The generator of high-power short terahertz pulses

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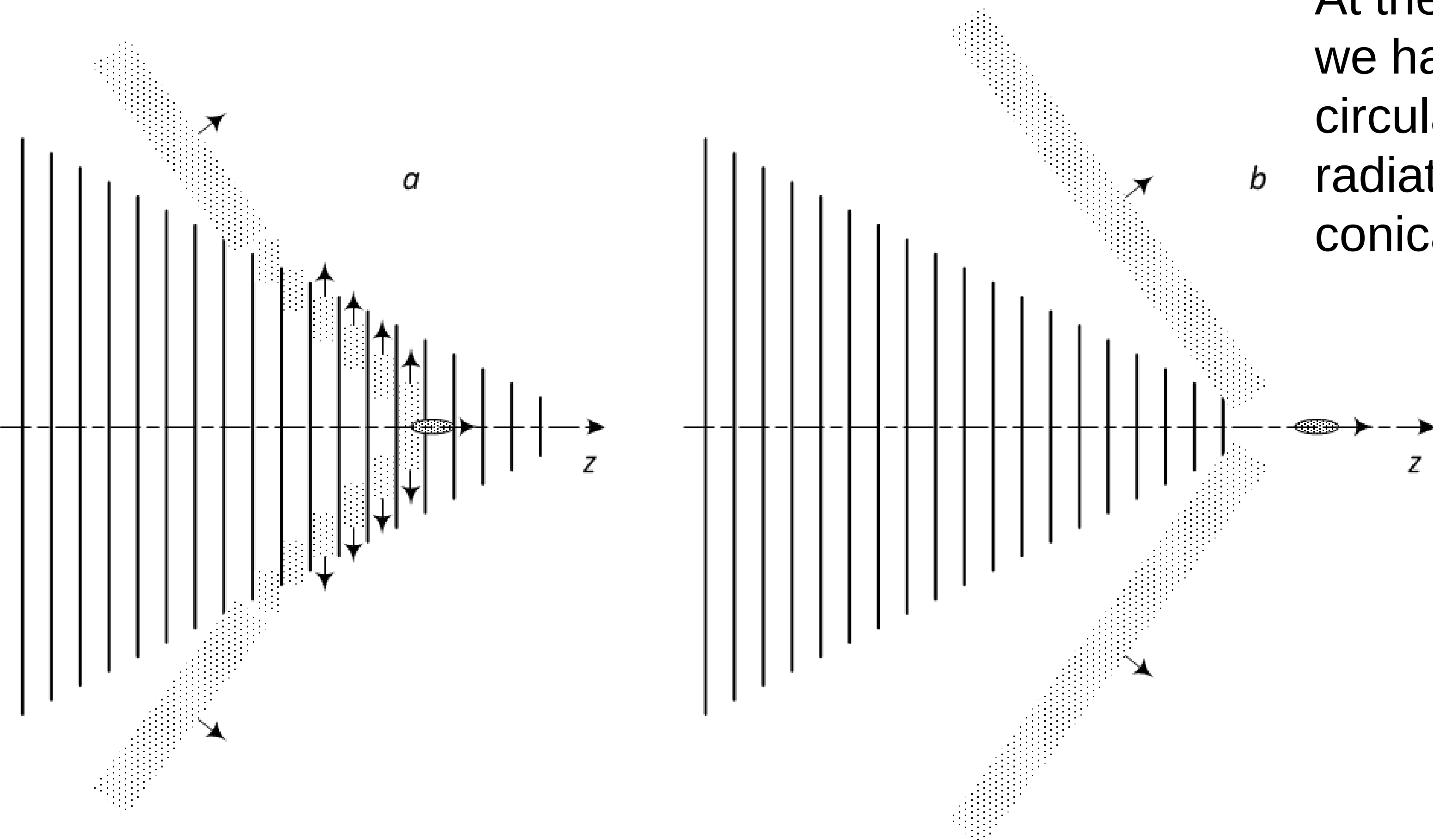
THz Pump
2RC
X-ray Probe

Contents

I. Introduction

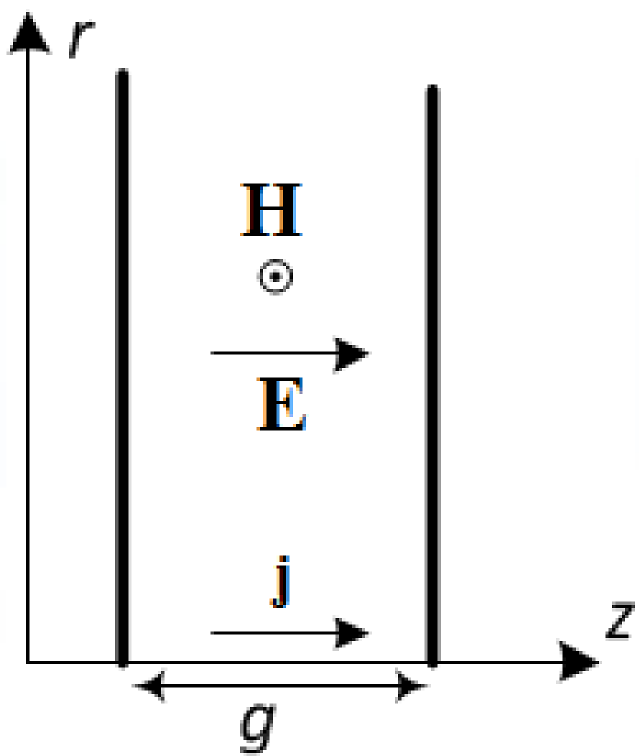
A round flat conducting foil plates with successively decreasing radius are stacked, comprising a truncated cone with axis *z*. Passing through each gap between foils, the bunch emits some energy into the gap. After that the radiation pulse s propagates radially, as it is shown in Fig. 1a.

At the cone outer surface we have synchronized circular radiators. Their radiation field forms the conical wave (see Fig. 1b).



II. The single gap excitation

TEM waves, having only the longitudinal electric E_z and the azimuthal magnetic H_α fields, which do not depend on z :



$$\frac{1}{r}\frac{\partial}{\partial r}(rH_\alpha) = \frac{4\pi}{c} j_{av} + \frac{n^2}{c}\frac{\partial}{\partial t}E_z$$

$$\frac{\partial}{\partial r}E_z = \frac{1}{c}\frac{\partial}{\partial t}H_\alpha$$

$$j_{av} = \int_{-g/2}^{g/2} j_z dz \Big/ g$$

-the beam current density, averaged over the gap length *g*. *n* is the refraction index.

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The solution

$$E_\omega = -\frac{\pi Q\omega}{c^2} F_\omega H_0^{(1)}(kr) \approx -\frac{Q\omega}{c^2} F_\omega \sqrt{\frac{2\pi}{ikr}} e^{ikr}$$

where

$$Q = \int_{-\infty}^{\infty} \int_0^{\infty} j_z 2\pi r dr dt$$ - the bunch charge

$$F_\omega = \frac{2c}{\omega g} \sin\left(\frac{\omega g}{2c}\right) \frac{2\pi}{Q} \int_0^\infty j_\omega(r) J_0(kr) r dr$$ - the form factor

For the Gaussian charge distribution

$$j_z = \frac{Qc}{(2\pi)^{3/2} a^2 l} e^{-\frac{r^2}{2a^2} - \frac{1}{2l^2}(z-ct)^2}$$

$$F_\omega = \frac{2c\sin\left(\frac{\omega g}{2c}\right)}{\omega g} e^{-\frac{\omega^2}{2}\left(\frac{l^2}{v^2} + \frac{n^2 a^2}{c^2}\right)} \approx 1 - \frac{\omega^2 l_{eff}^2}{2c^2}$$

where $l_{eff} = \sqrt{l^2 + n^2 a^2 + g^2 / 12}$

The field time dependence

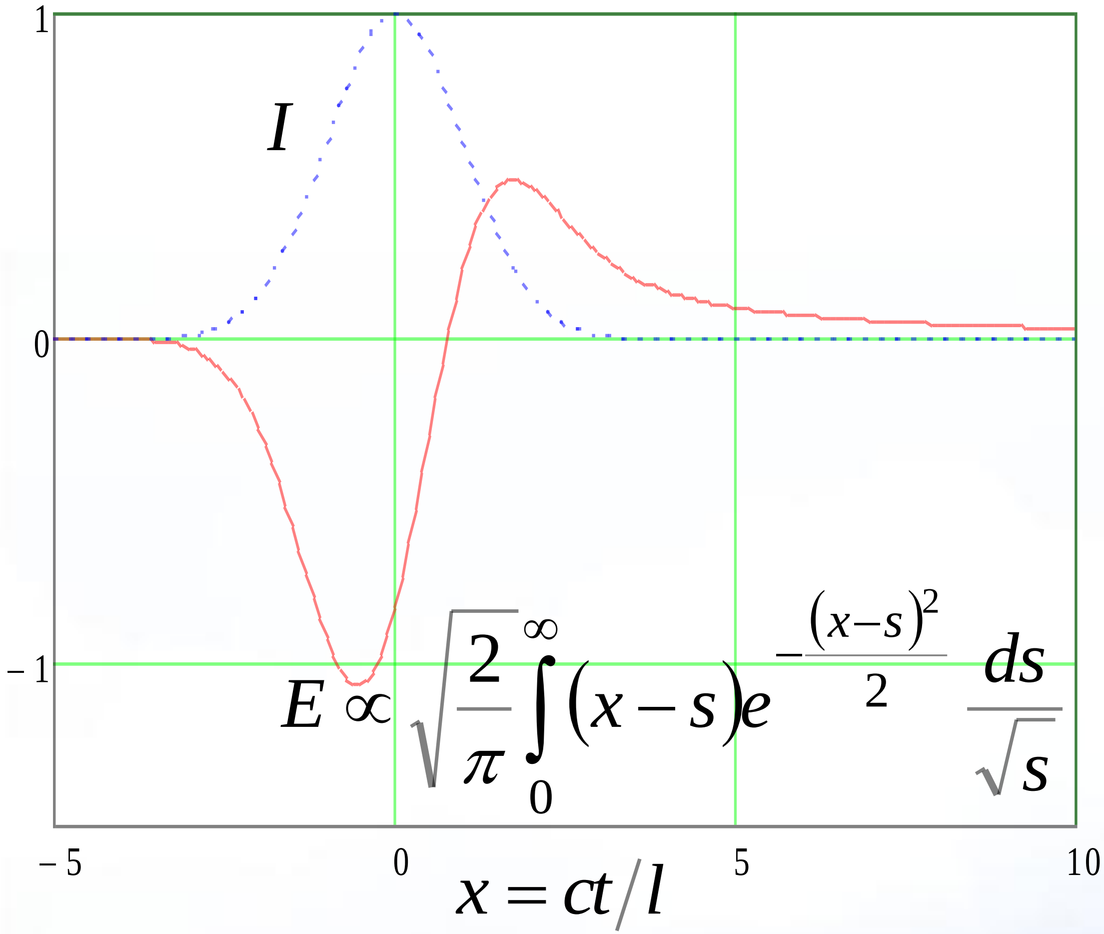
$$E(r,t) \approx -\frac{Q\sqrt{2}}{c^{3/2}\sqrt{\pi n r}} \mathrm{Re}\int_0^\infty F_\omega e^{-i\omega\left(t-\frac{nr}{c}\right)-i\frac{\pi}{4}}\sqrt{\omega}\,d\omega =$$

$$-\sqrt{\frac{2}{nrc^3}}\int_0^\infty \dot{I}_{eff}\left(t-\frac{nr}{c}-\tau\right)\frac{d\tau}{\sqrt{\tau}}$$

where

$$I_{eff}(t) = \int_{-\infty}^{\infty} F_\omega e^{-i\omega t}\frac{d\omega}{2\pi} \qquad I_{eff}(t) \to I(t) \quad for \quad l \gg \sqrt{n^2a^2 + g^2/12}$$

The field time dependence for the Gaussian bunch



$$E \propto \sqrt{\frac{2}{\pi}} \int_0^{\infty} (x-s) e^{-\frac{(x-s)^2}{2}} \frac{ds}{\sqrt{s}}$$

The maximum field is

$$|E|_{\max} \approx \frac{Q}{\sqrt{2nrl^3}}$$

and the corresponding peak power is

$$P_{\max} = \frac{cn}{4\pi} |E|^2_{\max} 2\pi r L \approx \frac{cQ^2}{4l^3} L$$

The emitted energy

The radiation spectral density is

$$2\pi \frac{dW}{d\omega} = -g 2\pi r \frac{c}{2\pi} \mathrm{Re} E_\omega^* H_\omega = g \frac{2\pi Q^2 \omega}{c^2} |F_\omega|^2$$

Then the total radiated energy is

$$W = g \frac{Q^2}{c^2} \int_0^\infty |F_\omega|^2 \omega d\omega$$

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The emitted energy for the Gaussian bunch

$$W \approx g\frac{Q^2}{2l_{eff}^2}$$

The corresponding effective average decelerating field is

$$\langle E_z \rangle = \frac{W}{Qg} \approx \frac{Q}{2l_{eff}^2}$$

For Q = 0.5 nC and l_{eff} = 0.1 mm it is about 2 MV/cm. Then, for the cone height L = 2 cm, the radiated energy is 2 mJ, and P_{max} = 0.4 GW.

To have the transverse size smaller, than the bunch length, the beam e mittance has to be less than $l_{eff}^2/L = 5\cdot10^{-7}$ m.

The comparison with the coherent transition radiation

The radiated energy may be compared with the one of the coherent transition radiation (for a narrow beam, *a* << *l*)

$$W_{CTR} \approx \frac{Q^2}{l\sqrt{\pi}} \ln \frac{r_{\max}}{a}$$

where r_{max} depends on geometry and electron energy.
The ratio of these energies is

$$\frac{W}{W_{CTR}} \approx \frac{L}{l} \frac{\sqrt{\pi}}{2\ln\left(r_{\max}/a\right)} \gg 1$$

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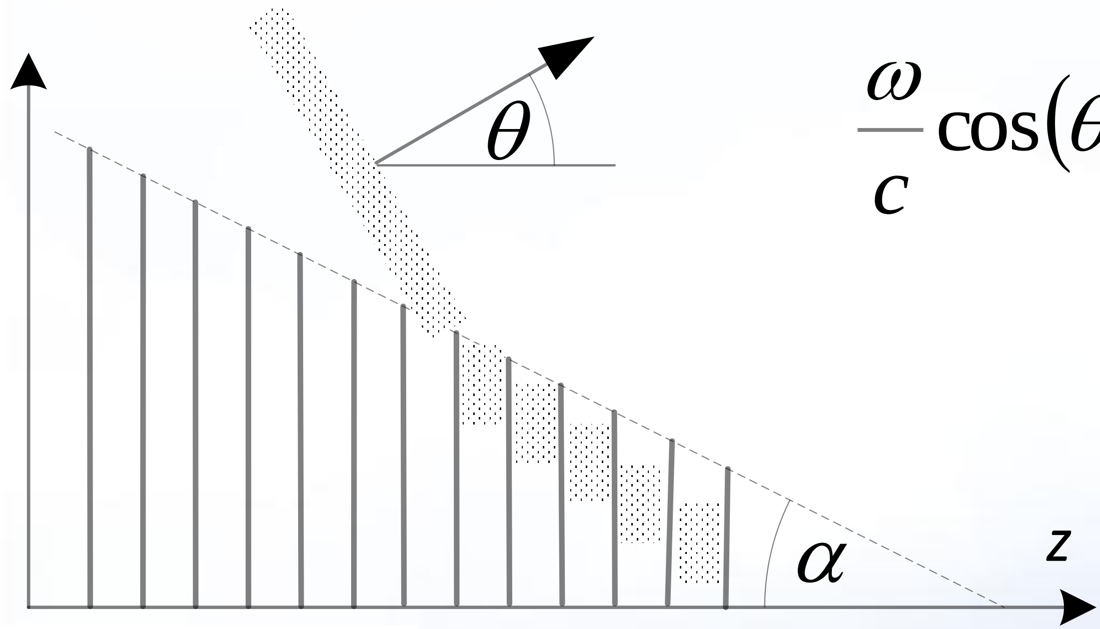
III. The refraction at the cone boundary

Inside the cone there are the waves $\exp\left(in\frac{\omega}{c}r + i\frac{\omega}{c}z - i\omega t \right)$

The tangent components of the wave vectors of this wave and the wave in the free space have to coincide at the boundary:



$$\frac{\omega}{c}\cos(\theta+\alpha) = -\frac{n\omega}{c}\sin\alpha + \frac{\omega}{c}\cos\alpha$$

Then

$$\theta = \mathrm{acos}(\cos\alpha - n\sin\alpha) - \alpha$$

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The reflection coefficient

$$R = \left(\frac{E_-}{E_+}\right)^2 = \left[\frac{n\sin(\theta+\alpha)-\cos\alpha}{n\sin(\theta+\alpha)+\cos\alpha}\right]^2 = \left[\frac{n\sqrt{(1-n^2)\tan^2\alpha+2n\tan\alpha-1}}{n\sqrt{(1-n^2)\tan^2\alpha+2n\tan\alpha+1}}\right]^2$$

It can be found from the boundary conditions

$$H = H_+ + H_-$$

$$E\sin(\theta+\alpha) = (E_+ + E_-)\cos\alpha$$

$$H_+ = -nE_+, \quad H_- = nE_-, \quad H = -E$$

Remark:

The foils form the anisotropic media with the diagonal permittivity tensor ε = diag($i\infty$,$i\infty$,n^2), and the radiation may be considered as the Cherenkov radiation in it.

There is no reflection for

$$\alpha_0 = \text{atan}\frac{n \pm \sqrt{n^2 - 1 + 1/n^2}}{n^2 - 1}$$

For $n = 1$ $\alpha_0 = \text{atan}(1/2) \approx 27^\circ$ and $R < 0.1$ for $10° < \alpha < 60°$.

It allows using different angles and not only cones, but other revolution surfaces, to control the wave front shape.

IV. The wave attenuation

The length of the *e*-time power attenuation is

$$\Delta r = cg \frac{|H_\alpha|^2}{4\pi n} \bigg/ \frac{2c\,\mathrm{Re}\,\zeta\,|H_\alpha|^2}{4\pi} = \frac{g}{2n\,\mathrm{Re}\,\zeta}$$

where ζ is the surface impedance.

The known normal-incidence absorption coefficients $4\mathrm{Re}\zeta$ in the THz range are typically less than one percent, but for small gaps the attenuation may be significant. Therefore it needs to choose the cone angle to be less than the value for zero reflection.

V. The multiple scattering

The multiple scattering of electrons on the atomic nuclei of foils (here we suppose the absence of matter between foils) increases the angle spread of electrons

$$\frac{d}{dz}\left\langle x'^2 \right\rangle = \frac{1}{X_0}\left(\frac{13.6MeV}{E}\right)^2$$

where X_0 is the radiation length of the foil cone material, E is the particle energy.

The corresponding growth of the beam transverse size a is

$$a^2 = a_0^2(z) + \frac{z^3}{3}\frac{d}{dz}\left\langle x'^2 \right\rangle$$

where a_0 is the size without multiple scattering.

The energy limitation

The transverse size has to be less than the bunch length. Therefore

$$\frac{d}{dz}\left\langle x'^2 \right\rangle < 3\frac{l^2}{L^3}$$

It may be expressed as the limitation for the electron energy

$$E > 13.6\,MeV\,\frac{L^{3/2}}{l\sqrt{3X_0}}$$

radiation length of graphite is about 0.2 m. Stacking the foils with thickness 10 micron and period 0.2 mm, one obtain the radiation length 4 m. Then for our example the minimum energy is 110 MeV.

Remarks

1. For high peak currents the focusing by the beam azimuthal magnetic field may reduce the beam size growth. It makes the energy limitation easier.

2. The small holes in the foils can eliminate the multiple scattering. In this case one has to substitute the hole radius divided by $\sqrt{2}$ instead of the r. m. s. transverse beam size *a* to the effective bunchlength l_{eff} .

VI. The on-axis field

$$E_\omega(0) = -\frac{2\pi^2\omega}{c^2}\int_0^\infty j_\omega(r) H_0^{(1)}(kr) r dr = -Z I_\omega$$

For Gaussian beam and low $\omega\sqrt{n^2a^2 + g^2/12} \ll 1$ frequencies

$$Z \approx \frac{\pi|\omega|}{c^2} + \frac{i\omega}{c^2}\left[\ln\left(\frac{\omega^2 n^2 a^2}{2c^2}\right) + 0.577\right]$$

The real part of this impedance gives the loss obtained before.
The imaginary part is almost inductive. This impedance may cause an additional bunching of the beam.

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Thank you