PROBING TRANSVERSE COHERENCE WITH THE HETERODYNE SPECKLE APPROACH: OVERVIEW AND DETAILS

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Abstract

Spatial Coherence properties of radiation produced by accelerated relativistic electrons are far from being trivial. The correct assessment of coherence of High-Brilliance X sources (3rd generation Synchrotron or FEL) is of crucial importance both in machine diagnostics and in experiment planning, in the case coherent techniques are used. Classical methods (Young's interferometer) provides a mild knowledge of the spatial coherence.

The Heterodyne Speckle Approach [1],[2] is a valuable alternative that exploits the statistical analysis of light scattered by spherical particles. The technique needs a very essential setup composed only by a water suspension of commercial colloidal particles and a CCD camera. Coherence information are retrieved from the Fourier analysis of the interference pattern generated by the stochastic superposition of the waves scattered by the particles and the unperturbed transmitted beam (heterodyne configuration). The technique a) provides a direct measure of transverse coherence without a-priori assumptions, b) provides full 2D coherence map with single-distance measures, c) has been proved to be capable of time-resolved measures with SR sources (ID06, ESRF), d) is potentially scalable over a wide range of wavelengths (tested 400nm, 0.1nm). It has been used for coherence measures both at the usage point and at the front-end of an undulator source (ID02-ESRF, Grenoble).

INTRODUCTION

Transverse coherence measurement are commonly performed looking at the quality of interference fringes of some kind of ad-hoc engineered device, being Young's interferometer the progenitor of these kind of techniques. The visibility of a given fringe system is proportional to the modulus of the so called Complex Coherence Function, defined as

$$\mu(\vec{r},\Delta\vec{r}) = \frac{\left\langle e_0(\vec{r})e_0^*(\vec{r}+\Delta\vec{r})\right\rangle}{\sqrt{\left\langle \left|e_0(\vec{r})\right|^2\right\rangle\left\langle \left|e_0^*(\vec{r}+\Delta\vec{r})\right|^2\right\rangle}} \qquad (1)$$

Here the brackets denote the ensemble average over many radiation pulses, since we are dealing with radiation produced by relativistic bunched electrons. In the case of natural thermal sources, like stars, fire and so on, brackets usually denote the time average, so that $\mu(\vec{r_0},\vec{r_0}) = I(\vec{r_0})$ is the classical definition of intensity at $\vec{r_0}$. Notice that the normalization by the mean intensity makes the coherence independent of the intensity profile of the beam, as it must be.

The fact that two points enter the definition of $\mu(\vec{r_0},\vec{r_0})$ means that the mutual degree of coherence of a certain field may vary both with the distance between the probing points $\Delta\vec{r}_{12} = \vec{r}_1 - \vec{r}_2$ and with the position across the beam $\vec{R}_{12} = (\vec{r}_1 + \vec{r}_2)/2$. For homogenous fields the coherence properties are constant all across the beam, so that μ is function of Δr only, regardless of which points are chosen. It must be stated, however, that coherence non-homogeneity can be introduced by beamline optics, so that dropping the dependence on $\vec{R}$ may not be the most recommended solution if one is concerned with optics testing (see for instance [3]).

In providing a 2D map of coherence, the main issue is the capability to explore a wide range of displacements $\vec{r}$ by means of a reasonable experimental setup. Young's interferometer is the simplest but even most inefficient setup, since it requires different pinhole separations and orientation. Other interferometric devices have been developed, in which the modulus of $\Delta\vec{r}$ can be selected by varying the observation distance and its direction by varying the test-plate orientation. But one can do even better, choosing a system in which there is no need to select $\Delta\vec{r}$. at all. The idea is just to write the coherence information corresponding to all the scale lengths on a 2D detector at once, then use Fourier analysis to separate the different spatial spectral contributions.

The system in question is a random distribution of scattering particles in solution. Particles used at visible and Angstrom wavelength are commercial (1um and 0.4um respectively) and are extremely cheap, and no kind of accurate positioning is required.

For a sort of conservation principle, all the efforts that in traditional approach are profused in the development and engineering of complex manufacts, the other one are dedicated to the study and comprehension of a simple physical system. More involved interferometric system than Young's one have been developed. But they all suffer the need of introducing a preferred direction of analysis (or two, in the best cases)

A SIMPLIFIED DESCRIPTION

One-particle and Many Particle

Information about coherence is completely contained in the interference fringes produced by the unperturbed

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transmitted beam and the scattered (almost) spherical wave by each single particle. This is shown in Figure 1:the spherical wave acts as a reference beam. At each radiation pulse fringes are written all over the plane, revealing features of the radiation (Figure 2, left). In the case of Synchrotron Radiation (SR) a single realization is not experimentally accessible (coherence time $\approx 10^{-14} s$) and only a time averaged measure is possible (Figure 2,right). According to principle of superposition of quasi-stationary fields, stable fringes are written at the generic point P'only when there is a high degree of coherence between the back-propagated point P and the particle position P_0. The intensity distribution has a straightforward interpretation and it turns that the visibility of fringes of (Figure 2,right) mimics the average shape of the coherence area of the incident field. This scheme, practically not feasible, is conceptually reminiscent of the point-diffraction interferometer.

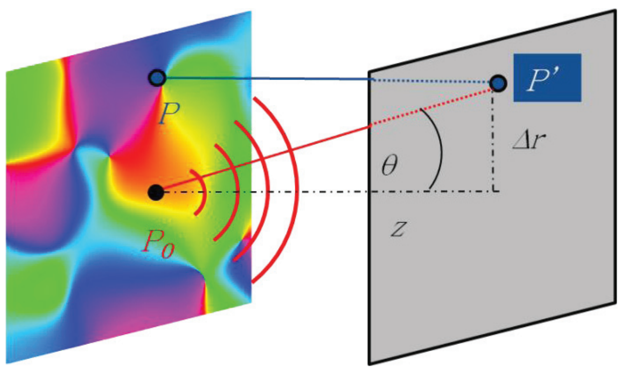

Figure 1: A single particle placed a P0 is illuminated with quasi-coherent radiation.

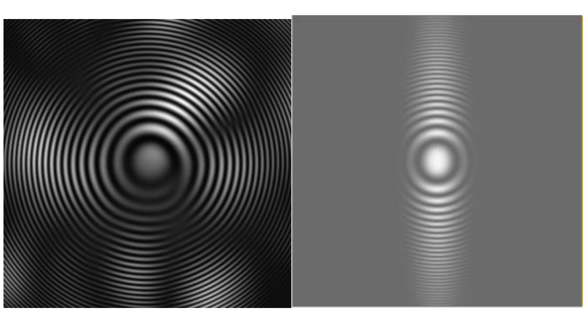

Figure 2: Simulated interference patterns in the case of Figure 1 with quasi-coherent radiation. LEFT: instantaneous fringes. RIGHT: time-averaged fringes.

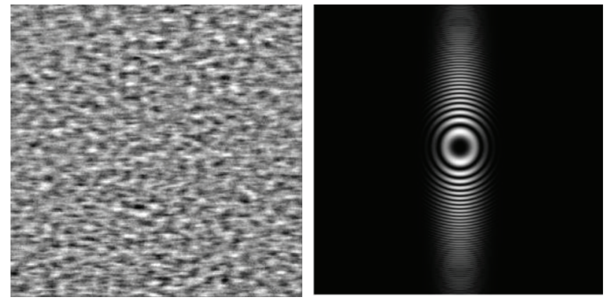

Figure 3: A Speckle field (left) and it's corresponding PS (right).

If many particles are used, the intensity distribution is a speckle pattern as in Figure 3 (left). Speckles are ubiquitous in optics and currently used in many X-wavelength techniques, and in coherence-probing as well. In many cases, however, their capabilities are undervalued, since information are extracted basing on the average contrast of the speckle pattern or on the contrast of a certain portion of it (sometimes accurately selected). But one must be aware that under well-definite conditions the Power Spectrum (PS) of the speckle field contains very rich information, yielding the squared modulus of $\mu(\Delta\vec{r})$ over an entire range of displacements $\Delta\vec{r}$. We require that:

a) Speckles satisfy the heterodyne condition
b) Light is collected in the Near-Field of the coherent area
c) Particles are much smaller than the coherent size σ_c and scatter light at an angle ϑ_s so that $\vartheta_s z >> \sigma_c$, where z is the observation distance.

Heterodyne and Near Field

Naming e_T the (unperturbed) transmitted field and e_S the field scattered by the *s-th* particle, the intensity at a point $\vec{r}$ is

$$i = | e_T |^2 + \sum_S^N 2\Re\, e_T{}^* e_S + \sum_{S,S'}^N 2\Re\, e_S{}^* e_{S'} \qquad (2)$$

If only a small portion of the incident beam is extinguished from the sample (negligible absorption and multiple scattering), the last term (homodyne) can be neglected. The second term (heterodyne) is a superposition of single-particle interference patterns as in Figure 2. Assuming that $| e_T |^2$ can be easily subtracted as constant background (Figure 4), we now compute the PS of the heterodyne term. If particles are random distributed, the summation of all the mixed contribution gives a uniform background, so that the PS is

$$I = \left| \sum_S^N \hat{e}_T \star \hat{e}_S \right|^2 \qquad (3)$$

where $\star$ is a convolution ^ and denotes the Fourier Transform. Now here comes the importance of being in

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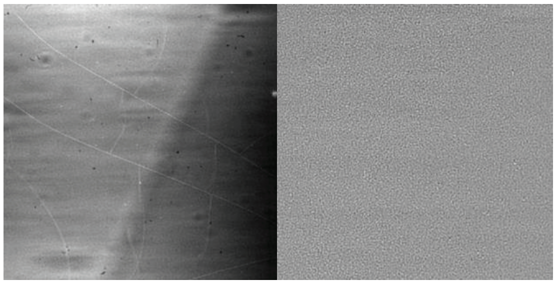

Figure 4: Speckle pattern before and after background subtraction (ID06, ESRF).

the Near-Field. In [2] (under preparation)is shown that, due to the highly oscillatory nature of patterns like that of Figure 2, the convolution of (3) can be approximated to a product if $z << \sigma_c^2 / ƛ$. Furthermore, one has that the PS I can be written as the product

$$I(\vec{q}) = F(\vec{q})T(\vec{q})H(\vec{q})C(\vec{q}) + K(\vec{q}) \quad (4)$$

where F is the particle form factor (it is recommended to choose particles with almost constant F over the range of interest), T is the Talbot contrast transfer function (well known in phase contrast methods, but with some specific analytical variation including the dependence on detector size [2],[3]), H is the imaging system transfer function (mainly the response of scintillator if X-rays are used), K is shotnoise and $C(q) = |\mu(\vec{q})|^2$ is the squared coherence factor. q wavevectors can be mapped in the real space by means of the scaling law

$$\vec{q} \mapsto \Delta\vec{r} = ƛ z \vec{q} \quad (5)$$

Again, notice that the direct relation between real and Fourier space (conflicting with the more common expectation that would state $q \sim 1/\Delta r$) is a direct consequence of the fact that we use spherical waves as reference beam and light is collected in the Near Field of the scattered light. From Eq. (5) one has that the accessible q range can be easily selected by changing the observation distance z, provided that the Near Field condition keeps fulfilled. Measures at different distances must overlap, thus providing a very effective self-consistence test (see Figure 5). In the case they don't, either temporal decoherence effects or peculiar optics mechanical instabilities are revealed.

About Temporal Coherence

Intrinsically, as any interferometric technique, even this one is simultaneously sensitive to transverse (spatial) and longitudinal (temporal) coherence. Longitudinal effects are negligible if the maximum extra-path ℓ between corresponding to the maximum transverse size of the experimental setup is such that $\ell < \Delta\lambda / 2\lambda^2$. In the case of many-pulse radiation (SR), longitudinal properties are usually dictated by monochromator bandwidth. The superposition of many pulses ($10^{14} - 10^{17}$) within a typical integration time (1-50ms) leads to a reduction of contrast of the speckle field, but not affecting its statistical properties (viz. the speckle size does not change). The case of single-pulse radiation (FEL) is slightly different, since a single wave-packet hits the detector. Since the spectrum of such a radiation is made up of number of spikes (longitudinal modes), the resulting speckle field can be regarded as the superposition of as many speckle fields as the longitudinal modes. The 2nd-order statistics of speckles (PS) still yields the information about transverse coherence. The 1st-order spatial statistics can, in principle, be related to the number of longitudinal modes. The same result can be obtained by means of 1st-order temporal statistics, which follows the well known Gamma distribution [4]. Notice that the number of modes obtained by 1st order statistics is the product of longitudinal and transverse mode $M = M_L \times M_T$. By means of Heterodyne Speckles one first directly obtain M_T from the knowledge of $\mu(\Delta\vec{r})$, then finds M_L. This holds for both SASE and seeded FEL.

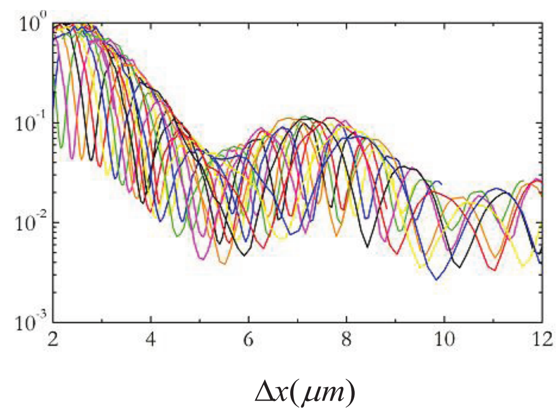

Figure 5: Overlap of PS C T obtained at different observation distances (ID02, ESRF). Horizontal profiles of 2D PS are considered. Dense oscillations are due to the T factor in (4). They have been preserved to show that maxima and minima of different curves form an upper and a lower enveloping curve. The C factor can be computed as the average of these two. Loose oscillations are due to the coherence area.

CURRENT CAPABILITIES

The most outstanding capability of the Heterodyne Speckle Approach is that of providing 2D coherence maps. In Figure 6 (left) a single PS is shown, revealing how a one-shot measure is capable of mapping a coherence area of non-trivial functional form. Circular oscillations are due to $T(\vec{q})$ transfer function in Eq. (4) and can be numerically removed so to give the modulus of the coherence factor shown in Figure 6(right). Oscillations are due to diffraction, while the Gaussian

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shape in the vertical direction is determined by the source only. For the measure a 2048x2048 pixel matrix was used. Since PS are symmetric, only one quadrant is relevant, which means that an equivalent measure with Young's interferometer would require over 10^6 measures to be performed. Some preliminary test have been conducted, such as the survey of coherence degradation due to the introduction of obstructions across the beam path (graphite foil, Figure 7) or to monochromator mechanical instability.

The direct consequence of full 2D mapping is the possibility of performing time-resolved measures. At ID06-ESRF the effects introduced by instabilities of the detector have been surveyed. This is expected to be one of the breakthroughs of the techniques, especially with FEL sources, where single pulses are accessible. Simultaneous characterisation of longitudinal and transverse coherence of FEL radiation has been recently performed [4].

As a side result, the technique has been used to provide high-quality characterization of the transfer function of phosphorous scintillators used in X-ray imaging. This is possible even with a low degree of spatial coherence, provided that the beam is monochromatic. All this results, as well as the complete mathematical formalism explaining the technique, will be published in [2].

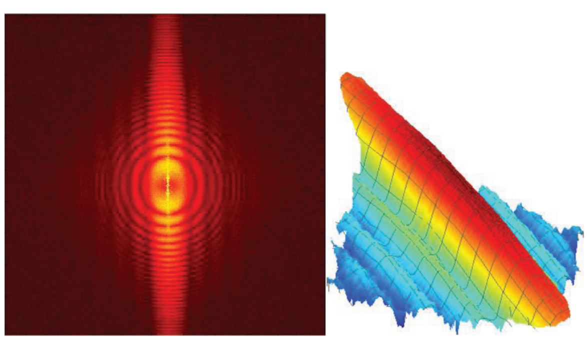

Figure 6: Left: Power Spectrum showing C(q)T(q). Right: corresponding coherence function $\sqrt{C(q)}$ (ID06, ESRF]).

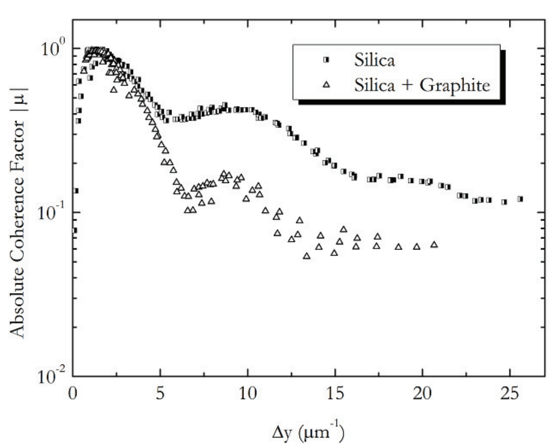

Figure 7: Coherence function of undisturbed beam and graphite obstructed beam. Measures at 3 different distances are merged together.

CURRENT ISSUES

The most delicate aspect of the technique is the background subtraction, by means of which the heterodyne the pure heterodyne term is extracted from a raw image. This is done by exploiting the fact that particles are free to move, so that heterodyne term varies from shot to shot, while background does not. With SR (which the technique has been mainly developed with) this is straightforward, since SR facilities deliver very stable beam.

With FEL radiation this may be not exactly the case, because intrinsic fluctuation in beam intensity distribution are expected. Measurement have been proved to be still possible, but a more skilled data analysis is required. See Ref. [5].

The technique is in principle exportable to any range of wavelengths provided that proper scatterers and proper (non absorbing) medium can be found. Magnification optics is required for coherence areas which are comparable to pixel size, making the use of scintillator screen compulsory.

The information of PS is the same as that of two-point correlation functions. It would be interesting to implement a spectral analysis based on higher-order correlation function in order to inspect non-homogeneity in transverse coherence.

CONCLUSION

The Heterodyne Speckle Approach provides an excellent mean for high-quality, time-resolved and instrumentally simple coherence measurements. It is intended to be both a useful mean for machine diagnostics and a valuable support for theoretical research concerning FEL radiation properties.

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