Wir schaffen Wissen – heute für morgen

Paul Scherrer Institut

Sven Reiche, Eduard Prat

Growth Rates and Coherence Properties of FODO-lattice based X-ray Free Electron Lasers

Outline

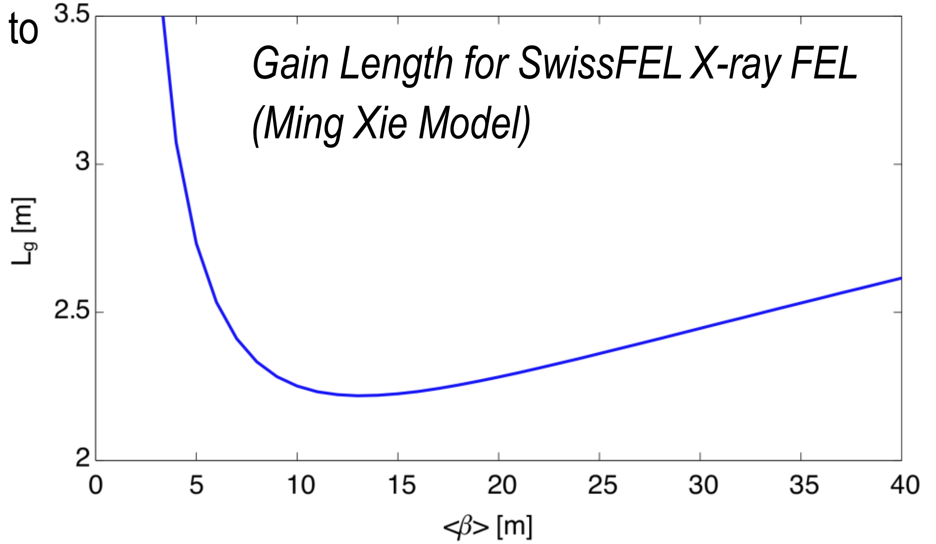
- Focusing and FODO Lattices
- FEL Eigenmodes
- Coherence
- FEL Performance of hard X-ray FEL (1 Ångstrom)
- FEL Performance of soft X-ray FEL (1 nm)
- Optimization of Focusing Strength

Focusing and FODO Lattice

•External focusing increases the FEL performance due to the higher electron density and larger FEL parameter.

•Stronger focusing also transfers kinetic energy into betatron oscillation, slowing down the electrons proportionally to its betatron amplitude.

→ **Optimizing the beta function**



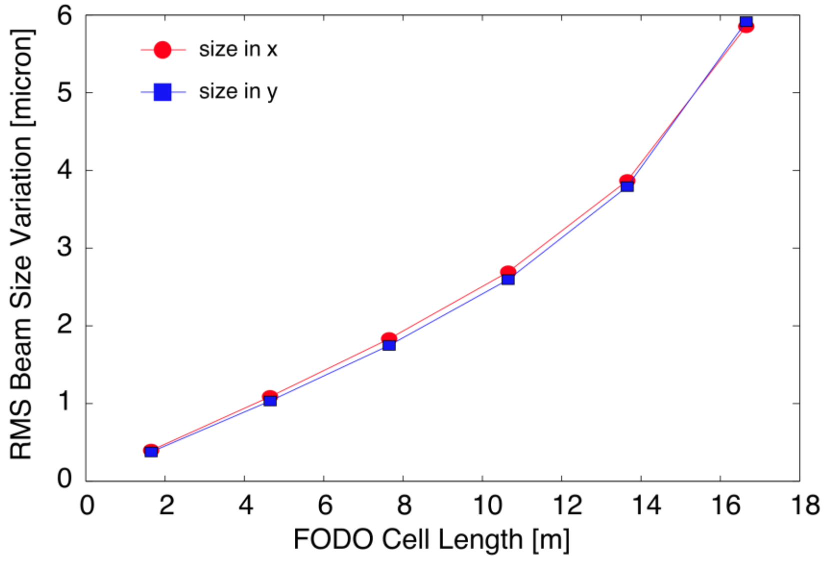
•FODO lattice to provide focusing

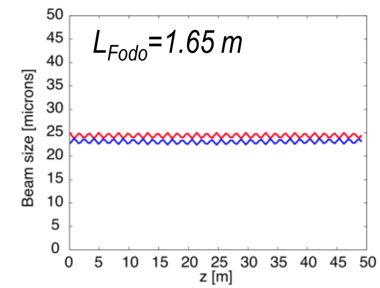
•Achievable beta-function is limited by the FODO cell length (instable limit)

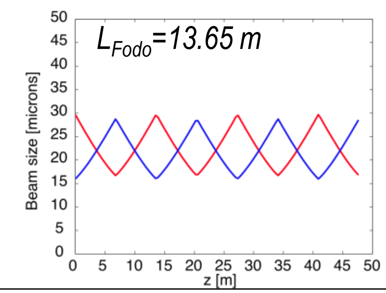
•Longer wavelength or low emittances pushes optimum beta-value below limit

•Strong variations in beam sizes when operating at focusing limit

Beam size variation (β=15 m)



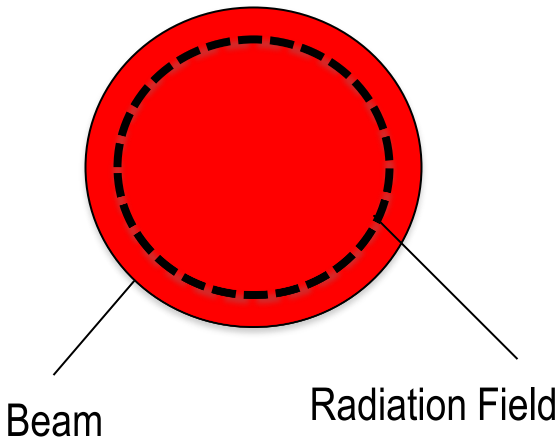




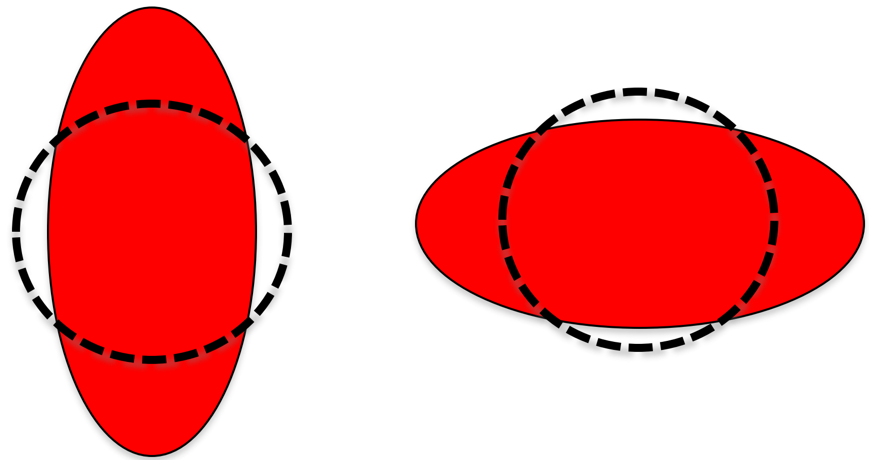
Mode and Beam Sizes

For low diffraction at hard X-ray FELs the optical mode stays inside the electron beam

Round, rigid electron beam



Beam Size Variation in FODO Lattice



- Electrons are pushed out of the radiation field, reducing the amount of electrons emitting into the FEL mode
- Effect should be more pronounced for strong variation, e.g. FODO cells at the stable transport limit

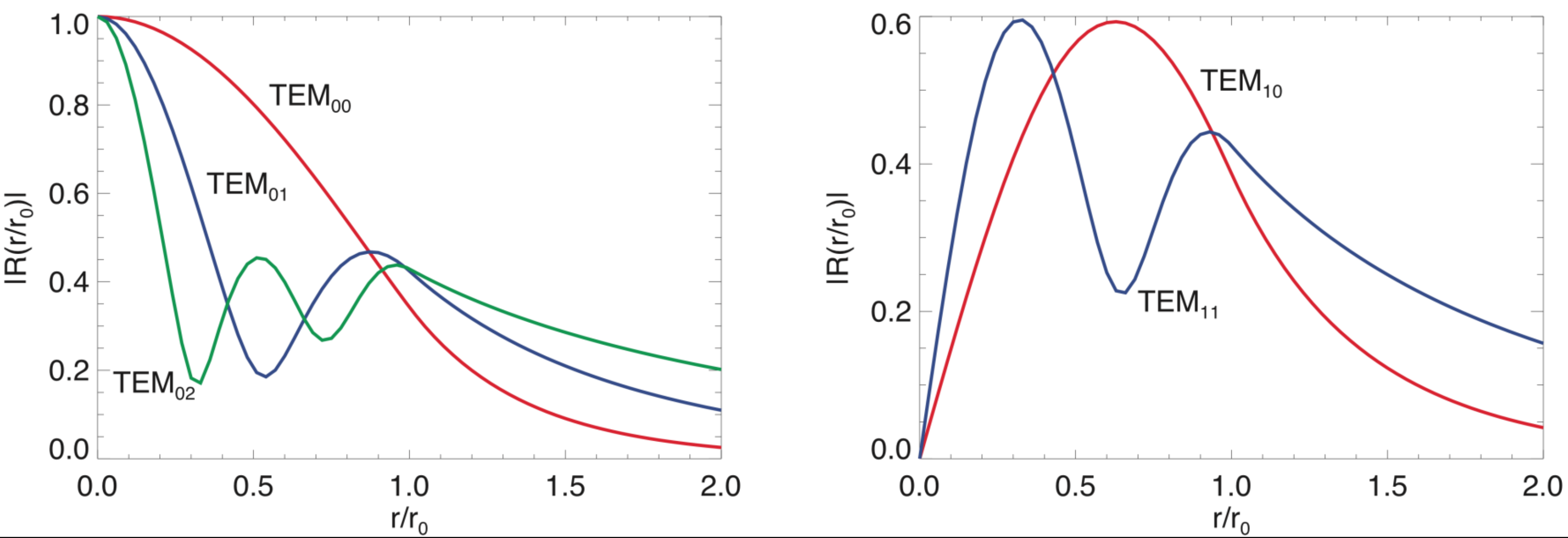
FEL Modes (Basic Idea of 3D FEL Theory)

- The FEL wave front with a growth rate Λ has to fulfill the field equation of the form:

$$\left[\nabla_\perp^2 + V(r,\Lambda)\right]A = 0$$

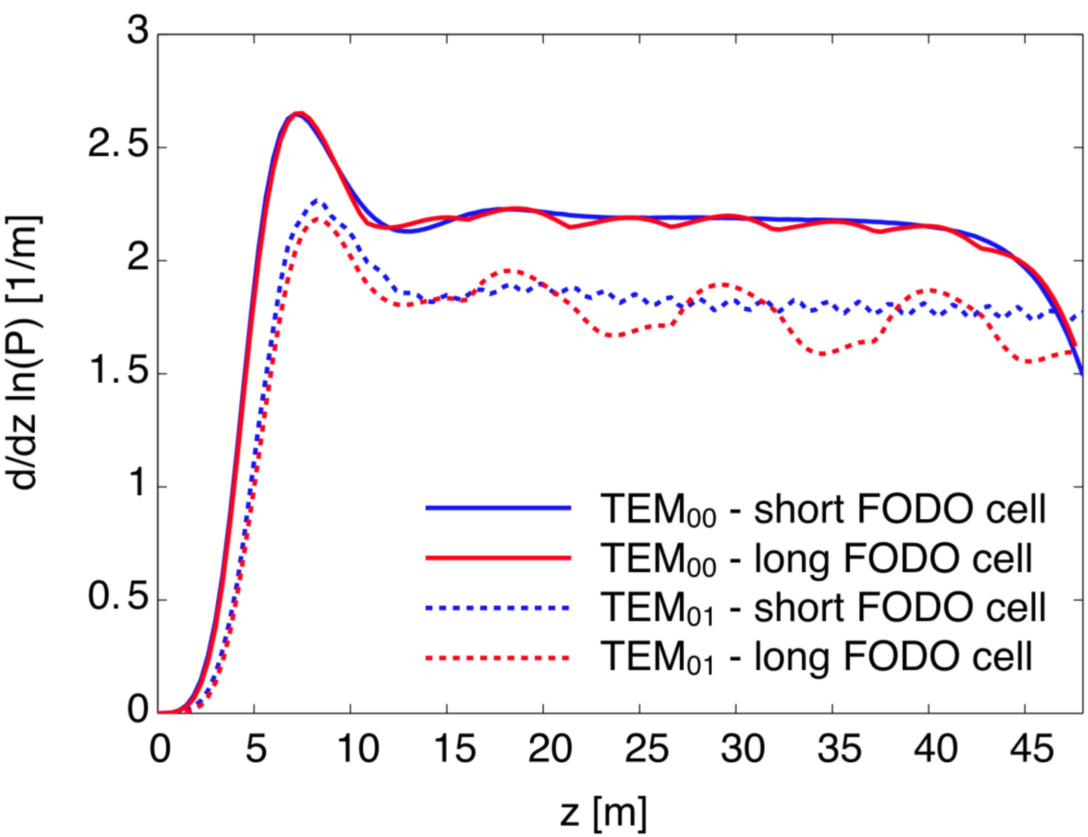
- The transverse electron distribution acts like a 2D potential in Quantum Mechanics.
- Note: $V(r,\Lambda)=0$ is the 1D dispersion equation with the given electron density at the radius *r*.
- Like in QM a valid solution for *A* has to be finite when $|A|^2$ is integrated over the transverse plane.
- Discrete solutions define growth rates Λ_{mn} for each FEL eigenmode.

Example: $A(r,\phi)=R_{mn}(r)e^{im\phi}$ for step profile with radius r_0

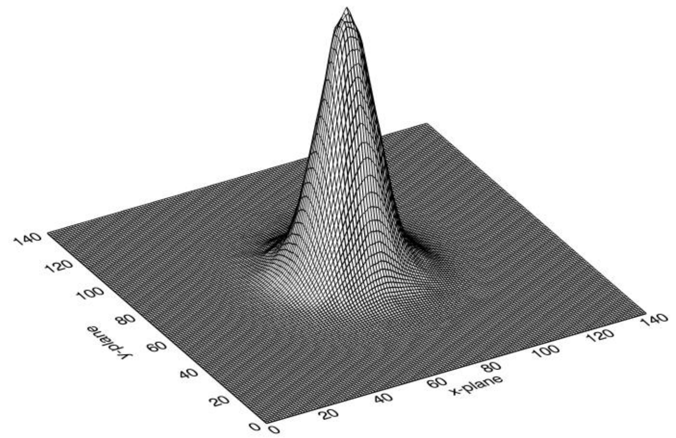


FEL Mode Simulations

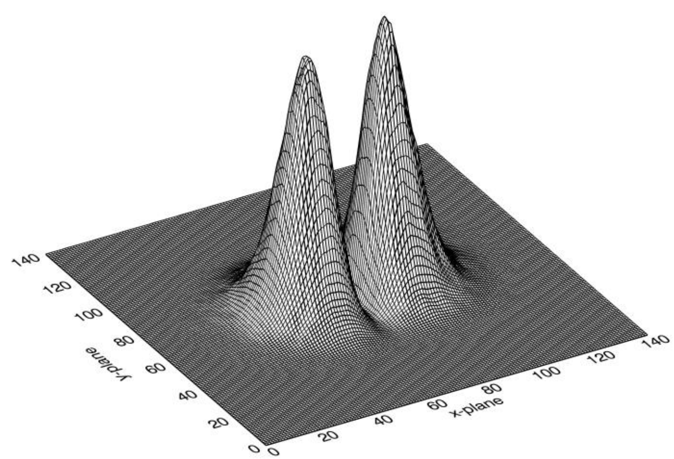
- Seeding with different Gauss-Hermite Modes
- Only TEM_{00} and TEM_{01} couple well to FEL eigenmodes
- TEM_{01} mode more sensitive to beam size variations
- Higher modes „leak“ into lower modes. Enhanced by stronger beam size variations



Fundamental Gauss Mode (TEM_{00})

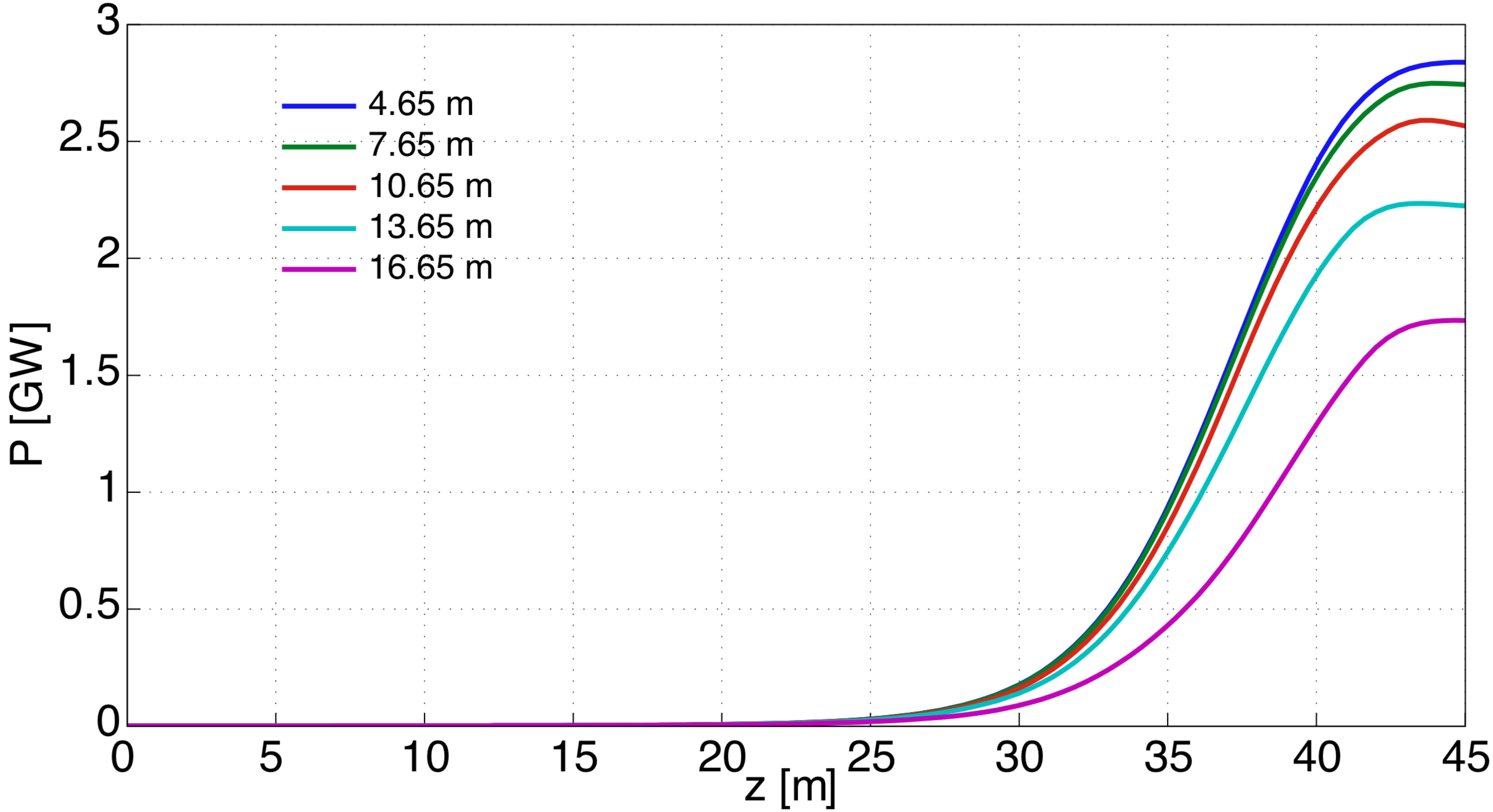


Gauss-Hermite Mode (TEM_{01})



FEL Performance for different FODO Cell Lengths

- Simulation with SwissFEL paramters at 1 Å for various cell length of the FODO lattice.
- Noticeable drop in saturation power for cell length of 10 m and longer, slight increase in saturation length



Transverse Coherence of a SASE FEL

- Coherence is the correlation between two points in space and time, averaged over many shots:

$$\Gamma_{12}(\tau) = \left\langle E(\vec{r}_1, t+\tau) \cdot E^*(\vec{r}_2, t) \right\rangle$$

- Considering only transverse coherence (t=0) the complex coherence function

$$\mu_{12} = \frac{\Gamma_{12}(0)}{\sqrt{\Gamma_{11}(0)\Gamma_{22}(0)}}$$

Is a measure coherence between two points (μ_{12}=1 → full coherence, μ_{12}=0 → incoherent).

- A quantity describing the entire beam is the intensity weighted average of the coherence function:

$$\varsigma = \frac{\int |\mu_{12}|^2 \Gamma_{11}\Gamma_{22} d\vec{r}_1 d\vec{r}_2}{\left[\int \Gamma_{11} d\vec{r}_1\right]^2}$$

Very impractical to evaluate ζ, computational and memory wise

Fast Evaluation of the Coherence Factor

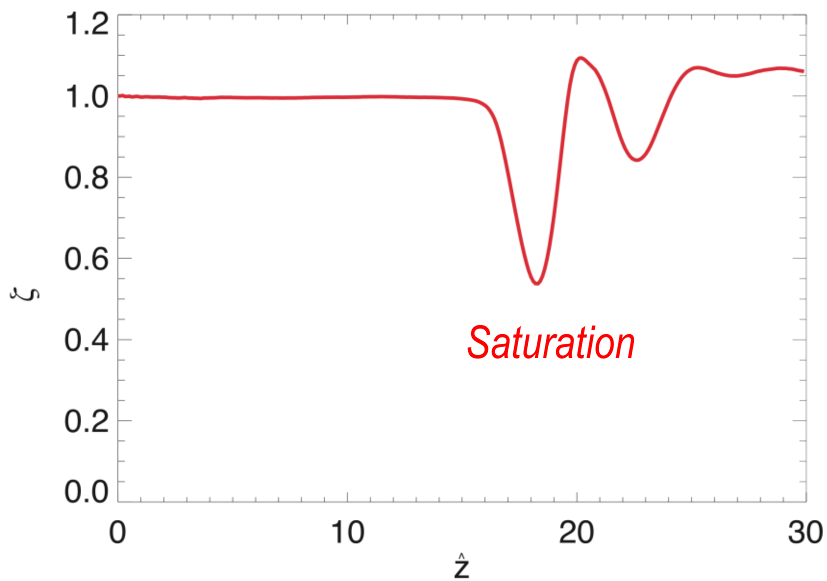
The SASE process is a stationary process and the average over many shots can be replaced by an average over a single long pulse.

Due to the seed by a “white noise” (shot noise) signal the coherence factor is identical to the fluctuation of the instantaneous power **within the linear regime** (see *SALDIN et a l*)

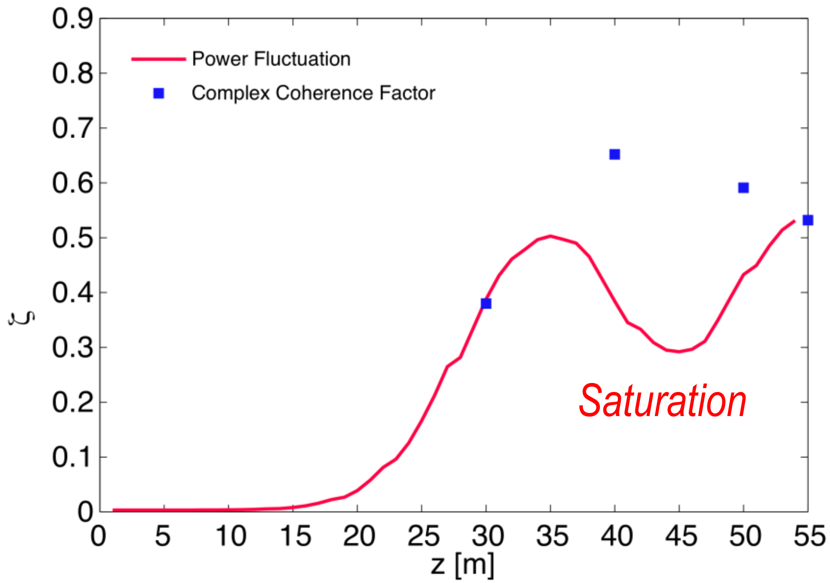
$$\varsigma = \frac{\left\langle \left(P - \langle P\rangle\right)^2 \right\rangle}{\langle P\rangle^2} = \frac{1}{M_L \cdot M_T}$$

M_L: Number of longitudinal modes
M_T: Number of transverse modes

1D Simulation (M_T=1)



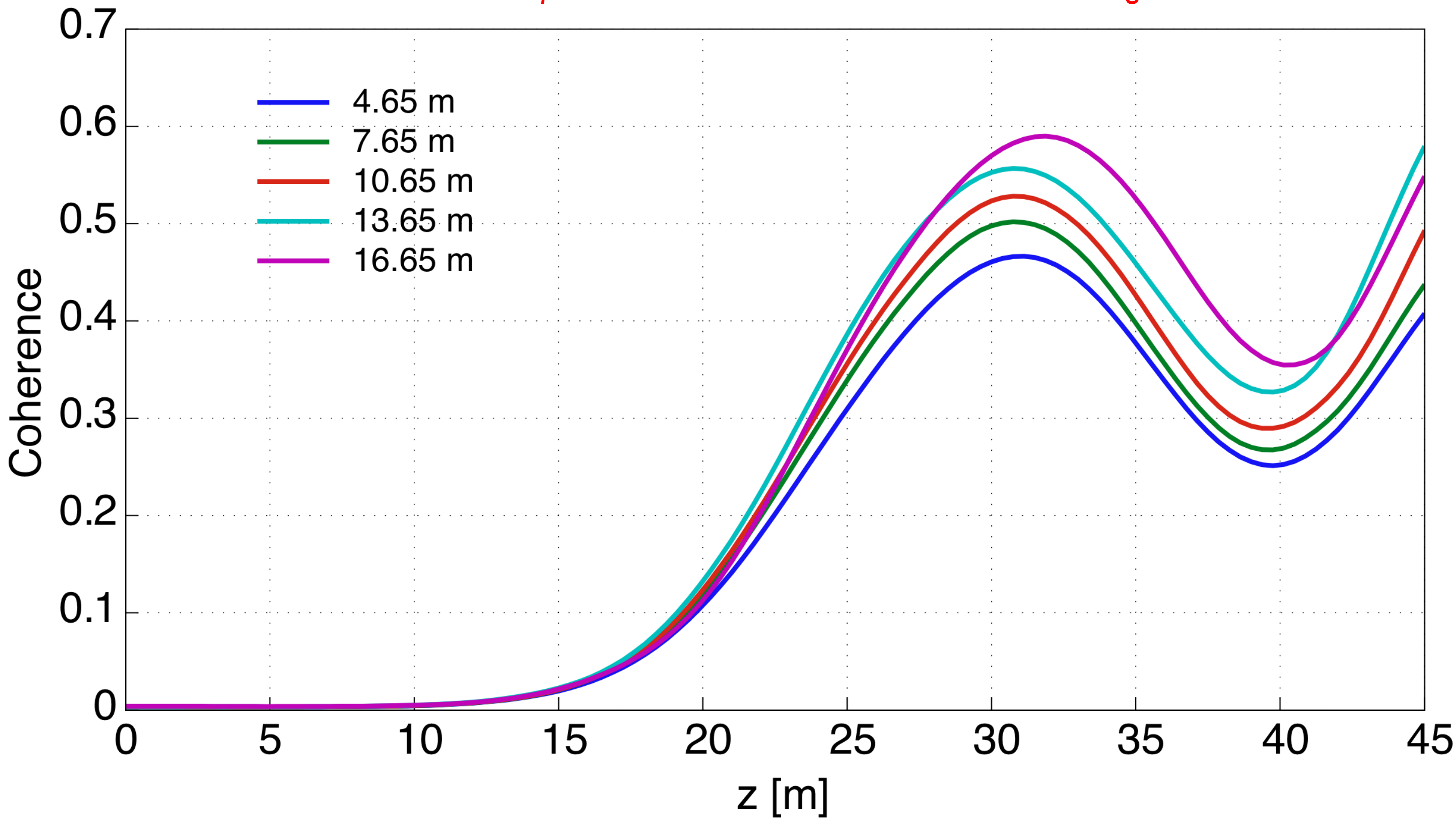
3D Simulation (Genesis 1.3)



Coherence

- Stronger growth in coherence for large beam size variations
- Higher modes more affected by FODO lattice than fundamental mode
- Dominance of fundamental mode grows faster → improved coherence

Difference of up to 30% between different FODO cell lengths

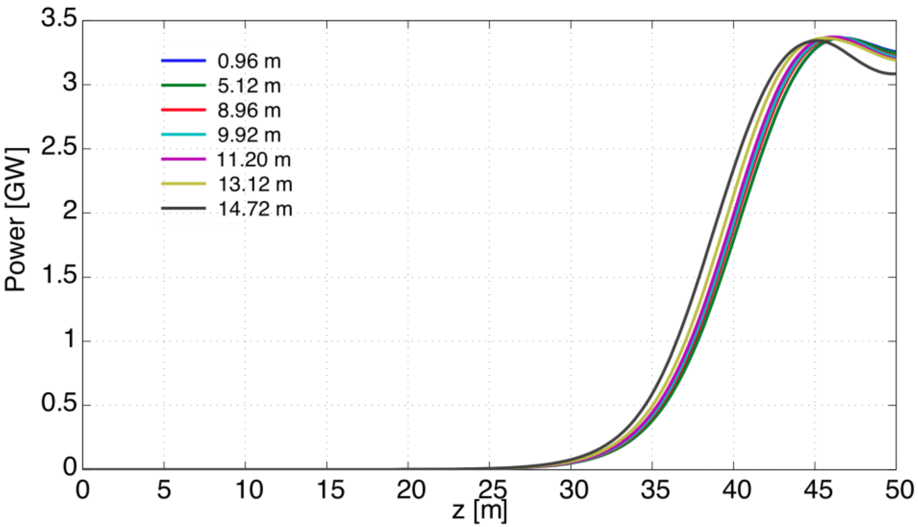


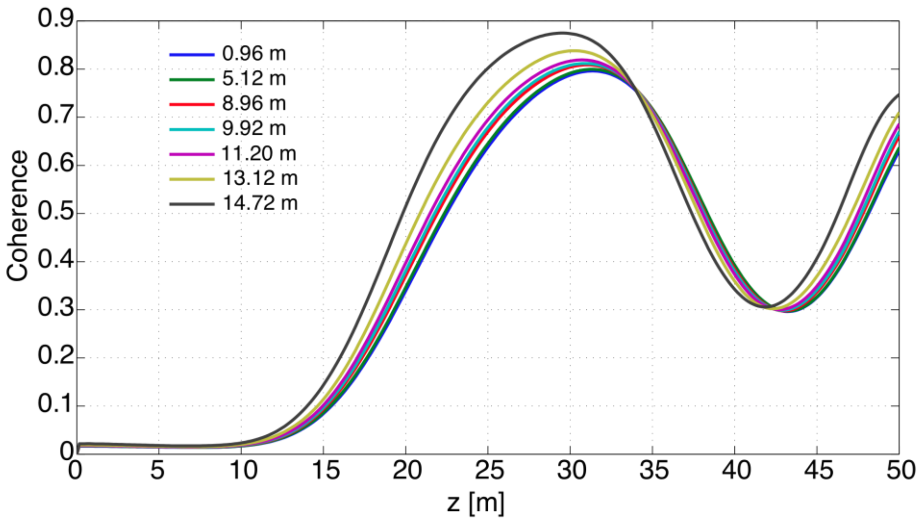
FEL Performance for Soft X-ray FEL at 1 nm

- Modeled after the SwissFEL soft X-ray beamline.
- In difference to hard X-ray case, the radiation mode (37 microns) is larger than electron beam size (32 microns).

- Very weak dependence of FEL performance on FODO cell length. Slight improvement arises from quadratic term of beta-function, which somehow reduces the average beam size.

- Still a noticeable dependence on coherence. A larger beam size variation improves upon coherence.





Betatron Scan

With given FODO lattice (9.5 m cell length) the beta function is optimized for best FEL performance.

Ming Xie Prediction for best gain length

$\beta = 12\ m$

Best gain length (Simulation)

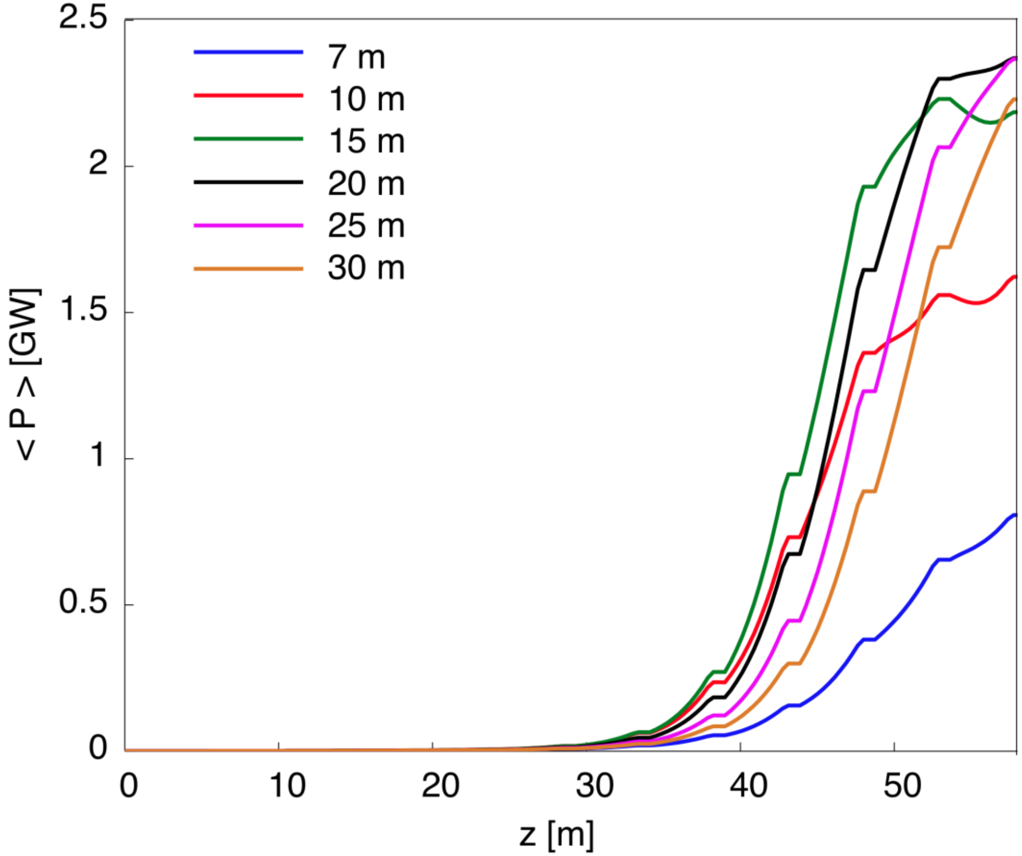
$\beta = 15\ m$

Best saturation power

$\beta = 20\text{-}25\ m$

Best coherence

$\beta = 10\ m$



Summary

FODO lattices are essential to optimize X-ray FELs

Strong variation of electron beam sizes:

- **Less saturation power and longer saturation length**
- **Improved coherence for SASE FEL**

Effect is not seen for longer wavelength (1 nm and longer)

Relaxed focusing for given FODO lattice can increase saturation power.