THE GENERATOR OF HIGH-POWER SHORT TERAHERTZ PULSES

N. Vinokurov#, Budker INP, Novosibirsk, Russia, and KAERI, Daejeon, S. Korea
Y. U. Jeong KAERI, Daejeon, S. Korea

Abstract

The multi-foil cone radiator to generate high field short terahertz pulses with the short electron bunches is described. A round flat conducting foil plates with successively decreasing radius are stacked, comprising a truncated cone with axis *z*. The gaps between foils are equal and filled by some dielectric (it may be vacuum). A short relativistic electron bunch propagates along the *z* axis. At high enough particle energy the energy losses and multiple scattering do not change the bunch shape significantly. Then, passing through each gap between foils, the bunch emits some energy into the gap. After that the radiation pulses propagate radially. For the TEM-like waves with longitudinal (along the *z* axis) electric and azimuthal magnetic field there is no dispersion in these radial lines, therefore the radiation pulses conserve their shapes (time dependence). At the cone outer surface we have synchronous circular radiators. Their radiation field forms the conical wave. The cone angle may be optimized; moreover, the nonlinear dependence of the foil plates radii on their longitudinal coordinate z may be used for the wave front shape control.

INTRODUCTION

The high field short terahertz pulses may be interesting for different applications [1, 2]. The multi-foil cone radiator to generate them using the short electron bunches is described in this proposal. The scheme under consideration is shown in Fig. 1. A round flat conducting foil plates with successively decreasing radius are stacked, comprising a truncated cone with axis *z*. The gaps between foils are equal and filled by some dielectric (it may be vacuum). A short relativistic electron bunch propagates along the *z* axis from left to right. At high enough particle energy the energy losses and multiple scattering do not change the bunch shape significantly. Then, passing through each gap between foils, the bunch emits some energy into the gap. After that the radiation pulses propagates radially, as it is shown in Fig. 1a. For the TEM-like waves with longitudinal (along the *z* axis) electric and azimuthal magnetic field there is no dispersion in these radial lines, therefore the radiation pulses almost conserve their shape (time dependence). At the cone outer surface we have synchronized circular radiators. Their radiation field forms the conical wave (see Fig. 1b).

THE SINGLE GAP EXCITATION

Let us find the radiation field in one gap. We are interested only in TEM waves, having only the longitudinal electric E_z and the azimuthal magnetic H_α fields, which do not depend on *z*. The Maxwell equations for such waves are

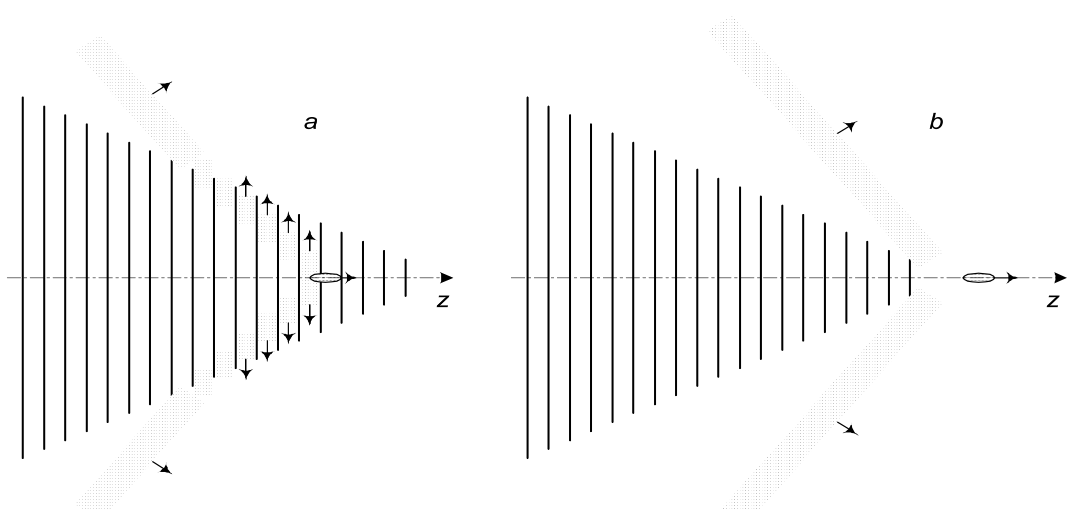

Figure 1: Short electron bunch passes through the conical foil stack. a – after the bunch passed a gap, the wave propagates radially. b – as the pulses reaches the foil boundaries, they are combined to a conical wave.

#vinokurov@inp.nsk.su

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$$\begin{aligned} &\frac{1}{r}\frac{\partial}{\partial r}\left(rH_\alpha\right) = \frac{4\pi}{c} j_{av} + \frac{n^2}{c}\frac{\partial}{\partial t}E_z \\ &\frac{\partial}{\partial r}E_z = \frac{1}{c}\frac{\partial}{\partial t}H_\alpha \end{aligned}, \tag{1}$$

where $j_{av} = \int_{-g/2}^{g/2} j_z dz \Big/ g$ is the beam current density, averaged over the gap length g, n is the refraction index, and round beam is assumed. The solution for the electric field Fourier transform outside the beam is

$$E_\omega = -\frac{\pi Q\omega}{c^2} F_\omega H_0^{(1)}(kr) \,. \tag{2}$$

where $Q = \int_{-\infty}^{\infty}\int_0^{\infty} j_z 2\pi r dr dt$ is the bunch charge,

$$F_\omega = \frac{2c}{\omega g}\sin\left(\frac{\omega g}{2c}\right)\frac{2\pi}{Q}\int_0^\infty j_\omega(r) J_0(kr) r dr \tag{3}$$

is the form factor, $k = n\omega/c$, the Hankel function $H_0^{(1)}(kr)$ $= J_0(kr) + iY_0(kr)$ is the combination of Bessel and Neumann functions. Here and below we suppose the bunch velocity to be equal to the light velocity c. The form factor modulus is less than one, and at zero frequency it is one. For the Gaussian charge distribution

$$j_z = \frac{Qc}{(2\pi)^{3/2} a^2 l} e^{-\frac{r^2}{2a^2} - \frac{1}{2l^2}(z-ct)^2} \tag{4}$$

$$F_\omega = \frac{2c\sin\left(\dfrac{\omega g}{2c}\right)}{\omega g} e^{-\frac{\omega^2}{2}\left(\frac{l^2}{v^2}+\frac{n^2a^2}{c^2}\right)} \approx 1 - \frac{\omega^2 l_{eff}^2}{2c^2} \,. \tag{5}$$

where $l_{eff} = \sqrt{l^2 + n^2a^2 + g^2/12}$. The corresponding time dependence at $kr >> 1$ is

$$\begin{aligned} E(r,t) &= -\frac{2\pi Q}{c^2}\mathrm{Re}\int_0^\infty F_\omega H_0^{(1)}(kr) e^{-i\omega t}\omega\frac{d\omega}{2\pi} \approx \\ &-\frac{Q\sqrt{2}}{c^{3/2}\sqrt{\pi n r}}\mathrm{Re}\int_0^\infty F_\omega e^{-i\omega\left(t-\frac{nr}{c}\right)-i\frac{\pi}{4}}\sqrt{\omega}d\omega = \\ &-\sqrt{\frac{2}{nrc^3}}\int_0^\infty \dot{I}_{eff}\left(t-\frac{nr}{c}-\tau\right)\frac{d\tau}{\sqrt{\tau}} \end{aligned}, \tag{6}$$

where I_{eff}/Q is the inverse Fourier transform of the form factor F_ω. For $l >> \sqrt{n^2a^2 + g^2/12}$ I_{eff} is the beam current.

According to Eq. (2) the radiation spectral density is

$$2\pi\frac{dW}{d\omega} = -g2\pi r\frac{c}{2\pi}\mathrm{Re}E_\omega^* H_\omega = g\frac{2\pi Q^2\omega}{c^2}\left|F_\omega\right|^2 \,. \tag{7}$$

Then the total radiated energy is

$$W = g\frac{Q^2}{c^2}\int_0^\infty \left|F_\omega\right|^2 \omega d\omega \,. \tag{8}$$

For the Gaussian bunch it gives

$$W \approx g\frac{Q^2}{2l_{eff}^2} \,. \tag{9}$$

The corresponding effective average decelerating field is

$$\left\langle E_z\right\rangle = \frac{W}{Qg} \approx \frac{Q}{2l_{eff}^2} \,. \tag{10}$$

For Q = 0.5 nC and l_{eff} = 0.1 mm it is about 2 MV/cm. Then for the cone height L = 2 cm the radiated energy is 2 mJ. To have the transverse size smaller, than the bunch length, the beam emittance has to be less than l_{eff}^2/L = $5\cdot 10^{-7}$ m.

The radiated energy may be compared with the radiated energy of the coherent transition radiation (for a narrow beam, $a << l$)

$$W_{CTR} \approx \frac{Q^2}{l\sqrt{\pi}}\ln\frac{r_{\max}}{a} \,, \tag{11}$$

where $r_{\max}$ depends on geometry and electron energy. The ratio of these energies

$$\frac{W}{W_{CTR}} \approx \frac{L}{l}\frac{\sqrt{\pi}}{2\ln\left(r_{\max}/a\right)} \tag{12}$$

can be large.

For $l >> \sqrt{n^2a^2 + g^2/12}$ the radiation field of the Gaussian bunch is

$$E(r,t) = \frac{-Q}{\sqrt{\pi n r l^3}}\int_0^\infty\left(\frac{ct-nr}{l}-s\right)e^{-\frac{1}{2}\left(\frac{ct-nr}{l}-s\right)^2}\frac{ds}{\sqrt{s}} \,. \tag{13}$$

The function $\sqrt{\frac{2}{\pi}}\int_0^\infty (x-s)e^{-\frac{1}{2}(x-s)^2}\frac{ds}{\sqrt{s}}$, which describes the field time dependence in the units of the electron bunch r. m. s. duration, is shown in Fig. 2 together with the Gaussian bunch shape.

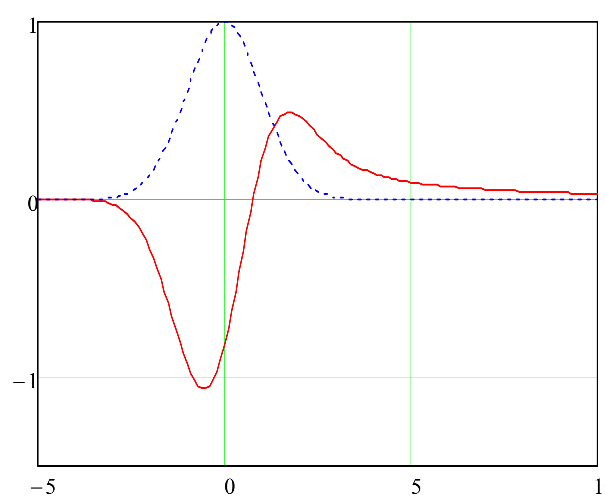

Figure 2: The function $\sqrt{\frac{2}{\pi}}\int_0^\infty (x-s)e^{-\frac{1}{2}(x-s)^2}\frac{ds}{\sqrt{s}}$ (solid line) and Gaussian exponent $\exp(-x^2/2)$ (dashed line).

The maximum field is

$$\left|E\right|_{\max} \approx \frac{Q}{\sqrt{2nrl^3}} \,, \tag{14}$$

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and the corresponding peak power is

$$P_{\max} = \frac{cn}{4\pi}|E|^2_{\max} 2\pi r L \approx \frac{cQ^2}{4l^3}L. \quad (15)$$

For our example it is 0.4 GW.

THE REFRACTION AT THE CONE BOUNDARY

Let us consider the wave refraction at the cone boundary; the cone angle is α (see Fig. 3).

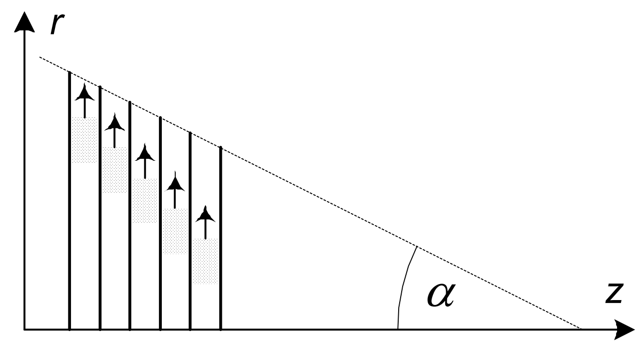

Figure 3: The refraction geometry.

Let the gaps are much smaller than the pulse length. In this case the foils form the anisotropic media with the diagonal permittivity tensor $\varepsilon = \mathrm{diag}(i\infty, i\infty, n^2)$, and the radiation may be considered as the Cherenkov radiation in it. Then inside the cone there are the waves $\exp(ikr + i\omega z/c - i\omega t)$. The tangent components of the wave vectors of this wave and the wave in the free space have to coincide at the boundary

$$\frac{\omega}{c}\cos(\theta+\alpha) = -\frac{n\omega}{c}\sin\alpha + \frac{\omega}{c}\cos\alpha,$$

where

$$\theta = \mathrm{acos}(\cos\alpha - n\sin\alpha) - \alpha \quad (16)$$

is the angle between the wave vector of radiation in free space and the z axis. The necessary condition for radiation $\tan(\alpha/2) < 1/n$ follows from Eq. (16).

The reflection coefficient

$$R = \left(\frac{E_-}{E_+}\right)^2 = \left[\frac{n\sin(\theta+\alpha) - \cos\alpha}{n\sin(\theta+\alpha)+\cos\alpha}\right]^2 = \left[\frac{n\sqrt{(1-n^2)\tan^2\alpha + 2n\tan\alpha} - 1}{n\sqrt{(1-n^2)\tan^2\alpha + 2n\tan\alpha} + 1}\right]^2 \quad (17)$$

can be found from the boundary conditions

$$H = H_+ + H_-$$
$$E\sin(\theta+\alpha) = (E_+ + E_-)\cos\alpha \quad (18)$$
$$H_+ = -nE_+, \quad H_- = nE_-, \quad H = -E$$

for incident (E_+ and H_+), reflected (E_- and H_-) and transmitted (E and H) waves.

There is no reflection for

$$\alpha_0 = \mathrm{atan}\frac{n \pm \sqrt{n^2 - 1 + 1/n^2}}{n^2-1}. \quad (19)$$

For $n = 1$ $\alpha_0 = \mathrm{atan}(1/2) \approx 27^\circ$, and $R < 0.1$ for $10° < \alpha < 60°$. It allows using different angles and not only cones, but other revolution surfaces to control the wave front shape.

THE WAVE ATTENUATION

It needs also to take into account the attenuation due to the finite surface impedance ζ. According to the Leontovich boundary condition the radial electric field at the foil surface is ζH_α. Then the Poynting vector amplitude is $c|H_\alpha|^2\mathrm{Re}\zeta/(4\pi)$, and the length of the e-time power attenuation is

$$\Delta r = cg\frac{|H_\alpha|^2}{4\pi n} \Big/ \frac{2c\,\mathrm{Re}\zeta |H_\alpha|^2}{4\pi} = \frac{g}{2n\,\mathrm{Re}\zeta}. \quad (20)$$

The known normal-incidence absorption coefficients 4Reζ in the THz range are typically less than one percent, but for small gaps the attenuation may be significant. Therefore it needs to choose the cone angle to be less than the value for zero reflection, given by Eq. (19).

THE MULTIPLE SCATTERING

The multiple scattering of electrons on the atomic nuclei of foils (here we suppose the absence of matter between foils) increases the angle spread of electrons [3]:

$$\frac{d}{dz}\langle x'^2\rangle = \frac{1}{X_0}\left(\frac{13.6MeV}{E}\right)^2, \quad (21)$$

where X_0 is the radiation length of the foil cone material, E is the particle energy.

The corresponding growth of the beam transverse size a is

$$a^2 = a_0^2(z) + \frac{z^3}{3}\frac{d}{dz}\langle x'^2\rangle, \quad (22)$$

where a_0 is the size without multiple scattering. According to Eq. (5) the transverse size has to be less than the bunch length. Therefore

$$\frac{d}{dz}\langle x'^2\rangle < 3\frac{l^2}{L^3}. \quad (23)$$

It may be expressed as the limitation for the electron energy

$$E > 13.6\,MeV\frac{L^{3/2}}{l\sqrt{3X_0}}. \quad (24)$$

The radiation length of graphite is about 0.2 m. Stacking the foils with thickness 10 micron and period 0.2 mm, one obtain the radiation length 4 m. Then for our example the minimum energy is 110 MeV. To decrease the minimum energy one can decrease the cone height (and radiated power). For example, for L = 1 cm it is 40 MeV and the radiated power is about 0.2 GW.

For high peak currents the focusing by the beam azimuthal magnetic field may reduce the beam size

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growth, given by Eq. (22). It makes the energy limitation Eq. (24) easier.

The small holes in the foils can eliminate the multiple scattering. In this case one has to substitute the hole radius divided by $\sqrt{2}$ instead of the r. m. s. transverse beam size a to the effective bunch length l_{eff}.

THE ON-AXIS FIELD

The on-axis field can be found from Eq. (1) also. It is

$$E_\omega(0) = -\frac{2\pi^2\omega}{c^2}\int_0^\infty j_\omega(r_1)H_0^{(1)}(kr_1)r_1 dr_1 = -ZI_\omega . \quad (25)$$

For Gaussian beam and low ($\omega\sqrt{n^2a^2 + g^2/12} << 1$) frequencies

$$Z \approx \frac{\pi|\omega|}{c^2} + \frac{i\omega}{c^2}\left[\ln\left(\frac{\omega^2 n^2 a^2}{2c^2}\right) + 0.577\right]. \quad (26)$$

The real part of this impedance gives the same loss as Eq. (7). The imaginary part is almost inductive. This impedance may cause an additional bunching of the beam. Then a higher peak power may be achieved.

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REFERENCES

[1] K. Reimann. *Rep. Progr. Phys.* **70** 1597 (2007).
[2] M. Tonouchi. *Nature Photon.* **1** 97 (2007).
[3] D. H. Perkins. Introduction to high energy physics. Addison-Wesley, 1987.

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