SATURATION IN FREE ELECTRON LASER WITH QUADRUPOLE WIGGLER AND AXIAL MAGNETIC FIELD

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Abstract

In this paper, we study the nonlinear evolution of a helical quadrupole wiggler FEL, in Raman regime and in the presence of space-charge field. By using Maxwell's equations and nonwiggler averaged equation of motion of electron beam, a set of nonlinear first-order differential equations describing the evolution of the helical quadrupole FEL is derived in the slowly varying amplitude and wave number approximation and solved numerically by the Runge–Kutta method. The beam is cold and propagates with a relativistic velocity. The amplitude of wiggler field increases adiabatically from zero to a constant level. To focus the electron beam, we apply an axial magnetic field. Finally, the results of helical quadrupole wiggler are compared to an equivalent dipole wiggler.

INTRODUCTION

There are two principle directions for FEL development activities. One is to generate high coherent X-ray pulses, and the other is to generate high average power at infrared wavelength. A FEL which works at millimeter wavelengths has many applications such as telecommunication and measurement of solid state materials and semi conductor's properties.

In order to generate a quadrupole wiggler magnetic field, one can use a helical winding of four wires, with a current flow in in two wires in one direction, and in the oder two wires in the opposite direction. First works on quadrupole wiggler FEL were done by Levush *et al.* [1]. They proposed this kind of wiggler and showed that it can represent a new concept to obtain high power, coherent radiation in millimeter and sub millimeter regime. They examined the near axis orbit properties of a quadrupole wiggler without an axial guide field but including the effects of space charge and showed that it had improved beam stability when compared to a dipole wiggler. The effect of an axial guide field on the nonlinear stage of the dipole wiggler FEL interaction studied by Freund [2]

Antonsen *et al.* [6] examined the nonlinear theory of a quadrupole free-electron laser when the betatron frequency is close to the mismatch frequency and found that it can reduced the three-dimensional equations to an integrable one-dimensional equation. CHANG *et al.* examined the characteristics of a Compton regime quadrupole magnetic wiggler for a FEL amplifier neglecting space charge effects. They showed that Optimum efficiencies for the quadrupole case occur for lower beam voltages, larger guide radii, and shorter total wiggler lengths than for a comparable dipole case [7].

The purpose of this study is to consider a FEL with a helical quadrupole wiggler in Raman regime at millimeter wavelengths.

This code is based on the equations in which the space-charge effect is presented and then, there are some additional terms in the equations to show this effect. This is an improvement of CHANG *et al.* Formulation in which this effect is neglected [7]. In fact, we modified Freund *et al.* formulation [2], to examine the evolution of radiation amplitude for a helical quadrupole wiggler and finally we compare its results with an equivalent helical dipole case.

FIELDS STRUCTURE AND POTENTIAL EQUATIONS

The idealized, one dimensional helical quadrupole wiggler magnetic field may be described as [7]

$$\mathbf{B_W}(z) = B_W(z)k_W\mathbf{e}_x\left[x\cos(2k_Wz) + y\sin(2k_Wz)\right] + B_W(z)k_W\mathbf{e}_y\left[-y\cos(2k_Wz) + x\sin(2k_Wz)\right] \quad (1)$$

$$B_w(z) = \begin{cases} B_w\sin^2\left(\dfrac{k_wz}{4N_w}\right), & 0 \le z \le N_w\lambda_w \\ B_w & N_w\lambda_w \le z \end{cases} \quad (2)$$

Where B_W refers to the wiggler amplitude which presents an adiabatic injection of the electron beam and $k_W = 2\pi/\lambda_W$ is the wiggler wave number.

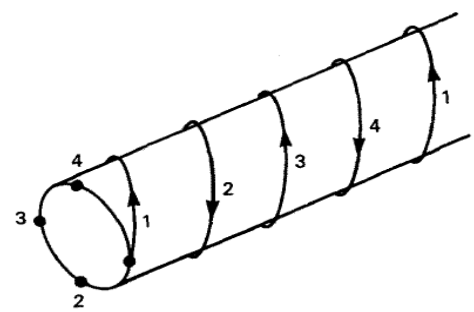

Figure 1: Quadrupole wiggler configuration

In addition to the wiggler filed, an axial magnetic field is used to focus the beam,

$$\mathbf{B}_0 = B_0e_z \quad (3)$$

The vector and scalar potentials of radiation and space charge may be in the form of [2], [5]

$$\delta\mathbf{A}(z,t) = \delta A(z)\left[\mathbf{e}_x\cos\alpha_+(z,t) - \mathbf{e}_y\sin\alpha_+(z,t)\right] \quad (4)$$

$$\delta\phi(z,t) = \delta\varphi(z)\cos\alpha(z,t) \quad (5)$$

$$\alpha_+(z,t) = \int_0^z dz'k_+(z') - \omega t \quad (6)$$

$$\alpha(z,t) = \int_0^z dz'k(z') - \omega t \quad (7)$$

ISBN 978-3-95450-123-6

Where $k_+(z)$ is the wave number of radiation and $k(z)$ is the wave number of the space charge wave.

ELECTRON ORBIT DYNAMICS

In order to derive the dynamics of electrons, the relativistic equation of motion is used, [2]

$$\mathbf{F} = \frac{d\mathbf{P}}{dt} = -e\left(\delta\mathbf{E} + \mathbf{V}/_c \times (\mathbf{B_w} + \mathbf{B_0} + \delta\mathbf{B})\right) \quad (8)$$

δE and δB are electric and magnetic field due to δA and $\delta\phi$ which may obtain from Maxwell equations as follow:

$$\delta\mathbf{B} = \nabla \times \delta\mathbf{A} \quad (9)$$

$$\delta\mathbf{E} = -\nabla\delta\phi - 1/_c \frac{\partial \delta\mathbf{A}}{\partial t} \quad (10)$$

Substituting Eq. (4) and Eq. (5) into Eq. (9) and Eq. (10), yields

$$\delta\mathbf{B} = \left[\frac{d\delta\mathbf{A}}{dz}\sin\alpha_+ + k_+\delta\mathbf{A}\cos\alpha_+\right]\mathbf{e}_x + \left[\frac{d\delta\mathbf{A}}{dz}\cos\alpha_+ - k_+\delta\mathbf{A}\sin\alpha_+\right]\mathbf{e}_y \quad (11)$$

$$\delta\mathbf{E} = -\frac{\delta\mathbf{A}}{c}\omega\left[\mathbf{e}_x\sin\alpha_+ + \mathbf{e}_y\cos\alpha_+\right] - \left[\frac{d\delta\phi}{dz}\cos\alpha - k\delta\phi\sin\alpha\right]\mathbf{e}_z \quad (12)$$

Using dimensionless variables $\bar{x} = k_w x$, $\bar{y} = k_w y$, $\bar{z} = k_w z$, $\bar{u}_{x,y,z} = \frac{u_{x,y,z}}{c}$, $\bar{\omega} = \frac{\omega}{ck_w}$, $\bar{k} = \frac{k}{k_w}$, $\bar{k}_+ = \frac{k_+}{k_w}$, $\mathbf{u} = \mathbf{p}/mc = \gamma\boldsymbol{\beta}$ and changing integration parameter from t to z according to the relation $d/_{d\bar{t}} = \beta\, d/_{d\bar{z}}$, the three components of Eq. (8) will be as:

$$\frac{d\bar{u}_x}{d\bar{z}} = \frac{1}{\bar{v}_z}\left(\bar{\omega}\delta a\sin\alpha_+ - \bar{v}_y\bar{\Omega}_0\right) + \frac{d\delta a}{d\bar{z}}\cos\alpha_+ - \bar{k}_+\delta a\sin\alpha_+ + \bar{\Omega}_w\left(\bar{x}\sin(2\bar{z}) - \bar{y}\cos(2\bar{z})\right) \quad (13)$$

$$\frac{d\bar{u}_y}{d\bar{z}} = \frac{1}{\bar{v}_z}\left(\bar{\omega}\delta a\cos\alpha_+ + \bar{v}_x\bar{\Omega}_0\right) - \frac{d\delta a}{d\bar{z}}\sin\alpha_+ - \bar{k}_+\delta a\cos\alpha_+ + \bar{\Omega}_w\left(\bar{y}\sin(2\bar{z}) + \bar{x}\cos(2\bar{z})\right) \quad (14)$$

$$\frac{d\bar{u}_z}{d\bar{z}} = \frac{1}{\bar{v}_z}\left(\frac{d\delta\varphi}{d\bar{z}}\cos\alpha - \bar{k}\delta\varphi\sin\alpha\right) - \frac{\bar{v}_x}{\bar{v}_z}\left[\frac{d\delta a}{d\bar{z}}\cos\alpha_+ - \bar{k}_+\delta a\sin\alpha_+ + \bar{\Omega}_w\left(\bar{x}\sin(2\bar{z}) - \bar{y}\cos(2\bar{z})\right)\right] + \frac{\bar{v}_y}{\bar{v}_z}\left[\frac{d\delta a}{d\bar{z}}\sin\alpha_+ + \bar{k}_+\delta a\cos\alpha_+ + \bar{\Omega}_w\left(\bar{y}\sin(2\bar{z}) + \bar{x}\cos(2\bar{z})\right)\right] \quad (15)$$

Differential equations which describe the evolution of α_+ and α are

$$\frac{d\alpha_+}{d\bar{z}} = \bar{k}_+ - \frac{\bar{\omega}}{\beta_z} \quad (16)$$

$$\frac{d\alpha}{d\bar{z}} = \bar{k} - \frac{\bar{\omega}}{\beta_z} \quad (17)$$

Moreover,

$$\frac{d\bar{x}}{d\bar{z}} = \frac{\bar{u}_x}{\bar{u}_z} \quad (18)$$

$$\frac{d\bar{y}}{d\bar{z}} = \frac{\bar{u}_y}{\bar{u}_z} \quad (19)$$

Are the differential equations governing the transverse electrons motion where

$$\bar{\Omega}_{0.w} = \frac{eB_{0,w}}{mc^2k_w},\ \delta a = \frac{e\delta A}{mc^2}, \delta\varphi = \frac{e\delta\phi}{mc^2},\ \bar{\omega}_b^2 = \frac{2\pi e^2 n_0}{mc^2k_w^2}$$

WAVES DYNAMICS

To complete the formulation, the equations which describe the evolution of waves should be considered. By selecting Coulomb gauge, Maxwell equations can be written as

$$\left(\frac{\partial^2}{\partial\bar{z}^2} - \frac{\partial^2}{\partial\bar{t}^2}\right)\delta\mathbf{a} = 4\pi\delta\mathbf{J}_\perp(z,t) \quad (20)$$

$$\frac{\partial^2}{\partial\bar{z}\partial\bar{t}}\delta\varphi(z,t) = -4\pi\delta J_z(z,t) \quad (21)$$

$$\frac{\partial^2}{\partial\bar{z}^2}\delta\phi(z,t) = -4\pi\delta\rho(z,t) \quad (22)$$

Where δJ is nonlinear current density which may be written as an average over entry time t_0[4],

$$\delta\mathbf{J}(z,t) = -n_0|e|v_{z0}\int_{-\infty}^{\infty}\frac{\sigma(t_0)\mathbf{v}(t_0,t)}{v_z(t_0,t)}\delta\left(t - \tau(t_0,t)\right)dt_0 \quad (23)$$

And $\delta\rho$ is the nonlinear charge density which is given by Eq. (22) when $\mathbf{v}_z$ is removed. By substitution Eq. (23) in Maxwell Eq. (20), a set of first order coupled nonlinear differential equations are derived for δa, k_+ and Γ_+, where Γ_+ is the growth rate of radiation. These equations are as follow: [5]

$$\frac{d\delta a}{dz} = \Gamma_+\delta a \quad (24)$$

$$\frac{d}{dz}\Gamma_+ = -\left(\frac{\omega^2}{c^2} + \Gamma_+^2 - k_+^2\right) + \frac{\omega_b^2}{c^2}\frac{\beta_{0z}}{\delta a}\left\langle\frac{v_x\cos\alpha_+ - v_y\sin\alpha_+}{|v_z|}\right\rangle \quad (25)$$

$$\frac{d}{dz}k_+ = -2k_+\Gamma_+ - \frac{\omega_b^2}{c^2}\frac{\beta_{0z}}{\delta a}\left\langle\frac{v_x\sin\alpha_+ + v_y\cos\alpha_+}{|v_z|}\right\rangle \quad (26)$$

Eq. (21) and Eq. (22) yield

ISBN 978-3-95450-123-6

$$\frac{d}{d\bar{z}}\delta\varphi = -2\frac{\bar{\omega}_b^{\,2}}{\bar{\omega}}\beta_{z0}\langle\sin\alpha\rangle \qquad (27)$$

$$k = -2\frac{\bar{\omega}_b^{\,2}}{\bar{\omega}\delta\varphi}\beta_{z0}\langle\cos\alpha\rangle \qquad (28)$$

Here, the average operator is defined over the initial ponderomotive phase $\psi_0 = \omega t_0$ as

$$\langle\langle\cdots\rangle\rangle \equiv \frac{1}{2\pi}\int_{-\pi}^{\pi} d\psi_0\sigma(\psi_0)(\cdots) \qquad (29)$$

Where ponderomotive phase is $\psi = \alpha_+ + k_w z$.

NUMERICAL PROCEDURE

Equation (13)-(19) together with equation (24)-(29) constitute a set of 7N+6 nonlinear and self consistent first order differential equations which governing the interaction evolution of the waves and electrons' motion and maybe solved numerically by the first order Runge-Kutta method where N is the number of electrons. It is the initial phase that appears in the averaging operator and discretizes the electrons. The integrations over the beam cross section are done with Simpson's method to discretizes electrons. The initial condition for phase distribution of electrons is $0 \le \psi_0 \le 2\pi$.The common parameters, which are used, correspond to a situation in which $\bar{\Omega}_0/\gamma = 2.6$, $\bar{\Omega}_w/\gamma = 0.05$, the relativistic factor $\gamma = 3.5$, and an entry taper of $N_w = 10$ wiggler periods. Wave numbers of the vector and scalar potential are chosen to satisfy the dispersion relation for a circularly polarized electromagnetic wave [5],

$$\bar{\omega}^2 - \bar{k}_+^2 - \frac{\bar{\omega}_b^{\,2}\left(\bar{\omega} - \bar{k}_+\beta_{z0}\right)}{\gamma_0\left(\bar{\omega} - \bar{\Omega}0/\gamma_0 - \bar{k}_+\beta_{z0}\right)} = 0 \qquad (30)$$

In the presence of axial magnetic field and the dispersion relation for the negative energy space-charge wave

$$\left(\bar{\omega} - \bar{k}\beta_{z0}\right) = -\kappa_b\beta_{z0} \qquad (31)$$

Since the steady-state amplifier model is considered, the initial amplitude of the vector potential can be selected arbitrarily to represent the amplitude of the injected signal which is selected to be $\delta a(\bar{z}=0) = 10^{-7}$ and the initial value of the growth rate (logarithmic derivative), is chosen to be zero at the entrance to the wiggler, as mentioned in [2]. However, the scalar potential must be found from Eq. (28).

In Fig. 2, the evolution of radiation amplitude is plotted along wiggler length. With the specific values we considered for this problem, saturation takes place around $\bar{z} = 21.2$ and saturation amplitude is $\delta a = 0.0107$ in this case.

In continue, the evolution of radiation amplitude is compared for dipole and quadrupole wigglers. In Fig. 3, radiation amplitude is plotted for dipole wiggler (dashed line) and quadrupole wiggler (solid line) for normalized frequency $\bar{\omega} = 12.1$ corresponding to $\lambda \approx 3mm$. It is seen that saturation takes place at $\bar{z} = 48.05$ with $\delta a_{sat} = 0.00349$ for dipole wiggler and $\bar{z} = 35.8$ with $\delta a_{sat} = 0.00282$ for quadrupole wiggler. For this case, using dipole wiggler is more effective to increase saturation amplitude.

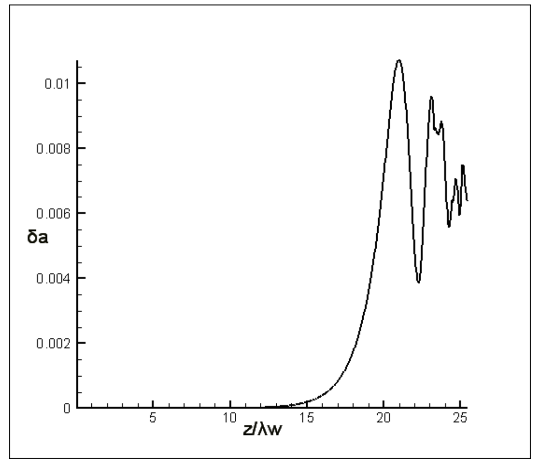

Figure 2: Evolution of the radiation amplitude vs. axial position for a quadrupole wiggler for $\bar{\Omega}_0/\gamma = 2.6$, $\gamma = 3.5$ $\bar{\Omega}_w/\gamma = 0.05$.

In Fig.4, the evolution of radiation amplitude along $\bar{z}$ direction is plotted for normalized frequency $\bar{\omega} = 6.9$ for dipole (dashed line) and quadrupole (solid line) wigglers.

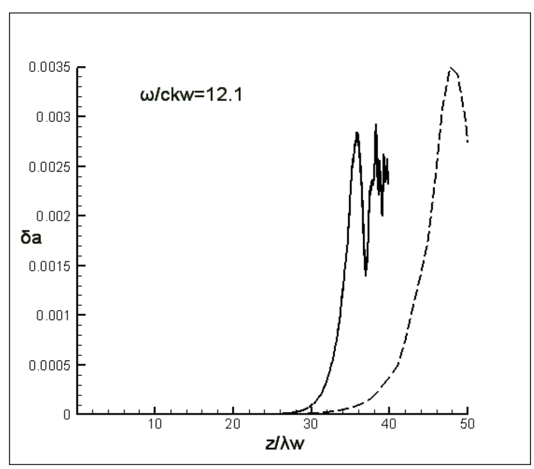

Figure 3: Evolution of the radiation amplitude vs. axial position for a quadrupole wiggler (solid line), and dipole wiggler (dashed line) for $\bar{\Omega}_0/\gamma = 2.6$, $\bar{\Omega}_w/\gamma = 0.05$,$\gamma = 3.5$. $\bar{\omega}$=12.1

ISBN 978-3-95450-123-6

In this case, the saturation point is at $\bar{z} = 42.1$ for dipole wiggler where $\delta a_{sat} = 0.0058$, and for quadrupole wiggler, saturation takes place at $\bar{z} = 21.2$ correspond to $\delta a_{sat} = 0.0107$. As it is seen, quadrupole wiggler is more effective to increase saturation amplitude in this case.

But according to the results of two different frequencies, quadrupole wiggler has reduced saturation length in both cases.

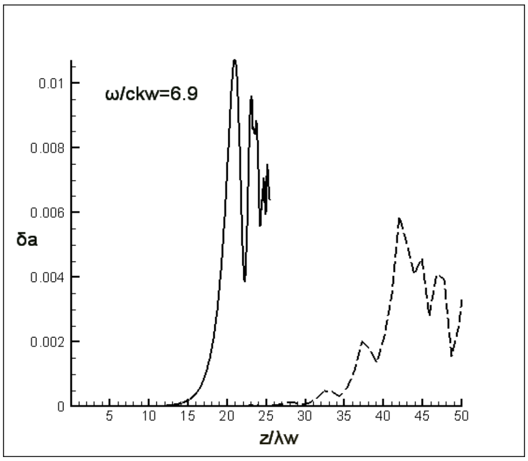

Figure 4: Evolution of the radiation amplitude vs. axial position for a quadrupole wiggler (solid line), and dipole wiggler (dashed line) for $\overline{\Omega_0} / \gamma = 2.6$, $\overline{\Omega} w / \gamma = 0.05$, $\gamma = 3.5$ $\bar{\omega}$=6.9

CONCLUSION

A new nonlinear configuration to describe the evolution of radiation amplitude for a helical quadrupole wiggler FEL in the presence of space-charge effect is investigated at millimeter wavelengths. The results showed that by applying a quadrupole wiggler, the saturation length may be decreased. It is also found out that this kind of wiggler is suitable for longer wave lengths in comparing with dipole wiggler and may extremely increase the saturation amplitude. Three dimensional formulation and shorter wavelengths will be considered in future studies.

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ISBN 978-3-95450-123-6