USE OF THE PROJECTED TORUS KNOT LATTICE FOR A COMPACT STORAGE RING FEL

S. Sasaki#, A. Miyamoto, Hiroshima Synchrotron Radiation Center, Hiroshima University, Higashi-Hiroshima, Japan

Abstract

We proposed a new scheme of lattice design for a compact storage ring in which a design orbit of electron beam closes after completing multiple turns. This new lattice can be made by placing necessary accelerator components at certain positions on a projected torus knot in the horizontal orbit plane. In this type of storage ring, the beam trajectory crosses in a bending magnet, i.e. each bending magnet accepts two beam orbits. For example, in the ring having the (11, 3) torus knot lattice with 11 bending magnets, a bunch goes through bending magnets 22-times to complete its 3-turn closed orbit. Since the maximum output laser power is proportional to the synchrotron radiation loss in the complete turn round a closed orbit starting from the optical resonator section, the maximum laser power from the projected torus knot storage ring FEL can be doubled for 2-turn lattice and tripled for 3-turn lattice compared with that from a conventional storage ring FEL. The new lattice scheme may contribute to more stable operation of a compact storage ring FEL.

INTRODUCTION

In general, the design orbit in a synchrotron accelerator or a storage ring closes in one turn around the ring, i.e. the length of closed orbit is equal to the circumference of the ring. On the other hand, a stellarator or a heliotron for a plasma confinement with magnetic fields has a donut (torus) shape with helically wound coils. Each coil winding does not close in one turn but closes after multiple turns around a torus. The shape of a coil winding on a torus is called torus knot. Similar to the coil winding of a stellarator, if the beam in an accelerator draws a torus knot trajectory, a design orbit closes after multiple turns.

The realistic multiple-turn lattice can be realized by placing conventional accelerator components on a projected torus knot in the horizontal plane [1]. The ring with this lattice may have larger maximum stored charge in multiple-bunch mode, and longer bunch-to-bunch interval in a single-bunch mode if compared with a conventional ring having the same footprint. Also, for an electron storage ring as the synchrotron light source or the ring FEL, a larger number of straight sections may accommodate with many insertion devices.

As a successive machine of current storage ring at Hiroshima Synchrotron Radiation Center, we have a plan to construct a third generation compact storage ring called HiSOR-II in order to serve brilliant synchrotron radiation (SR) from VUV to soft X-ray for SR users in the field of materials science, solid state physics, bio-molecular science, etc. Due to the requirement for the photon energy ranges from user community and the space limitation of construction area on the campus of Hiroshima University, the expected ring energy is 0.7 GeV and the ring diameter should be nearly equal to or smaller than 15 m. To meet all requirements including the oscillator FEL capability, construction of a compact storage ring with the lattice of torus knot architecture seems to be most plausible.

TORUS KNOT AND LATTECE

The Möbius strip is a one-sided nonorientable surface. It can be made from a band or a single strip by connecting both ends, so that one of the ends is half-twisted each other. As one can easily see by drawing a line on the surface of Möbius strip, the line closes after completing two turns (4π) around the ring. Analogically, if the top and bottom surfaces of a tall triangular prism are twisted $2\pi/3$ angle and then connected, the ridgeline closes after three turns (6π) [2]. These mathematical features are extended, generalized, and categorized as the group of torus knots [3]. Figure 1 shows an example of variations of (1,3) torus knot.

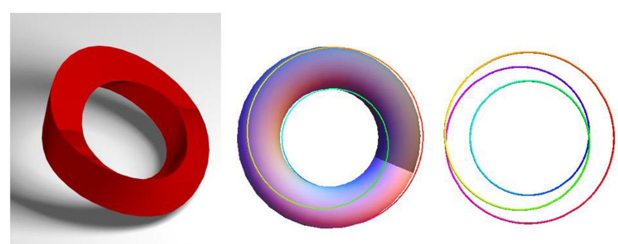

Figure 1: Left: Möbius triangular prizm, Middle: (1,3) torus knot wound around a torus, Right: (1,3) torus knot without torus.

Figure 2 shows some other torus knots.

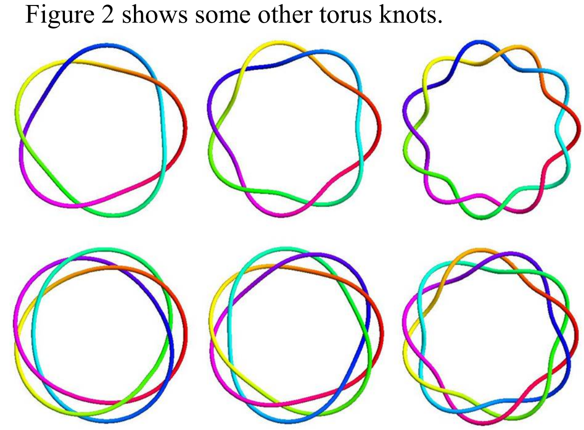

Figure 2: Examples of torus knots.

#sasakis@hiroshima-u.ac.jp

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The mathematical notations of these examples are: from top left to bottom right as (5,2), (7,2), (11,2), (5,3), (7,3), (11,3), respectively. Parameters p and q in the bracket (p,q) stand for the number of crossing of longitude and the number of crossing of meridian of the torus, respectively. Parameters p and q should be co-prime each other.

In the practical sense, the particle beam orbit of a storage ring lies on the horizontal plane. Therefore, in order to generate an accelerator lattice starting from one of the torus knot, it is a good idea to project a torus knot onto a flat plane. Figure 3 shows projected (5,2) and (11,3) torus knots and bending magnet positioning in the corresponding lattices.

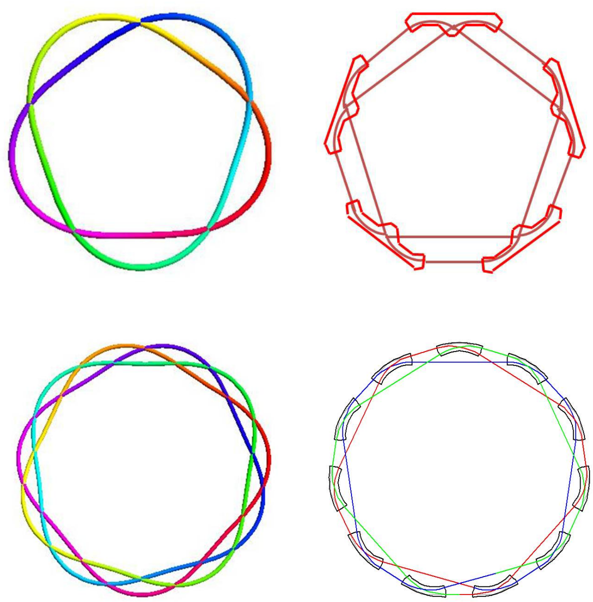

Figure 3: Examples of projected torus knots and bending magnet configurations of corresponding lattices.

As it is obvious from this figure, the basic structure of accelerator lattice can be made by placing a bending magnet at each crossing point of projected (5,2) torus knot, and at each outer crossing point of projected (11,3) torus knot. Although particle beam trajectory crosses in every bending magnet, the bending radius of either trajectory is the same. The orbit entered from inboard exit outboard and vice versa.

Whatever any torus knot as the starting point was chosen, one can construct a required lattice by adding necessary components after the position of each bending magnet is determined. As one can recognize by tracing the beam orbit, the unit cell of this type of lattice consists of two bending magnets, i.e. the double-bend unit lattice. The numbers of unit cells are five for geometrically 5-fold symmetry lattice, and eleven for the 11-fold symmetry lattice, respectively.

Figure 4 shows a realistic (11,3) torus knot lattice and a one-cell configuration of magnets. This lattice is planned to be used for the HiSOR-II storage ring [4, 5].

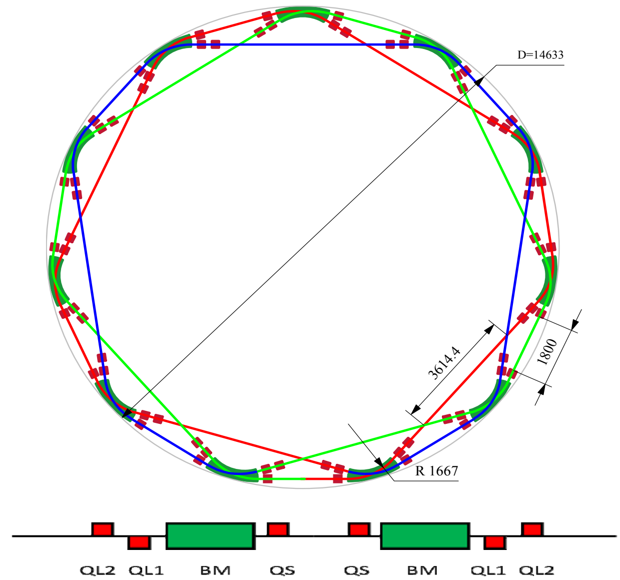

Figure 4: An example of (11,3) torus knot lattice for a compact light source and a unit cell.

This compact 11-fold symmetry light source ring has an outer perimeter of 46 m and a diameter of 14.63 m. The length of electron beam orbit is about 130.3 m. The length between crossing points of inner straight section is 3.6 m. The length of outer straight section is 1.8 m where a RF cavity and other necessary accelerator components and short insertion devices may be installed.

LATTICE DESIGN ISSUES

Low Emittance Lattices

Generally, Double Bend Achromat (DBA) is well known as a low emittance lattice, and it is often used for synchrotron light source rings. In recent years, the lattice which introduced dispersion into the straight sections is used to achieve lower emittance than that of DBA lattice.

The natural emittance is shown with radiation integrals as follows [6].

$$\varepsilon_x = C_q \gamma^2 \frac{I_5}{I_2 - I_4}$$

Where C_q is the classical radius of the electrons, and I_2, I_4 and I_5 are:

$$I_2 = \oint \frac{1}{\rho^2} ds,$$

$$I_4 = \oint \frac{D}{\rho}\left(\frac{1}{\rho^2} + 2K\right) ds,$$

$$I_5 = \oint \frac{\mathrm{H}}{|\rho|^3} ds.$$

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In MAX-III of MAX-lab, the combined function type bending magnet with QD field was adopted, and the lattice in that K in I_4 is negative value is used to achieve ultra low emittance. Learning form this fact, we investigated the possibility of adopting the MAX-III type lattice to our compact torus knot ring.

Crossed Orbit in Bending with Gradient Field

The combined type bending magnet with QD fields are necessary for MAX-III type low emittance lattice. However in our case, two beam orbits cross with a certain angle from the central axis in the bending magnet. This causes a slight change of quadrupole strength as well as bending radius along the beam orbit. But we found this problem can be solved [7].

Operating Point and Optical Functions

New ring has two bending magnets and two straight sections with different lengths in a unit cell. Therefore, this ring is categorized as the Double Bend system. We adopt the combined function bending magnets that we described previously. Furthermore, by using three families of quadrupole magnets, we can switch two kinds of lattice type i.e. MAX-III type and DBA.

Figure 5 shows the optical functions of 1/3 of the ring when the operating point of the ring is set at the low emittance mode and DBA mode each, and the natural emittance in each mode achieved 17.4 nmrad and 35 nmrad. The main parameters of (11,3) torus knot lattice designed for HiSOR-II storage ring is shown in Table 1.

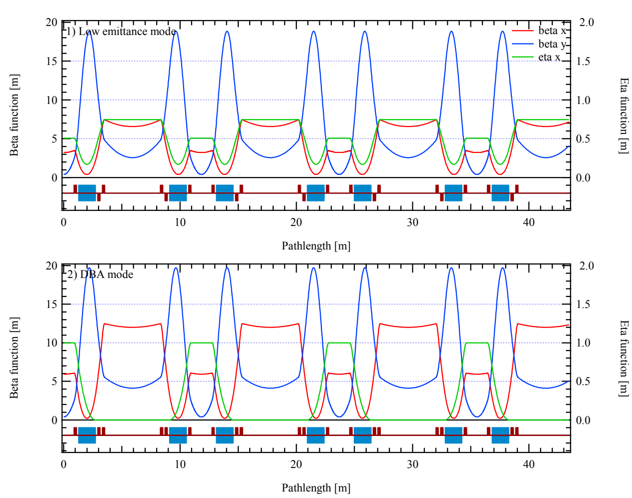

Figure 5: Optical functions on each operating point. 1):Low emittance mode, 2):DBA mode.

Table 1: The Main Parameters of (11,3) T-K Ring Designed for HiSOR-II Storage Ring

Perimeter	45.97 m
Orbit shape	(11,3) Torus knot
Orbit length	130.297 m
Beam energy	700 MeV
Straight sections	3.614 m ×11 1.800 m ×11
Harmonic number	88
RF frequency	202.474 MHz
Low emittance mode	
Betatron tune	(10.54, 6.67)
Natural emittance	17.4 nmrad
DBA mode	
Betatron tune	(10.78, 6.93)
Natural emittance	34.6 nmrad

OSCILLATOR FEL

As it is well known that the maximum output laser power of oscillator FEL is proportional to the synchrotron radiation loss in the complete turn round a closed orbit starting from the optical resonator section [8]. The maximum peak power of each micropulse from the projected torus knot storage ring FEL can be doubled for 2-turn lattice and tripled for 3-turn lattice compared with that from a conventional storage ring FEL. The new lattice scheme may contribute to more stable operation of a compact storage ring FEL.

Figure 6 shows a layout of oscillator section in the HiSOR-II ring.

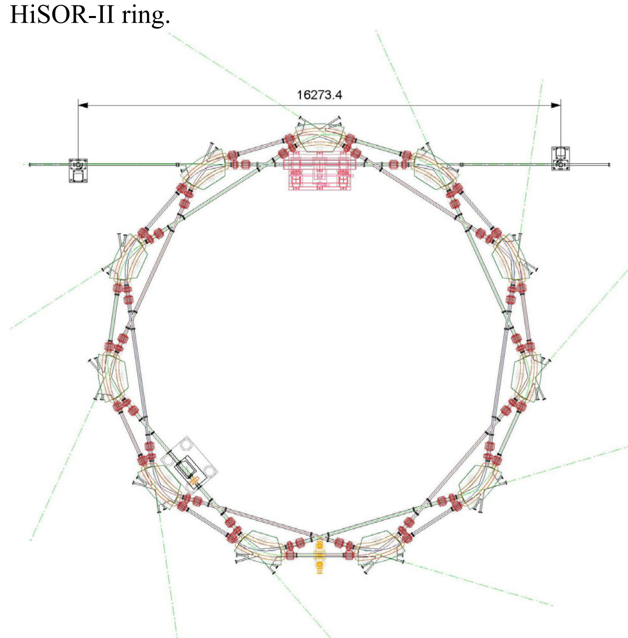

Figure 6: Oscillator FEL configuration in HiSOR-II ring.

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The orbit length of this ring is as long as 130 m though the perimeter of this ring is about 46 m. Therefore, the optical cavity length must be about 65 m if the ring is operated in a single bunch mode for FEL operation. In addition for the single bunch operation, the recirculating time becomes three times longer than that of one-turn ring, and hence the average power stays the same. To avoid these problems, we assume 4-bunch operation so that the cavity length should be about 16 m and the bunch-to-bunch length is 3/4. In other words, if the cavity length is the same, averaged laser power can be tripled compared with a conventional one-turn ring FEL.

Figure 7 shows the bird's eye view of HiSOR-II FEL complex.

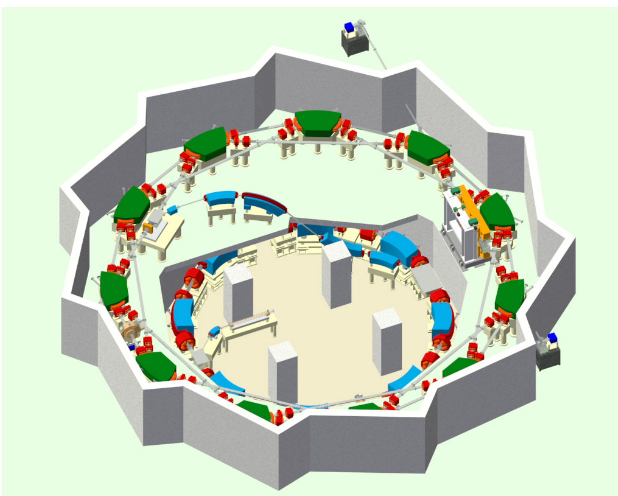

Figure 7: Designer's view of HiSOR-II ring with an oscillator FEL section.

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