NUMERICAL STUDY OF AN FEL BASED ON LWFA ELECTRONS AND A LASER-PLASMA WIGGLER *

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Abstract

Recent works [1] have suggested that laser-wakefield acceleration (LWFA) may be used to produce the electron beam of an FEL, thereby considerably reducing the size and cost of the device. However, when using conventional magnetic wigglers, the requirements on the beam quality are very stringent, and are still challenging with current LWFA beams.

An interesting alternative may be to use a laser-plasma wiggler (e.g. a plasma wave or a laser beam). Compared to a conventional wiggler, a laser-plasma wiggler has a field amplitude several orders of magnitude higher, as well as a correspondingly shorter wavelength - which may place lower constraints on the beam quality and eliminate the need for transport. Taking into account beam quality, beam transport and wiggler inhomogeneity, we evaluate the range of wiggler properties (field, wavelength) that would make the FEL radiation possible.

From this analysis, the counterpropagating laser wiggler [2] seems to be one of the most promising solutions. We therefore extend the widely-used Ming Xie formula [3] (which was derived for a static, magnetic wiggler) to a counterpropagating laser wiggler. We use this formula to evaluate the potential use of current state-of-the-art lasers.

INTRODUCTION

Over the past ten years, laser-wakefield acceleration (LWFA) has proven to be a successful solution to accelerate electrons to hundreds of MeVs over distances of a few millimeters, which is more than a thousand times shorter than the distance required with a conventional wiggler. As a consequence, replacing the accelerator of an FEL with a laser-wakefield accelerator could drastically reduce the size and cost of the device. Yet, LWFA electrons typically have larger energy spread and divergence than those from a conventional accelerator, so that coupling them to a conventional magnetic wiggler is still challenging. This is due partly to the tight requirements on the beam quality and partly to the difficulty to transport the divergent beam from the accelerator to the wiggler.

Alternatively, the LWFA electrons could be coupled to a wiggler generated by laser-plasma interaction. Previous studies proposed for instance to use a plasma wave [4] or a laser pulse [2] as a wiggler. Such a wiggler has several advantages. First, it can be placed, by proper synchronization, only a few microns behind the accelerator and thereby eliminate the need for transport. Second, its strong field (laser pulses can have magnetic fields in the kiloTesla range) and short wavelength (in the micron range) may place looser constraints on the beam quality. Finally, a laser-plasma accelerator coupled to a laser-plasma wiggler would only be a few millimeters long (without taking into account the size of the laser amplifiers) and therefore extremely compact.

This paper is organized in two sections. In the first one, we evaluate the range of wiggler period and wiggler field strength that would make the FEL mechanism possible, from the point of view of the Ming Xie formula [3]. This section is general and applies to any kind of laser-plasma wiggler. Then, in the second section, we evaluate in more depth the potential of a particular wiggler : that consisting of a CO_2 laser pulse. Further degrading effects like expansion of the electron bunch and longitudinal wiggler non-uniformity – that were not considered in the first section – are taken into account.

RANGE OF PARAMETERS IN WHICH THE FEL MECHANISM IS POSSIBLE

We consider either a static magnetic wiggler or a counterpropagating laser wiggler. In the case of the static wiggler, the fields are of the form

$$\mathbf{E}_{static} = \mathbf{0} \quad (1)$$
$$\mathbf{B}_{static} = a_w \frac{mck_w}{e}(\sin(k_w z)\mathbf{e_x} + \cos(k_w z)\mathbf{e_y}) \quad (2)$$

where a_w is the dimentionless wiggling parameter, c is the speed of light, m and e are the mass and charge of an electron, $k_w = 2\pi/\lambda_w$ with λ_w corresponding to the wiggler period, and where the electron beam propagates along the z-axis in the positive direction. In the case of a laser wiggler

$$\mathbf{E}_{laser} = a_w \frac{mc^2k_w}{2e}\left(-\cos\left(\frac{k_w(z+ct)}{2}\right)\mathbf{e_x} + \sin\left(\frac{k_w(z+ct)}{2}\right)\mathbf{e_y}\right) \quad (3)$$

$$\mathbf{B}_{laser} = a_w \frac{mck_w}{2e}\left(\sin\left(\frac{k_w(z+ct)}{2}\right)\mathbf{e_x} + \cos\left(\frac{k_w(z+ct)}{2}\right)\mathbf{e_y}\right) \quad (4)$$

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Notice that, if these fields correspond to those of a laser pulse, the wavelength of the laser is $\lambda_{laser} = 2\pi/(k_w/2) = 2\lambda_w$. It is important to remark that, with this notation, *the FEL equations*[1] *are the same for these two types of wiggler*. As pointed out in [5], the simple reason for this analogy is that the Lorentz force $\mathbf{E}+\mathbf{v}\times\mathbf{B}$ felt by relativistic electrons ($z \approx ct$) has the same expression in both cases. Thus, a laser wiggler is equivalent to a static magnetic wiggler with a period $\lambda_w = \lambda_{laser}/2$. In the rest of this article, we will take advantage of this fact in order to use results which are known for static wigglers and extend them to laser wigglers.

In particular, in both cases, the wavelength of the emitted radiation is

$$\lambda_r = \frac{(1+a_w^2)\lambda_w}{2\gamma^2} \tag{5}$$

and the FEL mechanism is characterized by the Pierce parameter

$$\rho = \left(\frac{I}{32\pi^2 I_A \sigma_x^2} \frac{a_w^2 \lambda_w^2}{\gamma^3} \right)^{1/3} \tag{6}$$

where γ is the Lorentz factor of the electrons, I the current of the electron bunch and σ_x is its transverse rms size, and $I_A = 4\pi\epsilon_0 mc^3/e$ is the Alfvén current.

Parameters of the Electron Bunch

According to equation (6), using a micrometer-scale wiggler period λ_w (as is the case for a laser-plasma wiggler) instead of a centimeter-scale period (conventional wiggler) leads to a lower value of ρ and thus to tighter requirements on the electron bunch. Yet, equation (5) indicates that, with a laser-plasma wiggler, a given value of λ_r corresponds to a lower value of γ than with a conventional wiggler. This partly compensates the effect of λ_w in the Pierce parameter. In this article, we choose $\gamma = 20$ (E = 10 MeV), as a reasonable compromise between lower wavelength and higher Pierce parameter.

The other parameters of the electron bunch are summarized in table 1. The value of the transverse rms size σ_x is typical for LWFA electron bunchs. While the values of the current I, the normalized emittance $\epsilon_{n,\perp}$ and the energy spread $\Delta\gamma$ may seem optimistic – since no such beam has been experimentally observed in this energy range – they are nonetheless consistent with some numerical simulations [2, 6]. It should also be noticed that current experiments mostly focus on higher electron energies (hundreds of MeV and more) and that a precise measurement of the emittance of LWFA electrons is still very challenging.

Table 1: Parameters of the Electron Bunch

γ	$\mathbf{\Delta\gamma}$	$\epsilon_{\mathbf{n},\perp}$	**I**	$\sigma_{\mathbf{x}}$
20	0.2	0.3 mm.mrad	10 kA	1 μm

[1] i.e. the motion equation for the electrons and the propagation equation for the emitted radiation

Parameters of the Wiggler

Although they are low here, the emittance and energy spread of the electrons will still degrade the FEL mechanism. Moreover, the small transverse rms size of the bunch may lead to degradation through diffraction. Yet the extent to which the FEL mechanism is affected depends on the values of a_w and λ_w, and may be evaluated using the Ming Xie formula :

$$L_{1D}/L_g = \frac{1}{1+\Lambda} \tag{7}$$

where L_{1D} is the ideal 1D gain length and L_g is the actual gain length. Λ quantifies the degradation of the FEL mechanism and is a growing function of $\Delta\gamma$, $\epsilon_{n,\perp}$ and $1/\sigma_x$.

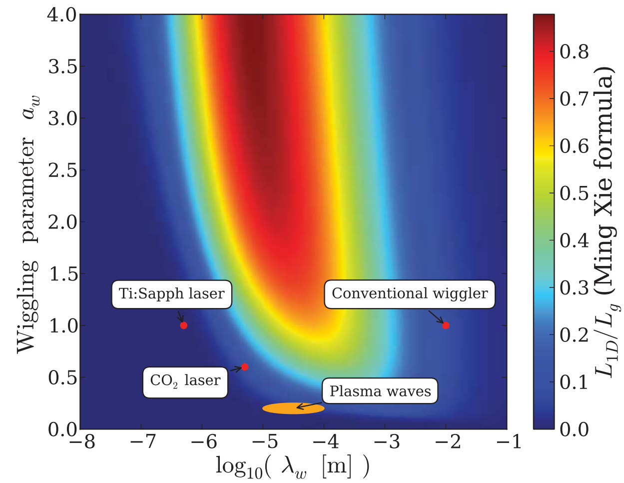

Figure 1: Plot of L_{1D}/L_g (as given by the Ming Xie formula) as a function of the wiggling parameter a_w and the wiggler period λ_w.

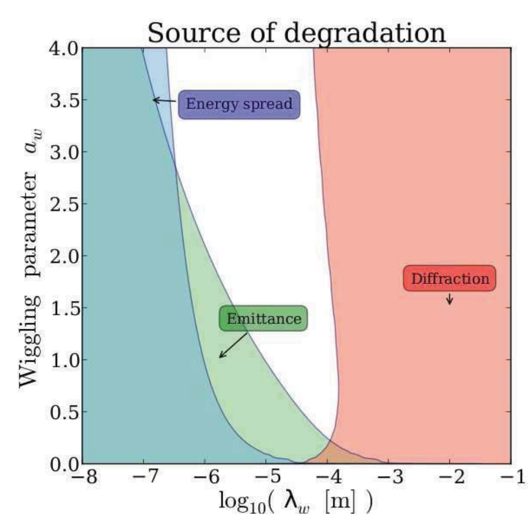

Figure 2: Range of parameters (in terms of wiggling parameter a_w and wiggler period λ_w) in which each source of degradation becomes important.

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Figure 1 is a plot of L_{1D}/L_g and shows in which range of parameter the FEL mechanism is least degraded. The positions of several possible wigglers have also been added to the plot, based on their typical values for λ_w and a_w, and using the relation $\lambda_w = \lambda_{laser}/2$ for laser wigglers. Figure 2 specifies the range of parameters in which each source of degradation (finite emittance, finite energy spread, diffraction) starts having a strong influence on the FEL mechanism.

Figure 1 and 2 show that, for existing potential wigglers (laser pulses or plasma waves), the FEL process is considerably degraded, and that this is mainly due to the finite beam emittance. Going towards higher wiggling parameters (e.g. increasing the intensity of the lasers) would clearly improve the situation (mainly because it increases the Pierce parameter ρ). Also, these figures show that a strong emphasis should be put on the improvement of the emittance of the beam, while keeping the energy spread low.

Still, with the parameters considered here and a CO_2 laser wiggler, the FEL process appears to be possible, athough it is degraded. We will consequently study this type of wiggler in more depth in the rest of the article.

THE POTENTIAL OF THE CO_2 LASER WIGGLER

FEL Properties

We consider a CO_2 laser ($\lambda_{laser} = 10\,\mu m$) of the kind of UCLA's "MARS" laser or BNL's CO_2 laser. Although their pulse durations are different (170 ps for MARS and 5 ps for BNL's CO_2 laser), both laser can be focused to a waist $w_0 \simeq 80\,\mu m$ over a Rayleigh length of 2 mm, reaching $a_w \simeq 0.6$ at the focal spot. The wavelength of the corresponding FEL radiation is 8.5 nm (146 eV), and the Ming Xie formula predicts a gain length of 150 μm.

While the effects of emittance and energy spread are properly taken into account by the Ming Xie formula, two more degrading effects which are not described by this formula may affect the FEL mechanism here. First, contrary to a conventional wiggler, a laser wiggler does not incorporate any structure that refocuses the electron bunch. As a consequence, the electron bunch will expand radially and the corresponding Pierce parameter will quickly decrease. Second, the wiggler field is not spatially uniform. The transverse non-uniformity of the wiggler is negligible since its waist (80 μm) is much larger than the bunch initial transverse size (1 μm), but its longitudinal non-uniformity may be important.

In order to assess the consequences of these two effects, we performed GENESIS simulations[2]. These simulations also allow us to obtain realistic results which consistently take into account the finite duration of the electron bunch and the saturation of the FEL. The simulations were run with a gaussian electron bunch whose parameters are summarized in table 2. For this gaussian bunch, the peak current is 12 kA, which is close to the current considered in the previous section (10 kA).

[2]Since GENESIS only features static wigglers, the simulations were performed on the equivalent static wiggler having $\lambda_w = 5\,\mu m$ and $a_w = 0.6$.

Table 2: Parameters of the electron bunch used in the time-dependent GENESIS simulations. Q and τ represent the total charge of the bunch and its duration, and their values are representative of those obtained for LWFA electrons with controled injection.

γ	$\Delta\gamma$	$\epsilon_{n,\perp}$	σ_x	Q	τ
20	0.2	0.3 mm.mrad	1 μm	100 pC	3 fs

Degradation due to the Expansion of the Bunch

Figure 3 shows the results of a simulation for which the electron bunch freely expands while going through a *uniform* wiggler. For comparison, figure 4 shows a simulation with the same parameters, but in which the bunch is periodically refocused, and keeps a small transverse size $\sigma_x \simeq 1\mu m$.

From these results, it appears that the expansion of the bunch drastically affects the FEL process : after 1.5 mm of propagation through the wiggler, FEL emission virtually stops. However, even in this case, the power of the emitted radiation remains substantial (100 MW peak power). Also, in the case of an expanding beam, the duration of the radiation is very short (2 fs rms duration vs. about 10 fs in the case of a periodically refocused beam), which may be an advantage for some applications.

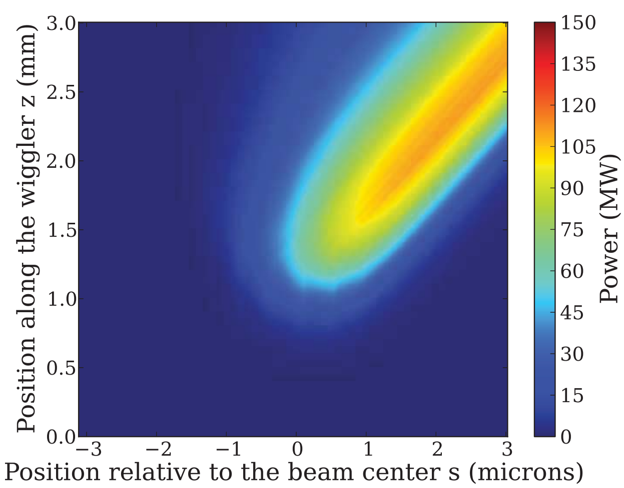

Figure 3: Radiation power along the wiggler in the case of a freely expanding electron bunch

Degradation due to the Non-uniformity of the Wiggler

The last effect to take into account is the non-uniformity of the wiggler. As the electron bunch travels along the laser

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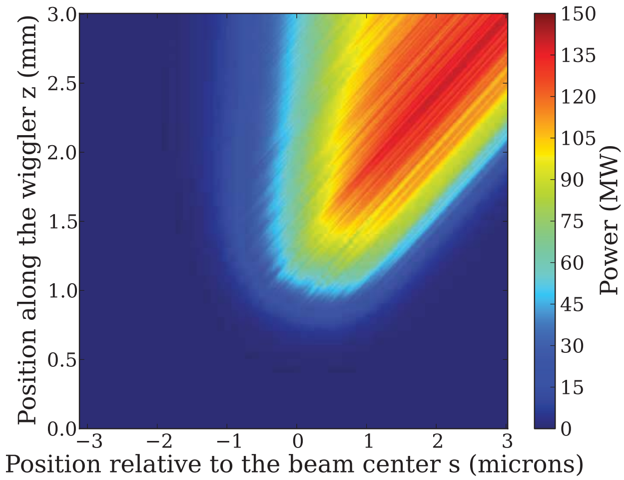

Figure 4: Radiation power along the wiggler in the case of a periodically refocused electron bunch

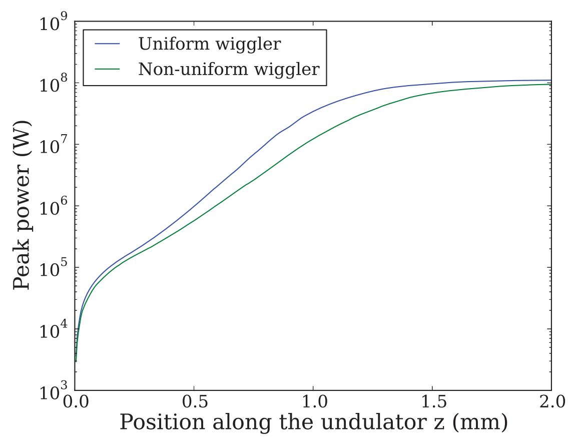

Figure 5: Evolution of the peak power of the FEL radiation along the wiggler for a uniform wiggler and non-uniform wiggler obeying equation (8) with $Z_R = 2\,\text{mm}$.

wiggler, the wiggling parameter a_w that it feels changes, due to two phenomena. First, the laser wiggler is a pulse with a finite duration and therefore its longitudinal profile is not uniform. Second, this pulse is strongly focused in order to reach high intensities. Yet, due to diffraction, the intensity drops before and after the focal plane, on a lengthscale given by the Rayleigh length.

While both effects are important in the case of BNL's CO_2 laser, the effect of diffraction dominates in the case of the "MARS" laser (the Rayleigh length is 2 mm long while the length of the laser pulse is about 50 mm). In the rest of this article, we will focus on this second case. Thus the expression of the wigging parameter is :

$$a_w(z) = \frac{a_{w,0}}{\sqrt{1 + (z - z_{foc})^2 / Z_R^2}} \tag{8}$$

where z is the position along the wiggler, z_{foc} is the position of the laser focal spot and Z_R is the Rayleigh length.

Figure 5 compares the emitted power for a uniform wiggler and for a wiggler obeying equation (8) with $Z_R = 2\,\text{mm}$. Both simulations were run with a freely expanding bunch. It appears that the non-uniformity has a minor impact in this case : it makes the gain length somewhat longer but keeps the final peak power almost unchanged.

To conclude with this section, in the case of the non-uniform wiggler, the output FEL radiation has a total energy of 1.2 μJ at 8.5 nm (5×10^{10} photons). Its rms duration is 2 fs and its peak power reaches 100 MW.

CONCLUSION

In this numerical study, we investigated the idea of an FEL based on a laser-plasma accelerator and a laser-plasma wiggler (e.g. a counterpropagating laser pulse or a plasma wave). Using the Ming Xie formula, we evaluated the range of parameter that would make the FEL process possible, and we compared it to existing potential wigglers. We then studied the case of a CO_2 laser wiggler in more detail, using GENESIS simulations and the analogy between a laser wiggler and a static wiggler.

This study shows that the idea of such a laser-plasma wiggler seems realistic. This scheme requires a challenging bunch quality ($\Delta\gamma/\gamma = 0.01$ and $\epsilon_{n,\perp} = 0.3$ mm.mrad for electrons at 10 MeV), which has not yet been observed in experiments. However, in the light of some simulations [2, 6], such a bunch quality does not seem beyond reach.

Provided that such a bunch can be produced, already existing CO_2 lasers (UCLA's "MARS" laser for instance) may be used as wigglers to successfully produce FEL radiation at 8.5 nm wavelength with a peak power of the order of 100 MW and a duration of the order of 2 fs. Importantly, in obtaining these results, we consistently took into account the main potential sources of degradation : beam emittance, beam energy spread, radiation diffraction, beam expansion, and wiggler non-uniformity.

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