $B < 0.28$. Moreover, the fifth harmonic grows faster than the third one when $B < 0.15$ and K is large. In fact, if the diffraction parameter for the fundamental harmonic is about 0.1 or less, there might a number of amplified harmonics with similar growth rates. We should note that this number can be reduced when the energy spread is included into consideration.

To find out how the value of B, at which the harmonics have the same gain length as the fundamental, depends on the undulator parameter K, one can use the Eqs. (4)-(6). We present the results for the third and the fifth harmonics in Fig. 3. The areas below the curves in Fig. 3 correspond to the case when corresponding harmonics grow faster than the fundamental frequency. We should stress that the condition $2\pi\epsilon/\lambda \ll 1$ is necessary but not sufficient for reach-

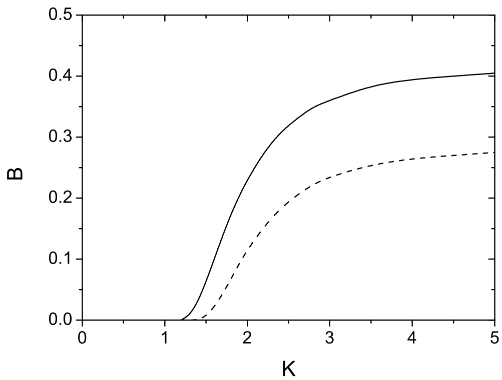

Figure 3: Diffraction parameter of the fundamental wavelength, for which the third (solid) and the fifth (dash) harmonics have the same gain length as the fundamental, versus the rms undulator parameter K. Below these curves harmonics have shorter gain lengths than the fundamental frequency.

ing this regime.

Let us discuss why the effect, considered in this Section, can only take place in the frame of 3D theory and in the limit of a thin beam. In 1D theory the gain factor (inversely proportional to the gain length) scales as $(A_{JJh}^2 \omega_h)^{1/3}$, if we keep only parameters that depend on harmonic number. The frequency here comes from the dynamical part of the problem, it reflects the fact that the beam gets bunched easier at higher frequencies. As for the electrodynamic part of the problem, the amplitude of the radiation field of charged planes does not depend on frequency. Since the product $A_{JJh}^2 h$ decreases with harmonic number for any K, gain length of harmonics is always larger than that of the fundamental frequency. Concerning the 3D theory, the solution of the paraxial wave equation shows that on-axis field amplitude is proportional to the frequency. So, both dynamical and electrodynamic parts contribute to the solution of the self-consistent problem with ω_h. That is why in the gain factor in Eq. (2) we have squared frequency $(A_{JJh}^2 \omega_h^2)^{1/2}$, i.e. it depends on harmonic number via the product $A_{JJh}^2 h^2$ which can increase with harmonic number if K is sufficiently large. Since in the case of a thin electron beam the function f_1 depends only weakly, in fact logarithmically, on the diffraction parameter (which is larger for harmonics), harmonics can grow faster than the fundamental frequency in some range of parameters B and K, as it is illustrated in Fig. 3.

So far we have discussed an exponential gain regime and did not consider an initial-value problem. In the simulations one can observe that the fundamental dominates saturation regime even if its gain length is slightly longer than that of harmonics. First, it has a higher effective start-up power due to a larger factor A_{JJ}. Second, in nonlinear regime the longitudinal phase space of the electron beam is affected stronger by the fundamental frequency. As a result, saturation power of harmonics in the case $B \simeq 0.1$ is weaker[1] than it would have been in the absence of the fundamental frequency (but still much higher than in the case of nonlinear harmonic generation). The bandwidth at saturation is inversely proportional to harmonic number (contrary to the case of nonlinear harmonic generation).

Let us present a numerical example for the European XFEL. An electron beam with the energy of 10.5 GeV lases in SASE3 undulator (which is placed behind the hard X-ray undulator SASE1) with the period 6.8 cm and the rms undulator parameter 7.4 at the fundamental wavelength 4.5 nm. We consider electron bunches with the charge of 100 pC: the peak current is 5 kA, averaged normalized slice emittance is 0.3 μm from start-to-end simulations [18]. It is assumed that SASE3 operates in "fresh bunch" mode, i.e. there is no lasing to saturation in SASE1. Slice energy spread (due to the quantum diffusion [19] in SASE1 and the active part of SASE3 undulators is added quadratically to the value obtained in start-to-end similations [18]. For the beta-function of 15 m we obtain from (1) that the diffraction parameter for the fundamental wavelength is 0.3, so that a simplified model, considered in this Section, suggests that the third harmonic can grow faster than the fundamental. However, harmonics are more sensitive to the energy spread than the fundamental frequency, therefore we use a general eigenvalue equation [23] that includes all the important effects. We find that the field gain length is 2.44 m for the fundamental harmonic, 2.42 m for the third harmonic, and 2.52 m for the fifth one. In Fig. 4 we present the results of numerical simulations. Even though the saturation power of harmonics is lower than it would have been in the absence of the fundamental, it is still by an order of magnitude higher than that expected from nonlinear harmonic generation [20]. The saturation power of the third (fifth) harmonic is 12% (3%) of the saturation power of the fundamental frequency. Thus, accurate calculation of harmonic lasing is necessary for planning of user experiments and X-ray beam transport.

Note that the method of brilliance improvement, described in [23], is especially attractive in the considered regime. Indeed, one can, in principle, use a high harmonic number so that the bandwidth reduction can be significant. Another useful application is the simultaneous lasing at the fundamental wavelength and at the third harmonic with comparable intensities that can be used in jitter-free pump-probe experiments making use of a split-and-delay stage [24]. For such an experiment one can, in principle, manipulate relative intensities with the help of phase shifters.

On the other hand, a high-intensity harmonic radiation can disturb some experiments, or may lead to an excessive power load on mirrors of X-ray transport. In this case the harmonics can be suppressed by different means. For example, one can increase the energy spread with the help of a laser heater [26–28] which is going to be a part of the standard design of an X-ray FEL accelerator complex. In the

[1] The third harmonic saturates earlier than the fundamental, and at a full expected power when diffraction parameter is on the order of 0.01.

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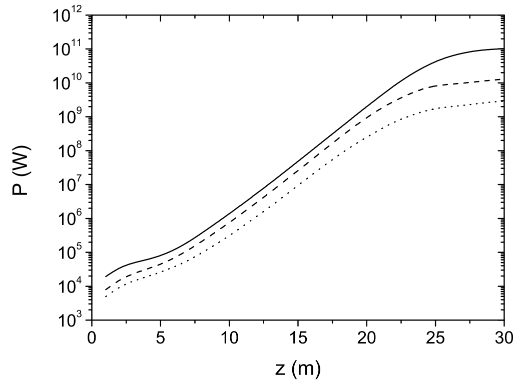

Figure 4: An example for the European XFEL. Averaged peak power for the fundamental harmonic (solid), the third harmonic (dash), and the fifth harmonic (dot) versus undulator length for SASE3 operating at 4.5 nm. Parameters are in the text. Simulations were performed with the code FAST.

above presented example, an increase of the energy spread up to 5 MeV would strongly suppress harmonic lasing, so that one would get an intensity level expected from nonlinear harmonic generation. Another method is the use of phase shifters, but now aiming at suppression of harmonics. In this case the phase shifts for the fundamental frequency could be below 1 rad while for harmonics they are h times larger, i.e. the suppression effect is stronger. Other options are an increase of the beta-function (what leads to an increase of the diffraction parameter) or the application of linear undulator taper [29, 30] that would have stronger effect on the amplification of harmonics.

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