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# Two-dimensional Coherence Measurements of FEL Radiation: The Heterodyne Speckle Approach

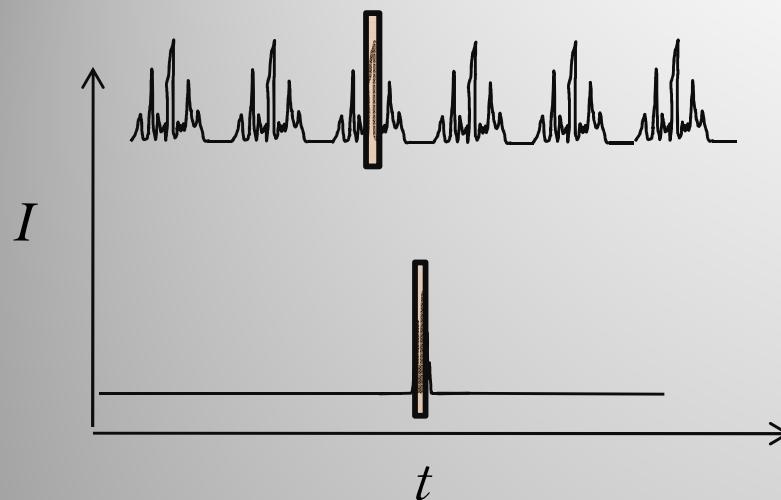
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**Matteo D. Alaimo**

**University of Milan & INFN**



# Transverse coherence

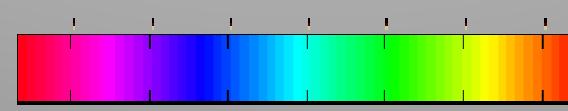
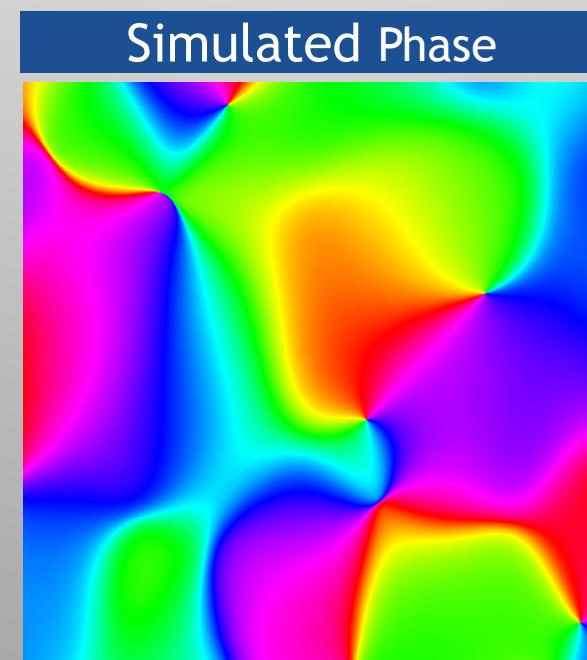
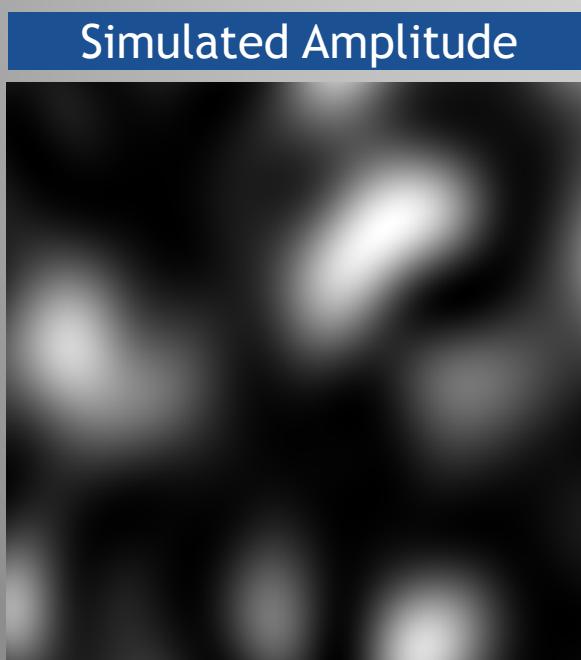


**Synchrotron Radiation**  
pulse duration 30 ps

**FEL Radiation**  
pulse duration 1 ps

## Complex Coherence Factor

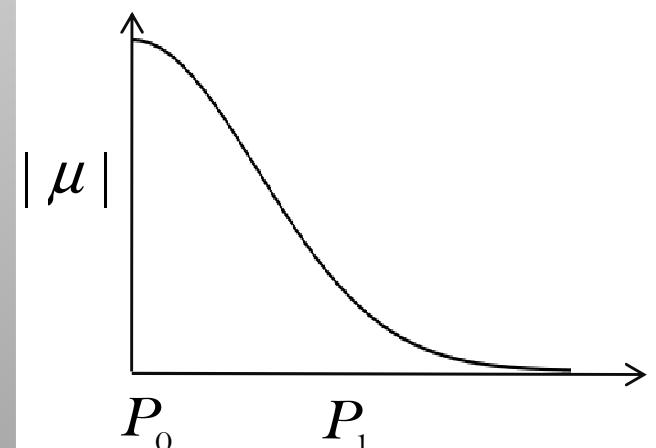
$$\mu(P_0, P_1) = \frac{\langle E(P_0)E^*(P_1) \rangle}{\sqrt{\langle I(P_0) \rangle \langle I(P_1) \rangle}}$$



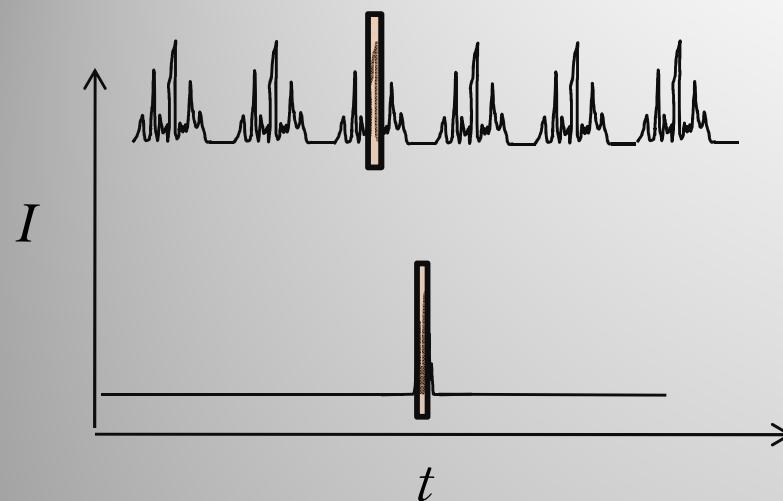
0

$2\pi$

## Example: gaussian Coherence factor



# Transverse coherence

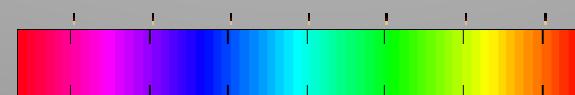
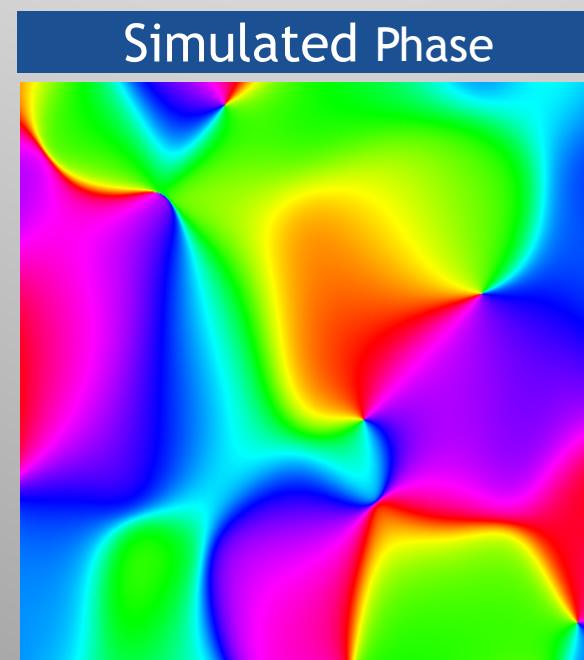
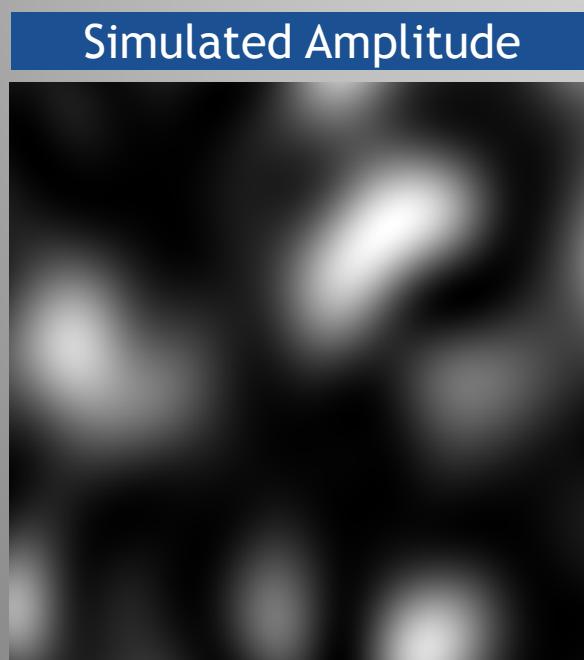


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pulse duration 1 ps

## Complex Coherence Factor

$$\mu(P_0, P_1) = \frac{\langle E(P_0)E^*(P_1) \rangle}{\sqrt{\langle I(P_0) \rangle \langle I(P_1) \rangle}}$$

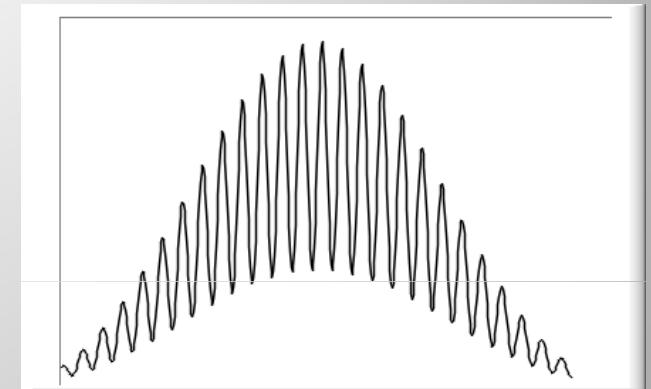
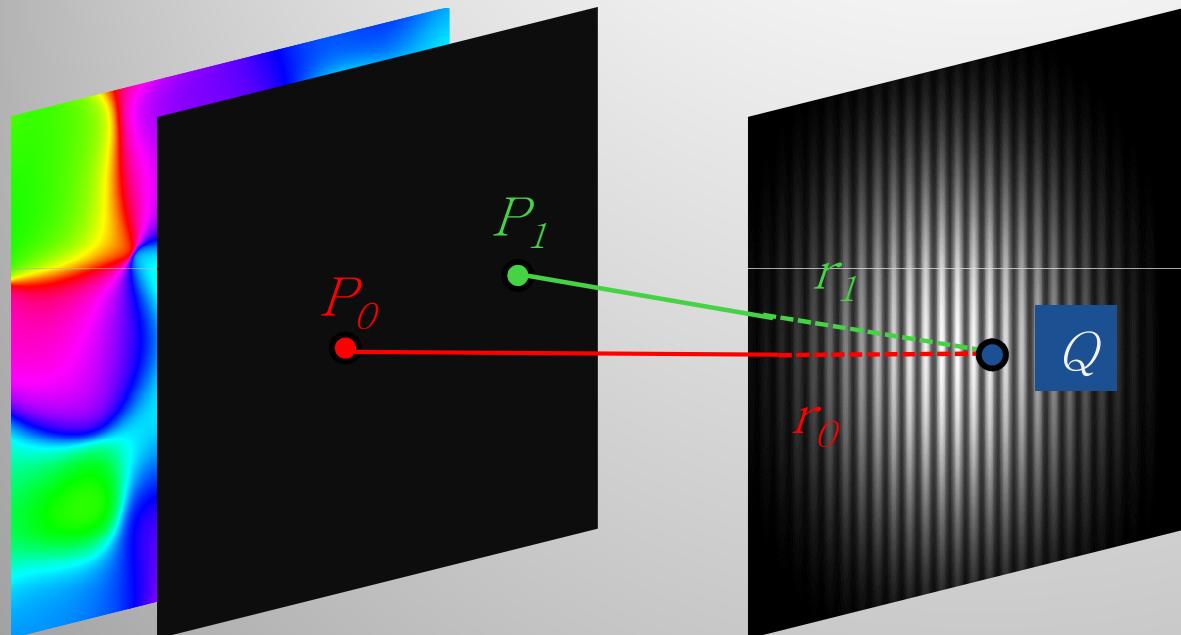


0  $2\pi$

## Why coherence is important?

- Phase contrast imaging
- Coherent Diffraction Imaging and Topography
- X-Ray Photon Correlation Spectroscopy

# Young's Interferometer

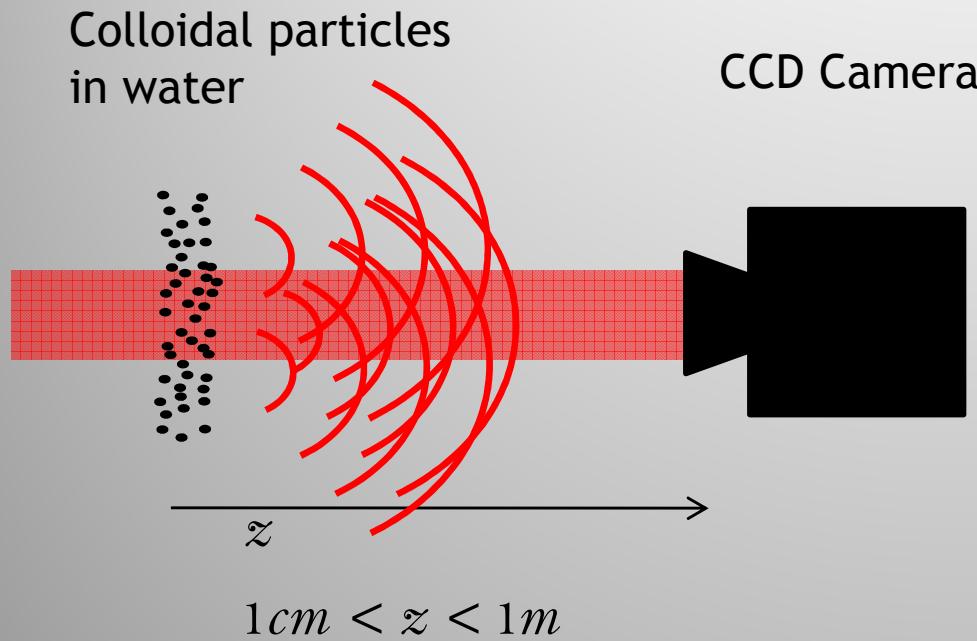


**Visibility:**

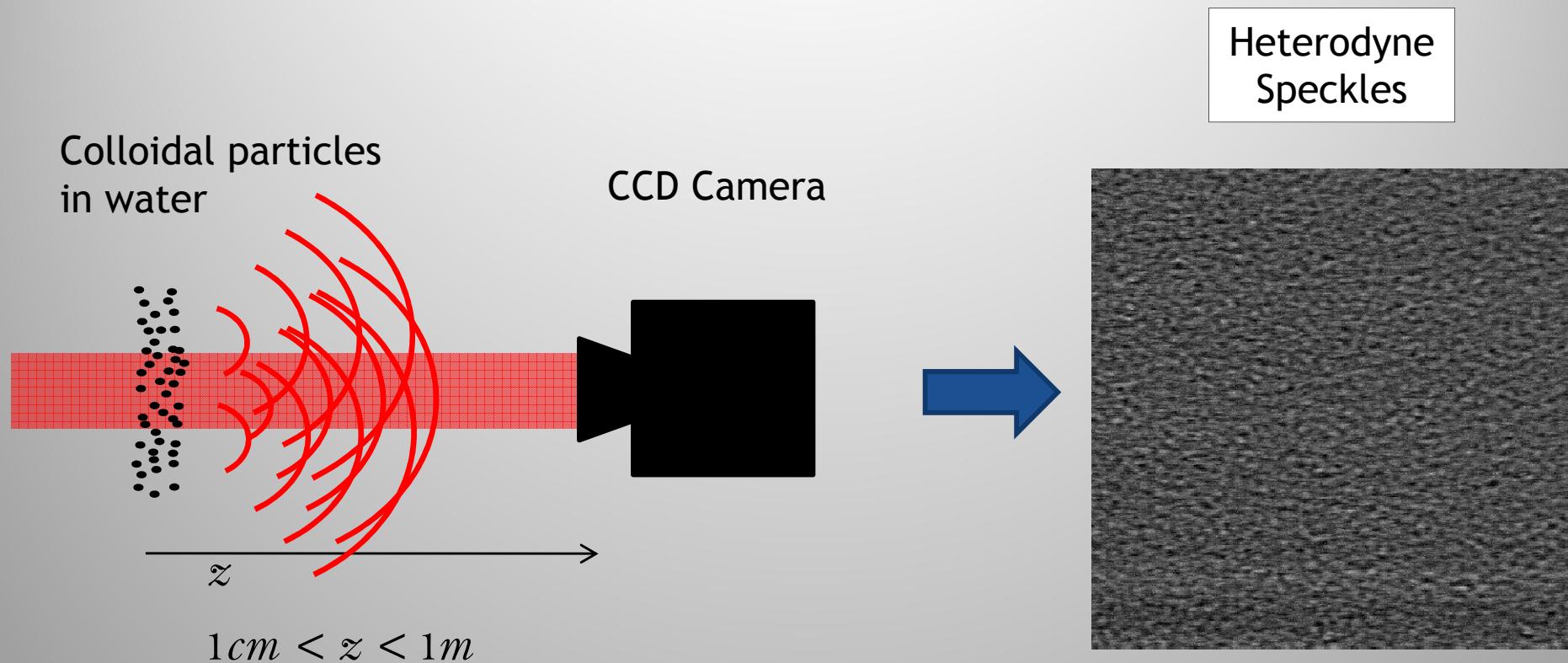
$$V = \left( \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} \right)$$

$$V \propto |\mu(P_0, P_1)|$$

# Heterodyne Speckle Approach



# Heterodyne Speckle Approach

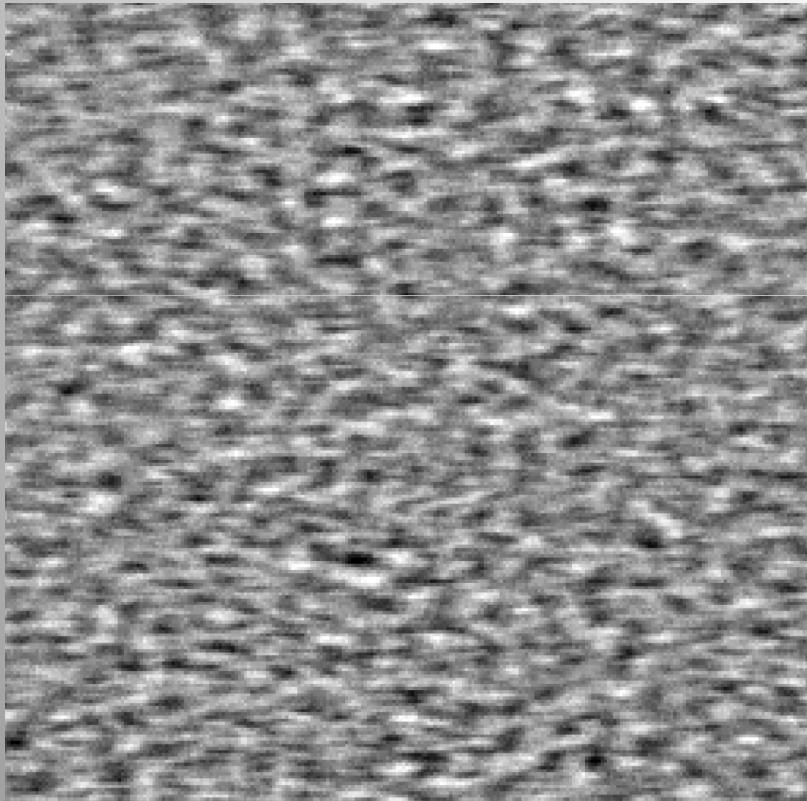


*Probing the transverse coherence of an undulator X-ray beam using brownian particles, PRL, M.D. Alaimo & al.*

*Heterodyne speckle approach for two-dimensional map of SR coherence, JSR (in submission), M.Manfredda & al.*

# Heterodyne Speckles

Speckles (ID06 – ESRF -  $\lambda=0.1$  nm)



$E_T$ : transmitted field  
 $E_S$ : scattered field

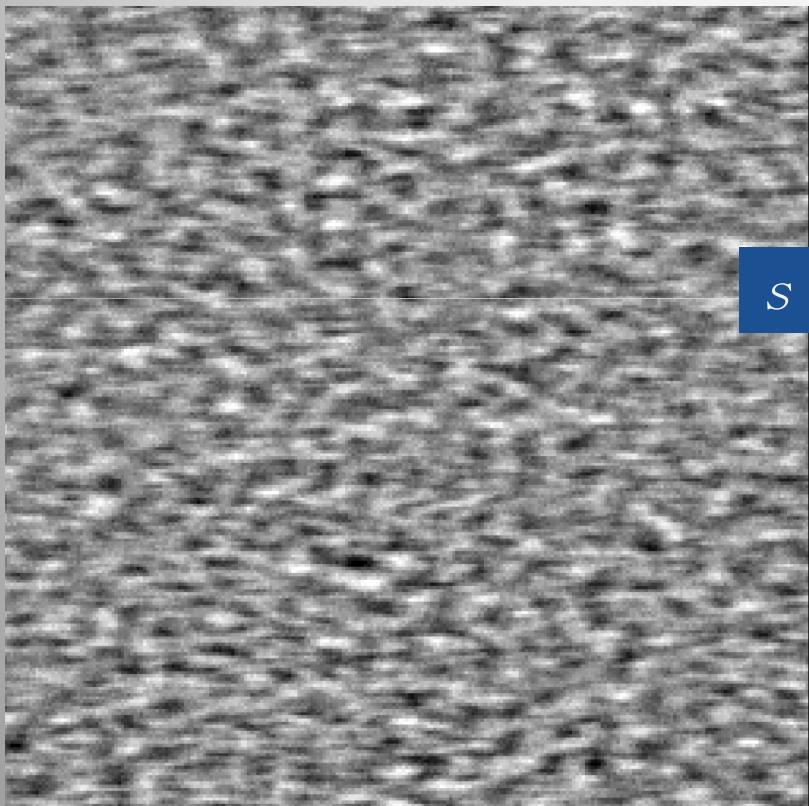
$$I = \left\langle \left| E_T + \sum_s^N E_s \right|^2 \right\rangle = \left\langle |E_T|^2 \right\rangle + \left\langle 2 \sum_s^N \Re E_T^* E_s \right\rangle + \left\langle 2 \sum_{s,s'}^N \cancel{\sum} E_s^* E_{s'} \right\rangle$$

Heterodyne Term

$$\sum_s^N E_s \ll E_0$$

# Heterodyne Speckles

Speckles (ID06 – ESRF -  $\lambda=0.1$  nm)



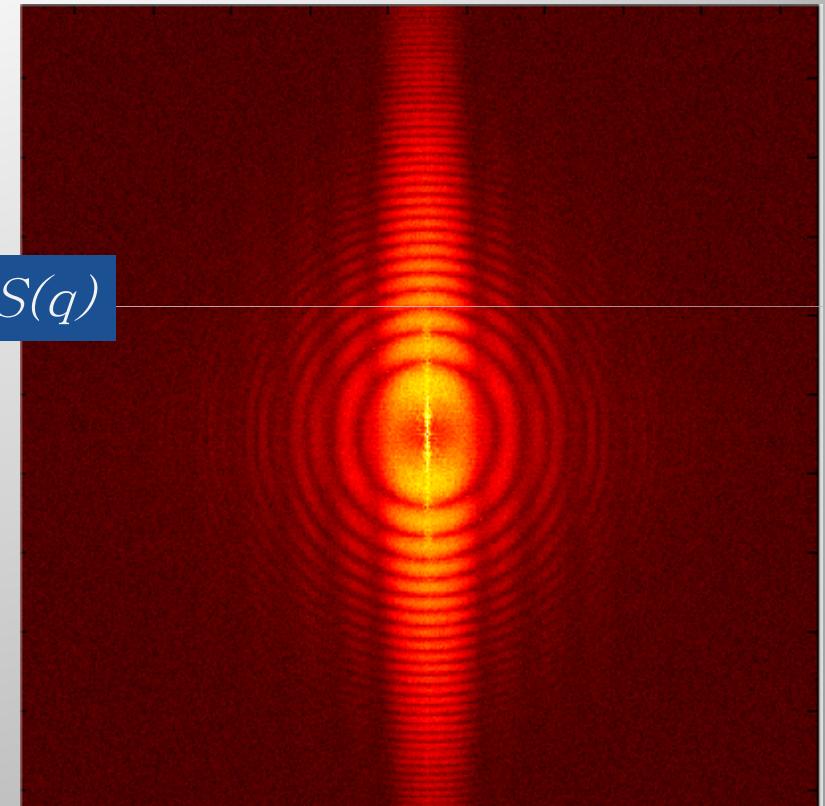
Spatial power spectrum

$\mathcal{F}$

$s(r)$



$S(q)$



$I(q)$ : particle form factor (flat)

$T(q)$ : Talbot trasfer function [  $\sin^2(q^2z/2 k)$  ]

$H(q)$  = Sensor transfer function

$$S(q) = I(q) T(q) H(q) C(q)$$

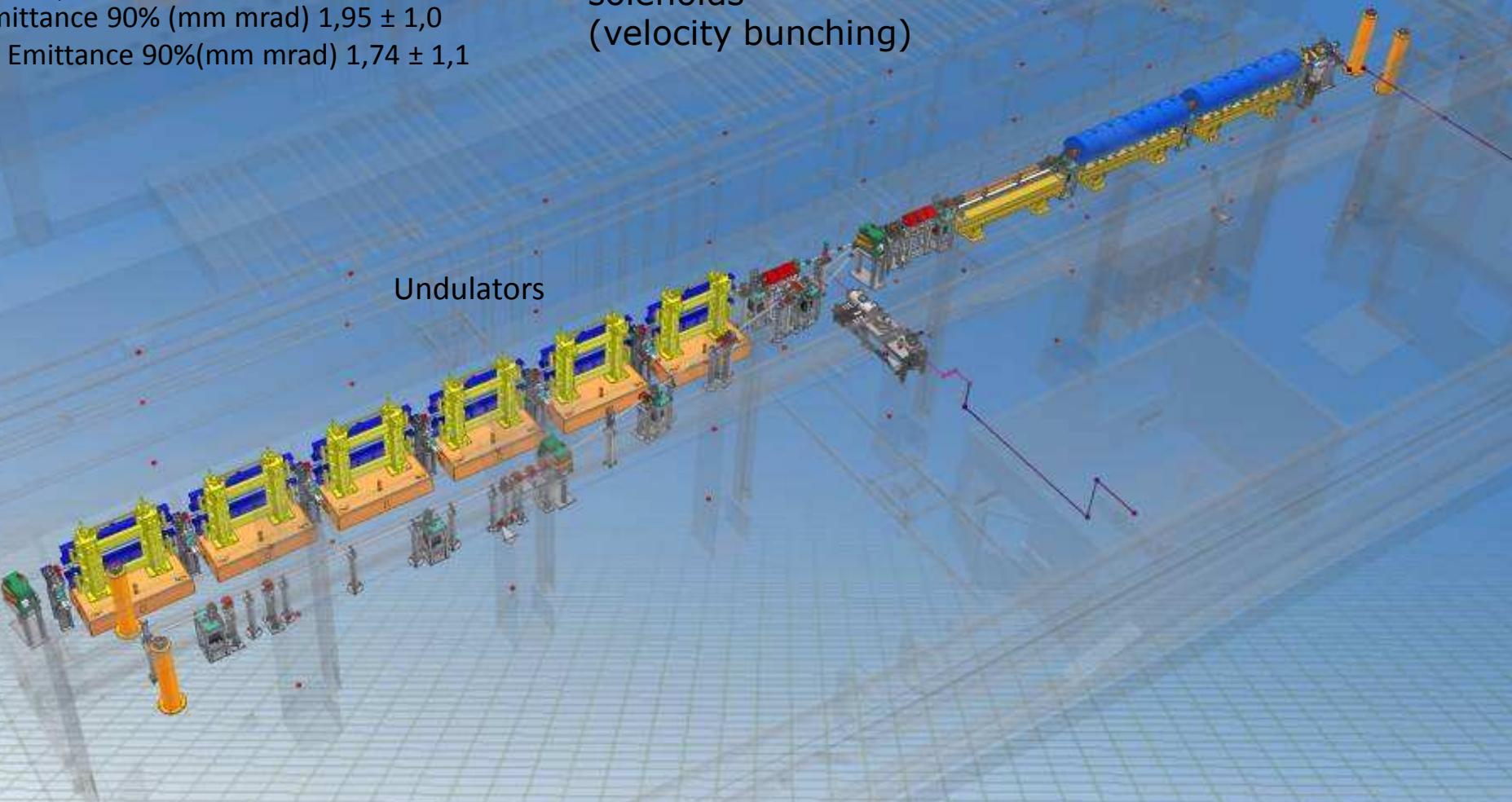
$$r = \frac{q}{k} z$$

$$C(r(q)) = |\mu|^2 \quad (\mu: \text{Complex Coherence Factor})$$

- **6 sections undulator; 77 periods per section**
- Magnetic Length 215 cm
- K factor: from 0.5 and 3
- Beam energy  $E_B$ (MeV)  $162.5 \pm 0.27$
- Beam charge(pC)  $312 \pm 16$
- Energy Spread (proj: %)  $0.2 \pm 0.015$
- Energy Spread (slice %)  $0.050 \pm 0.005$
- Length r.m.s. (ps)  $1.65 \pm 0.05$
- Beam current  $I_{peak}$ (A)  $75.63 \pm 3.5$
- Vertical Emittance 90% (mm mrad)  $1.95 \pm 1.0$
- Horizontal Emittance 90%(mm mrad)  $1.74 \pm 1.1$

1.6 cells RF injector  
UCLA/BNL/SLAC design  
120 MV/m

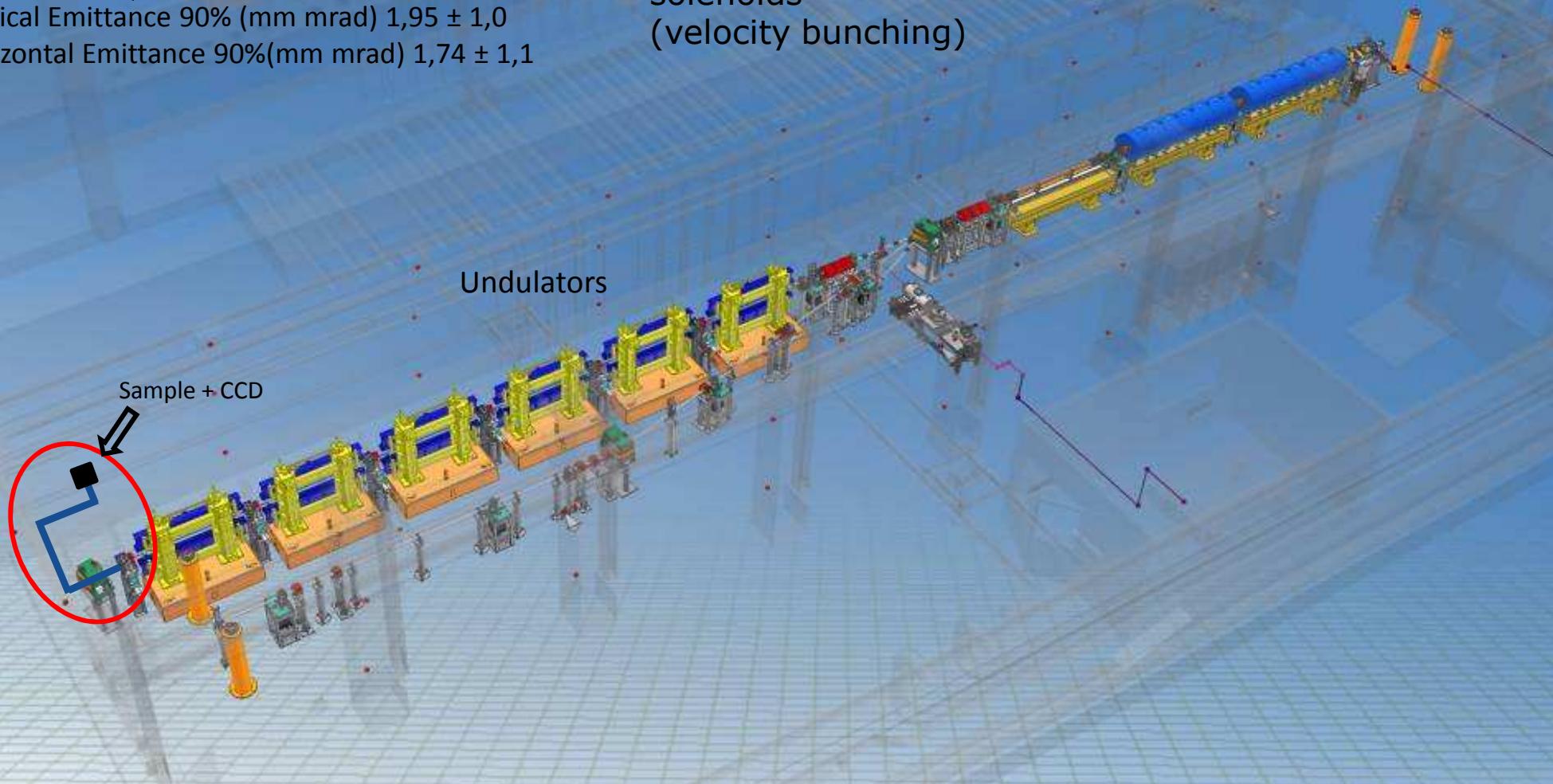
3 SLAC type Focalization  
solenoids  
(velocity bunching)



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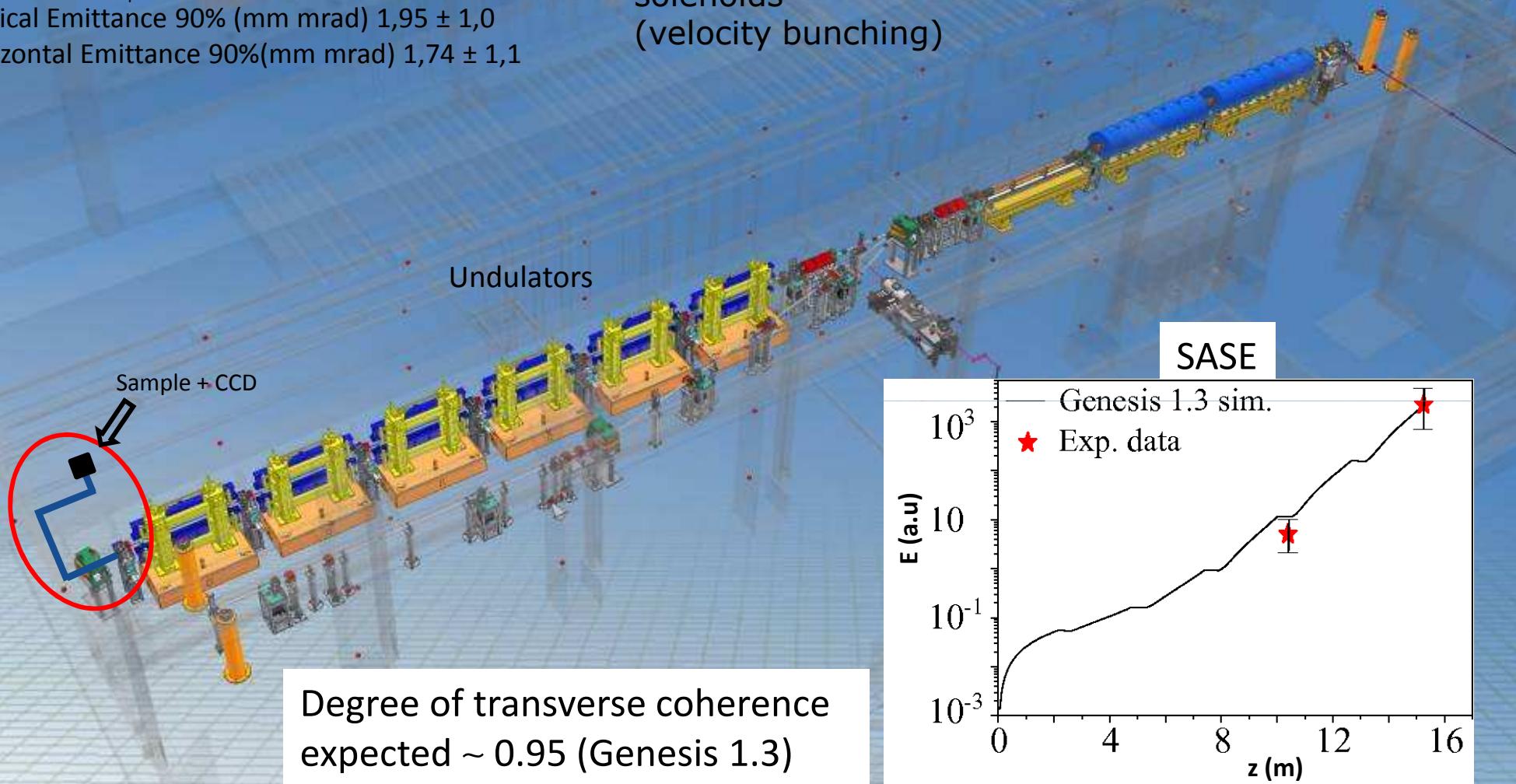
3 SLAC type Focalization  
solenoids  
(velocity bunching)



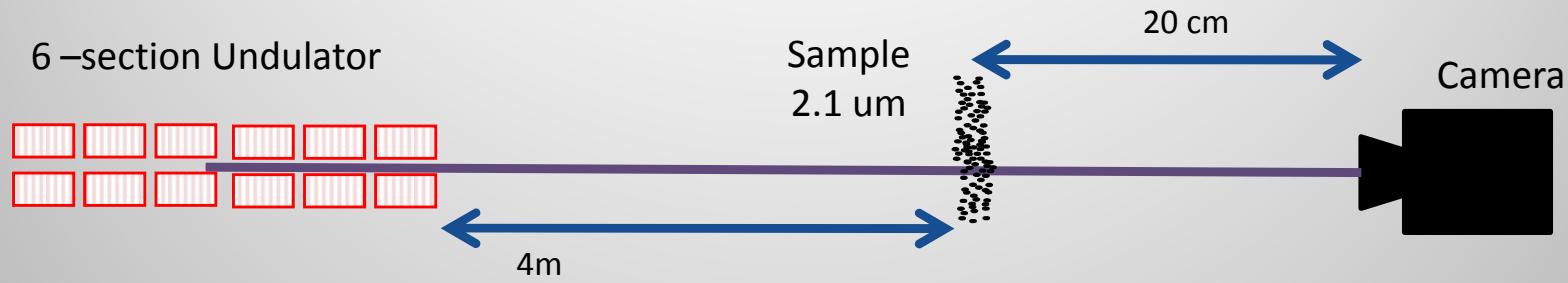
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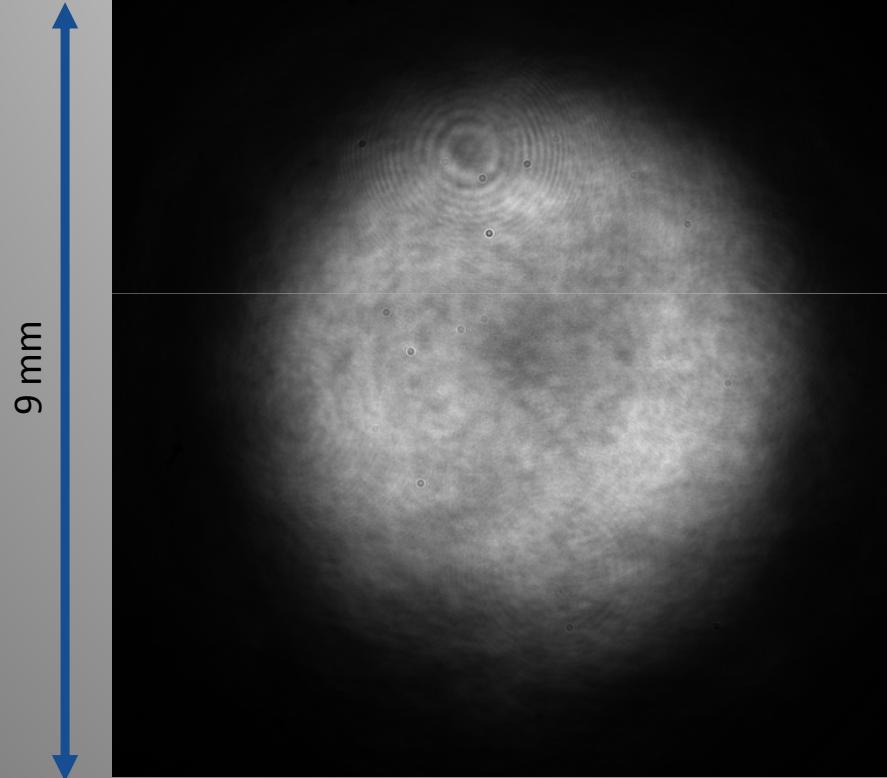
3 SLAC type Focalization  
solenoids  
(velocity bunching)



$$\lambda = 400\text{nm}$$

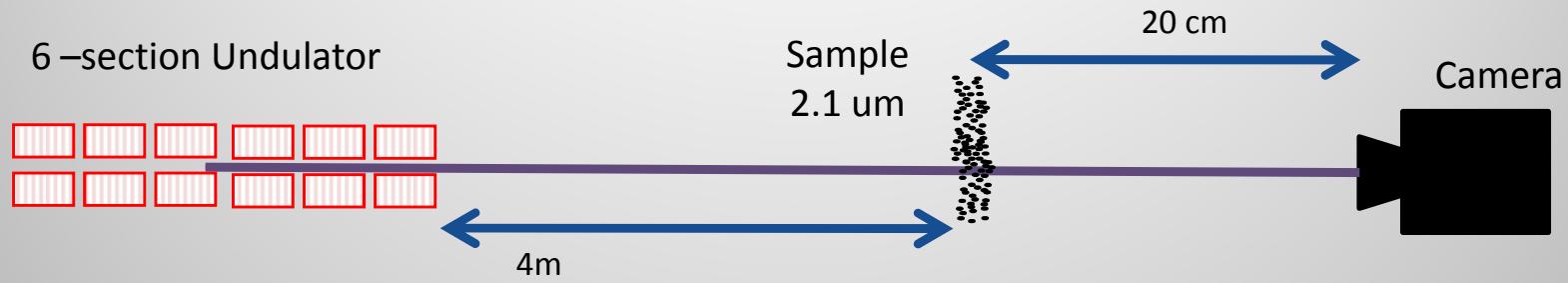


$I(t) : \text{image}$

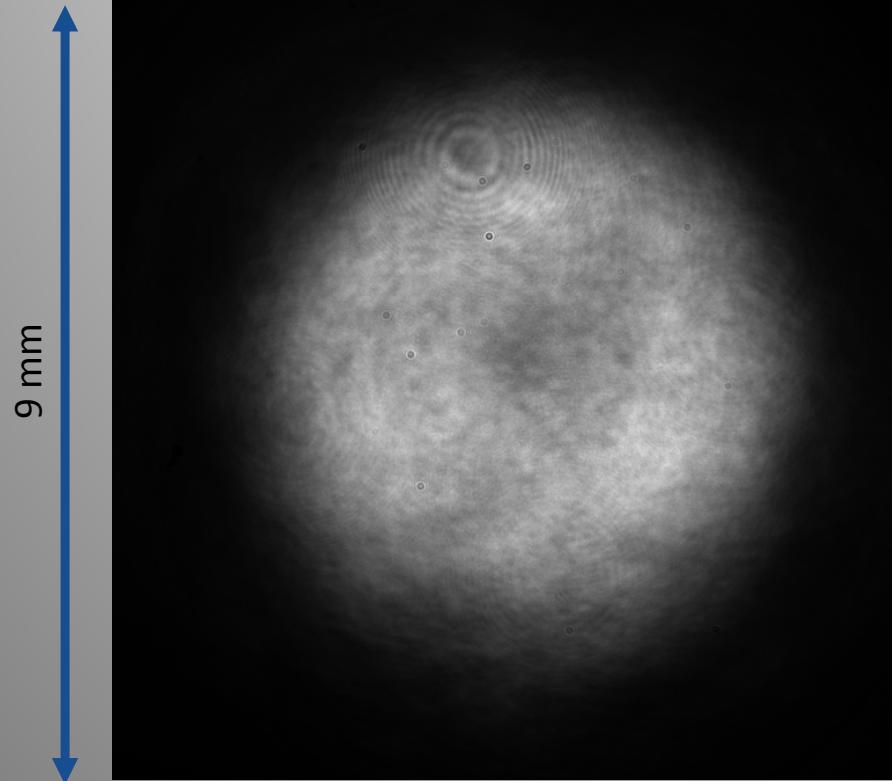


$$= \left\langle |E_T|^2 \right\rangle + \left\langle 2 \sum_s^N \Re E_T^* E_s \right\rangle$$

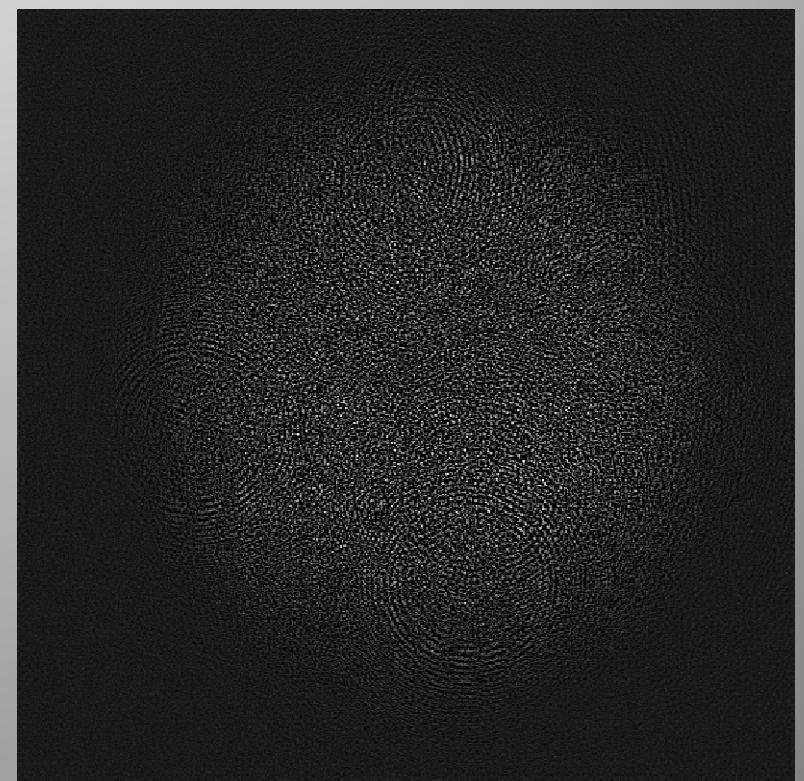
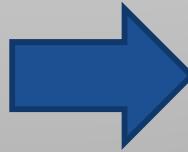
$$\lambda = 400nm$$

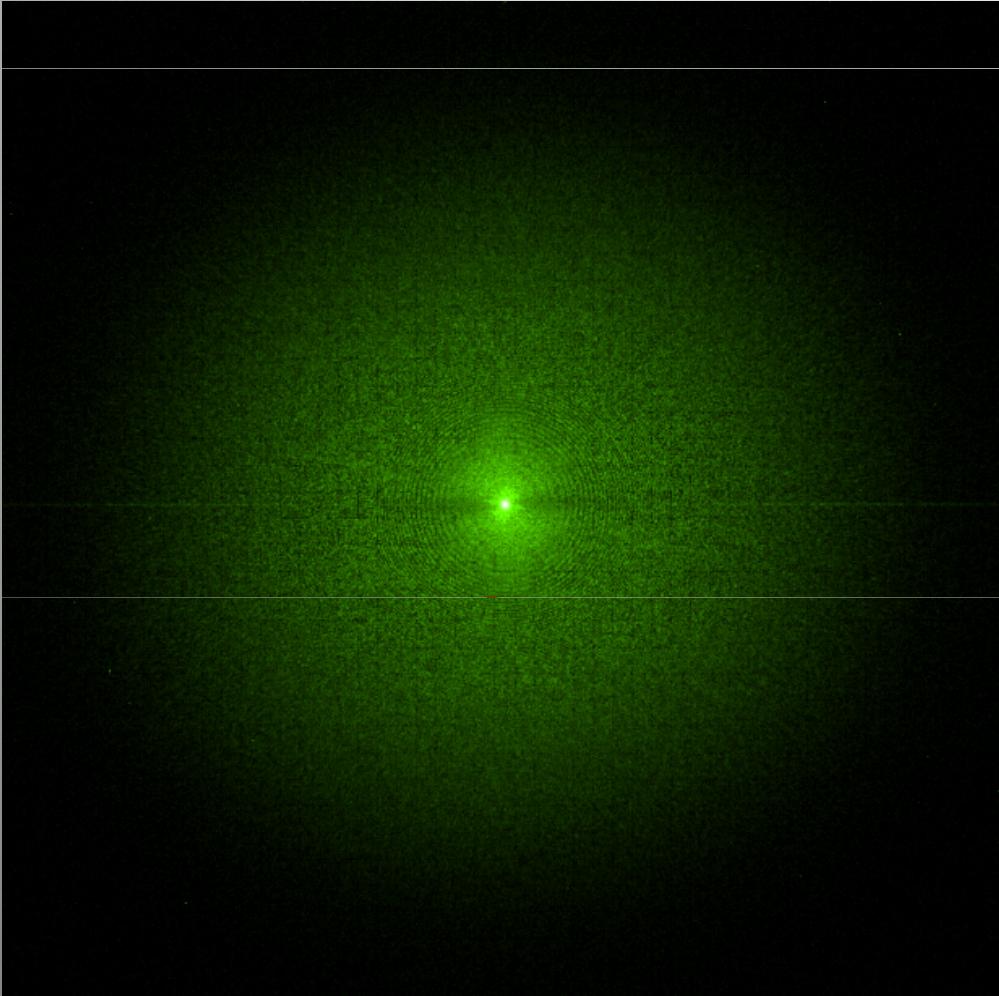


$I(t)$  : image



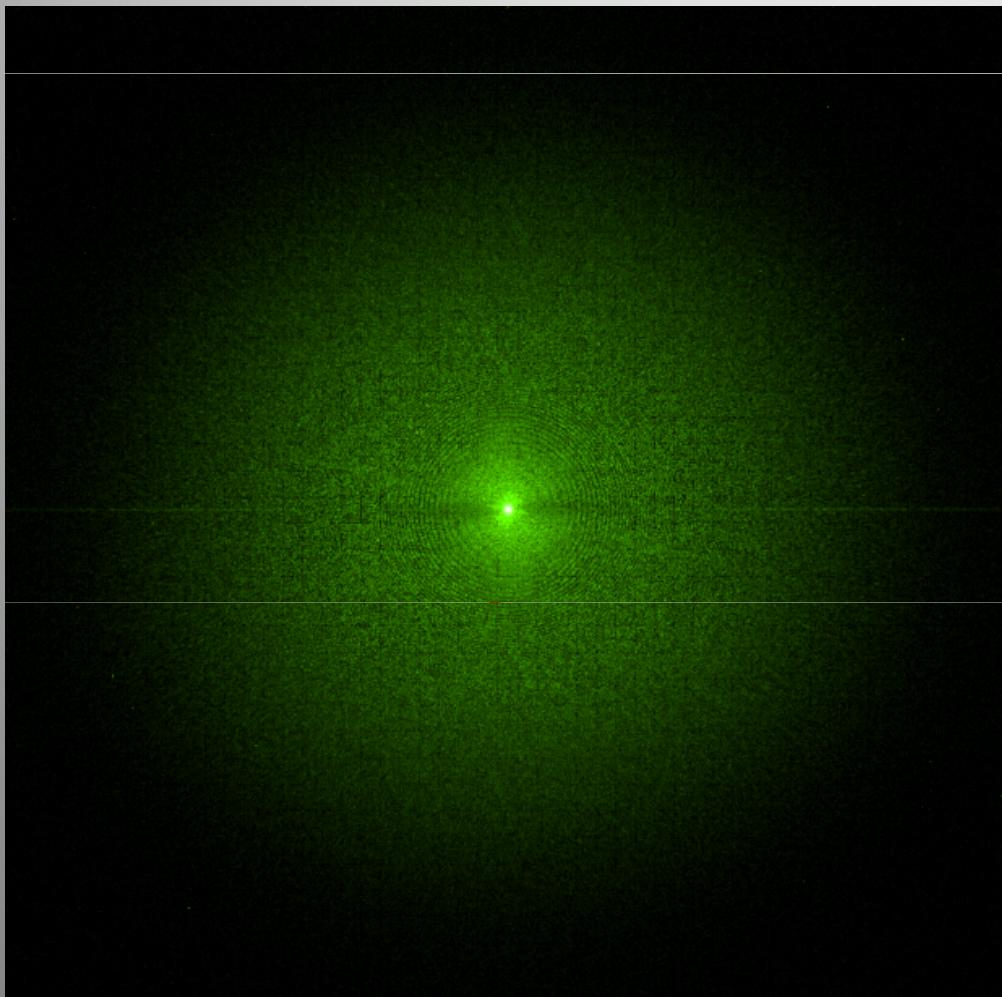
$I(t)-I(t+\tau)$ : image difference



**Full Coherence Factor****Typical 2D Power Spectrum**

Full Coherence Factor

Typical 2D Power Spectrum

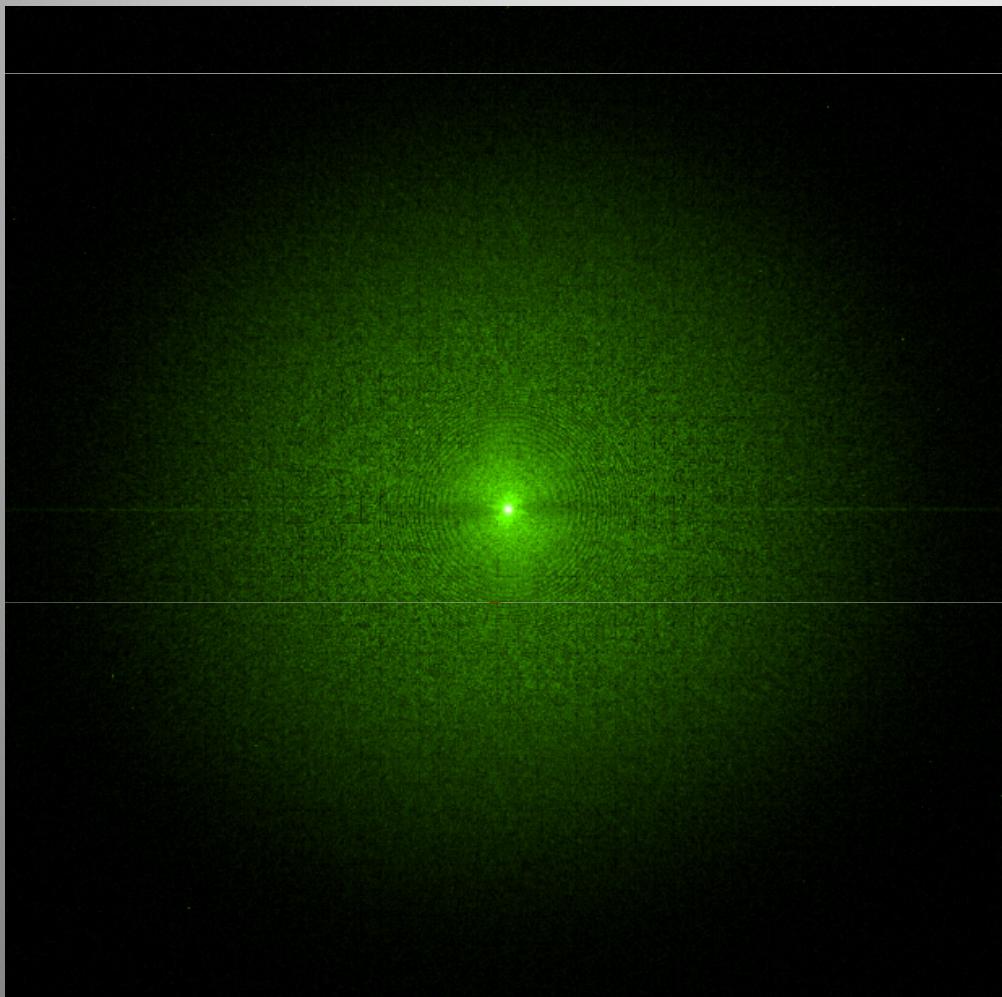


1200 px

600 px

**Full Coherence Factor**

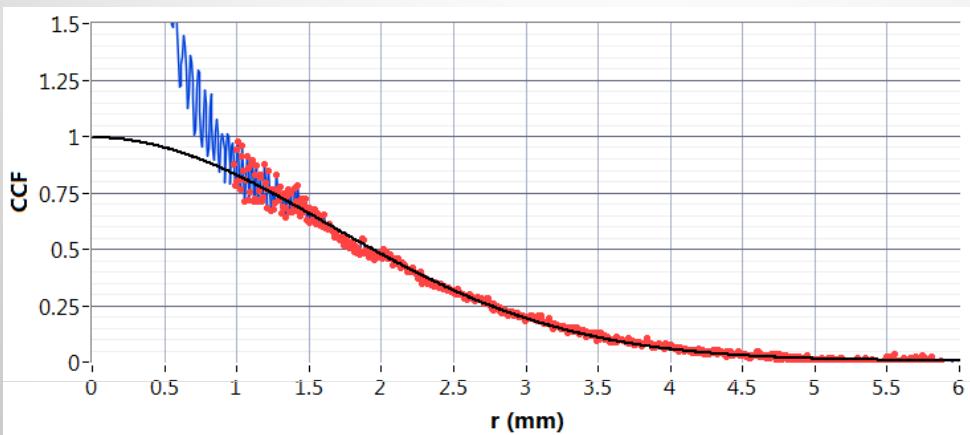
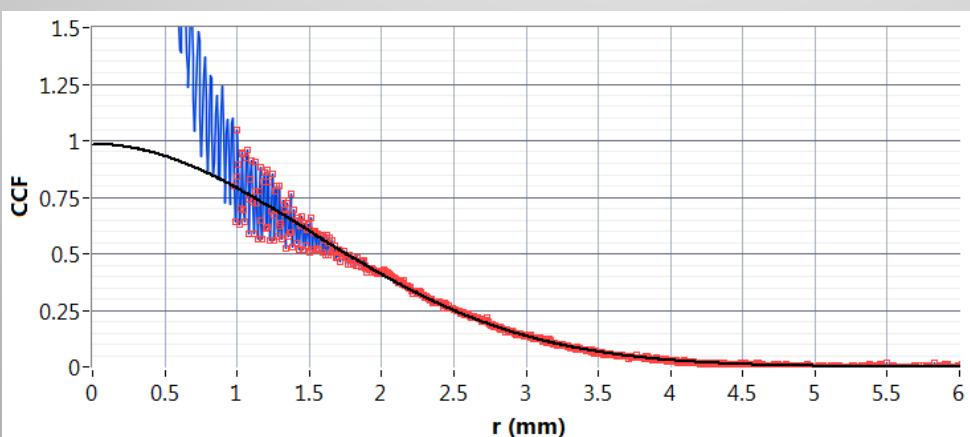
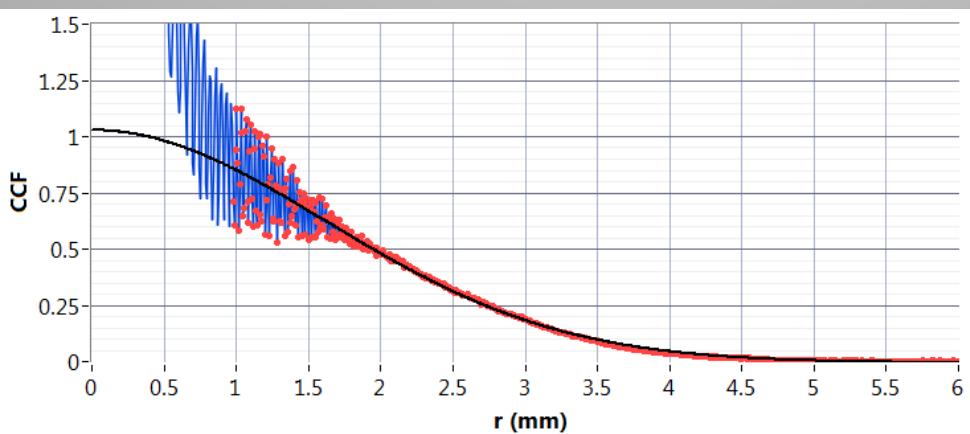
Typical 2D Power Spectrum



1200 px

600 px

720 000  
Young's double slit  
experiments

**4 UD Sections****5 UD Sections****6 UD Sections**

	$\sigma_{COH}$	$\sigma_{COH}/\sigma$
4U	1.64 mm	0.7
5U	1.50 mm	0.8
6U	1.62 mm	0.82

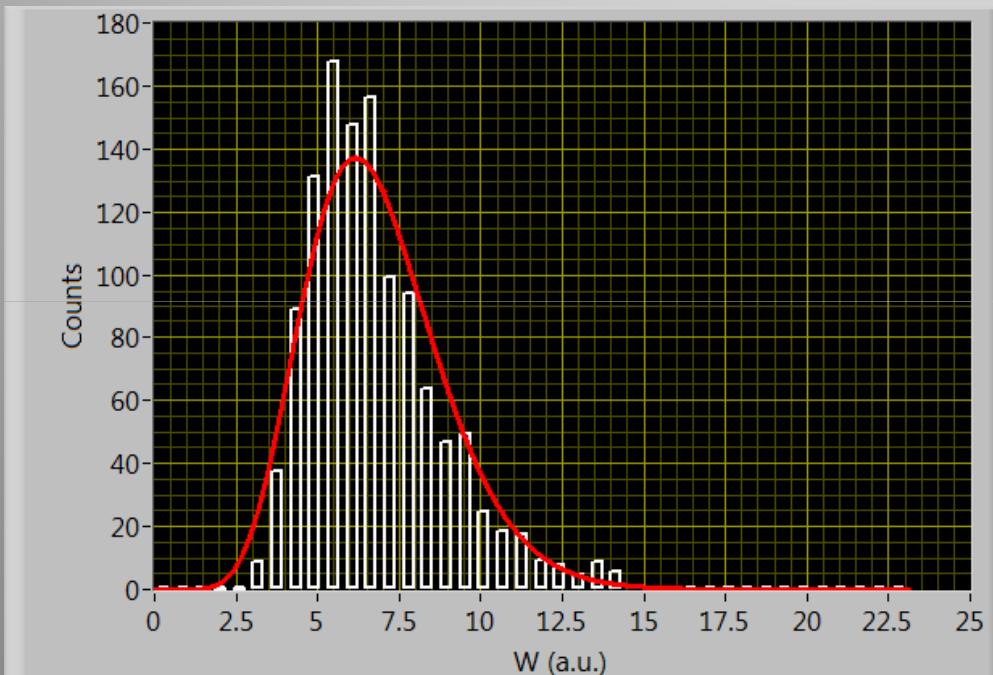
# Number of modes

## Gamma function

$$M = M_T \cdot M_L = \frac{1}{\sigma_w^2}$$

$$\sigma_w^2 = \frac{\sqrt{\langle (W - \langle W \rangle)^2 \rangle}}{\langle W \rangle}$$

4 undulators



4U  $M=10.5$

5U  $M=6.5$

6U  $M=3.2$

Statistical analysis on the  
same images used for  
coherence measurements

# Conclusions and perspectives

## Heterodyne speckle approach

Simple and direct interferometric setup

Poster **THPD066**  
Michele Manfredda

Two -Dimensional map

Valid for different wavelength

Time resolved

# Conclusions and perspectives

## Heterodyne speckle approach

Simple and direct interferometric setup

Poster **THPD066**  
Michele Manfredda

Two -Dimensional map

Valid for different wavelength

## Outlooks

Time resolved

Improvements in samples for different wavelength

Improving background subtraction

High order Correlations (asymmetric CCF)

*Thank you for your attention*

Special Thanks

**M. Manfredda, M. A. C. Potenza, M. Giglio and G. Geloni**  
for the strict cooperation



I thank all **ID02, ID06** and **ESRF** Staff for providing  
technical support and useful hints.



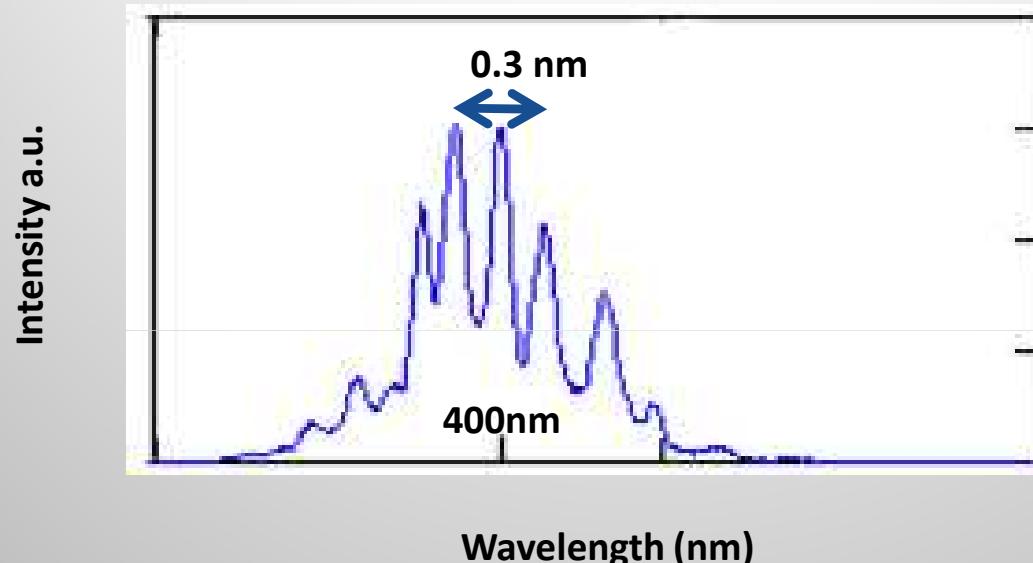
Thanks to **L. Serafini, V. Petrillo**

for the cooperation within INFN and SPARC.

Thanks to all the SPARC (LNF) and INFN staff who  
cooperated at this work.



# Temporal Coherence

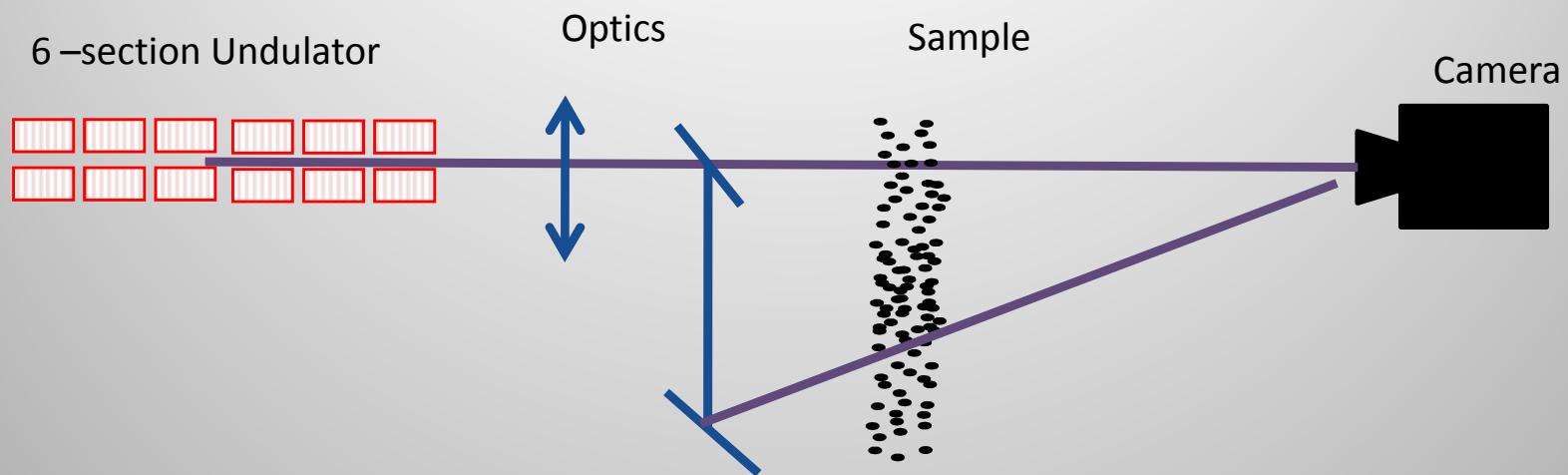


Coherence Length

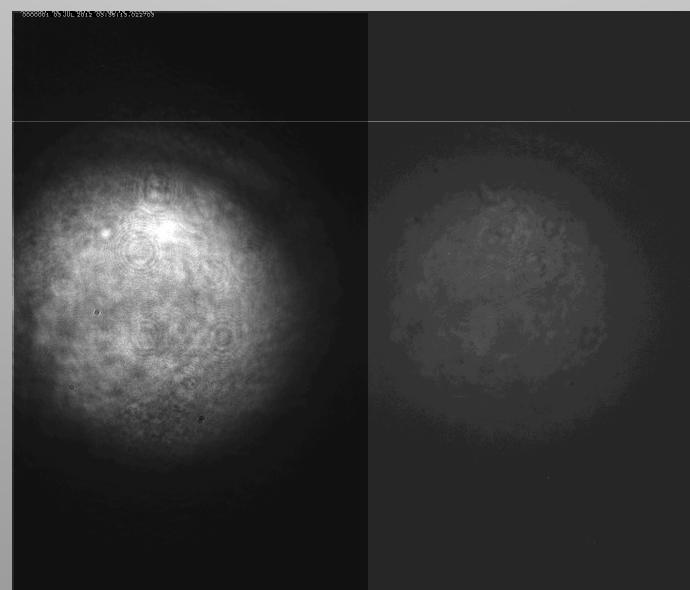
$$\ell_c = \frac{\lambda^2}{2\Delta\lambda} \approx 270 \mu m \quad \longrightarrow \quad \ell_c \gg \delta l$$

Maximum experimental  
transverse displacement

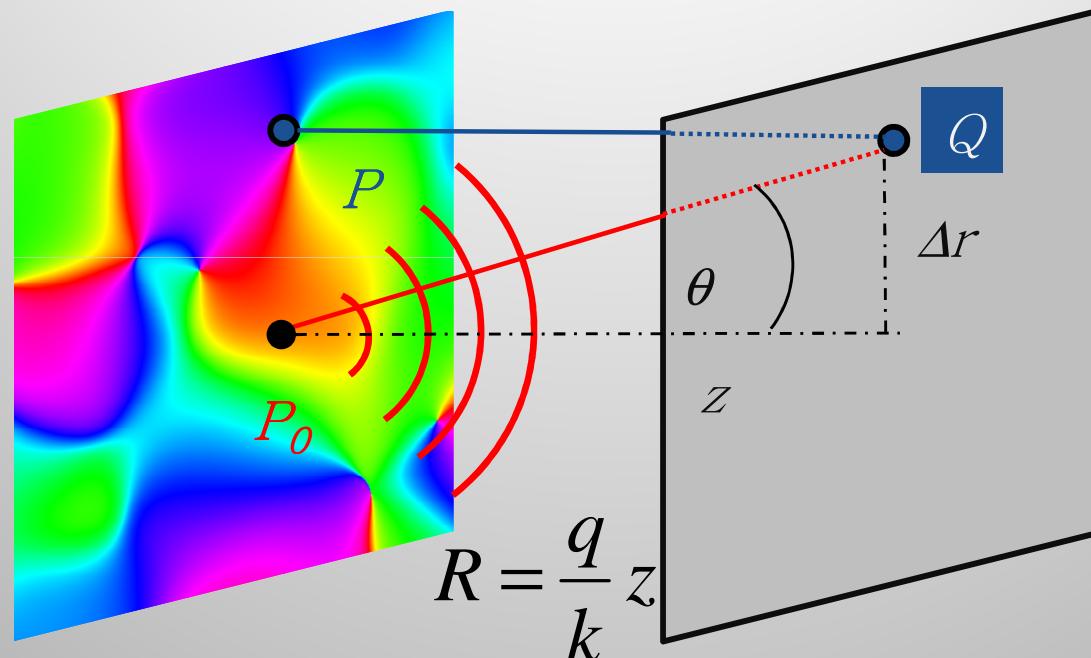
$$\delta l \approx 90 \mu m$$

$\lambda = 400nm$ 

Possible with visible wavelength only

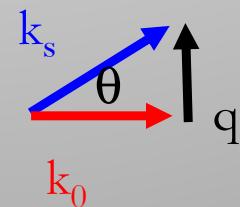


# In-Line Holography

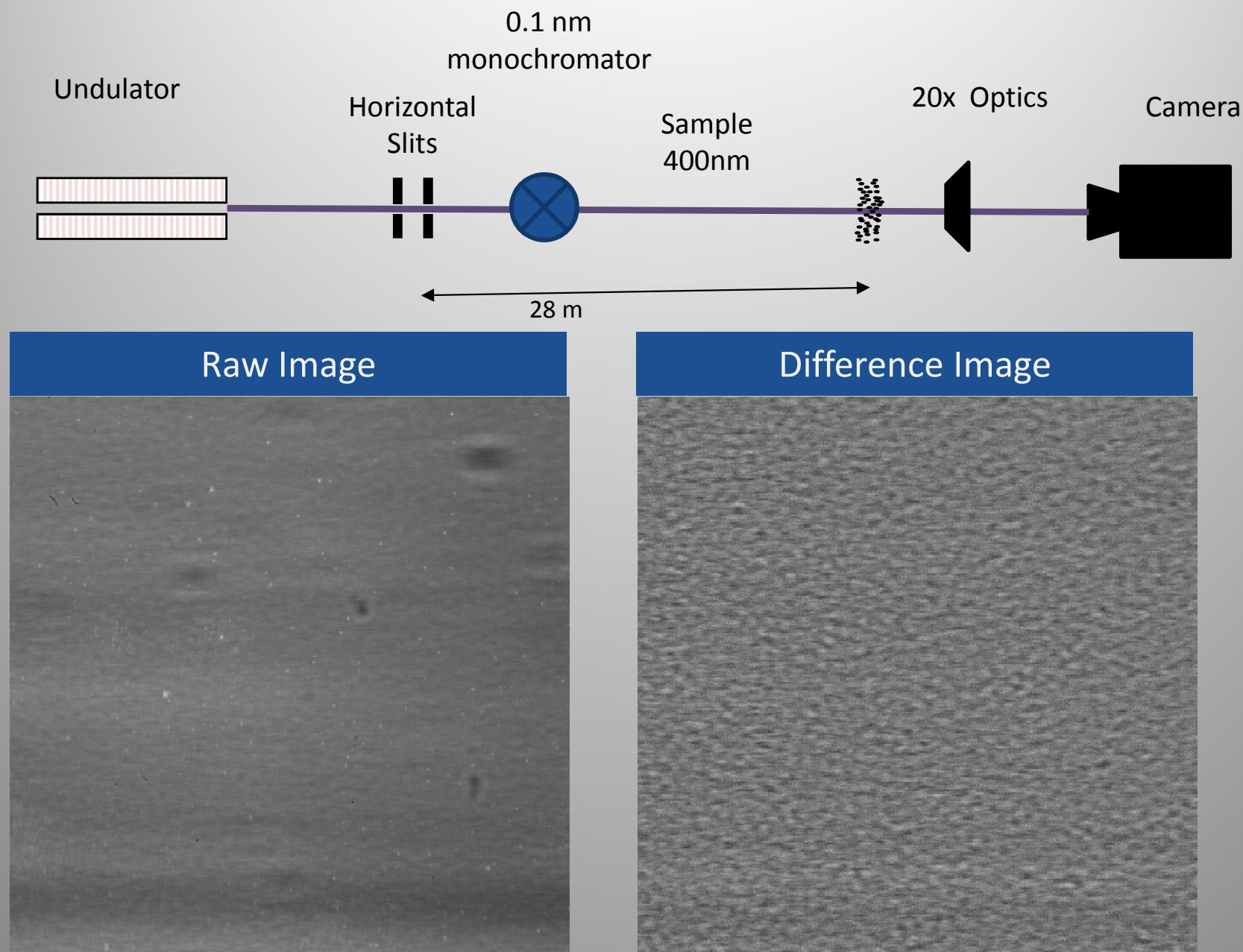


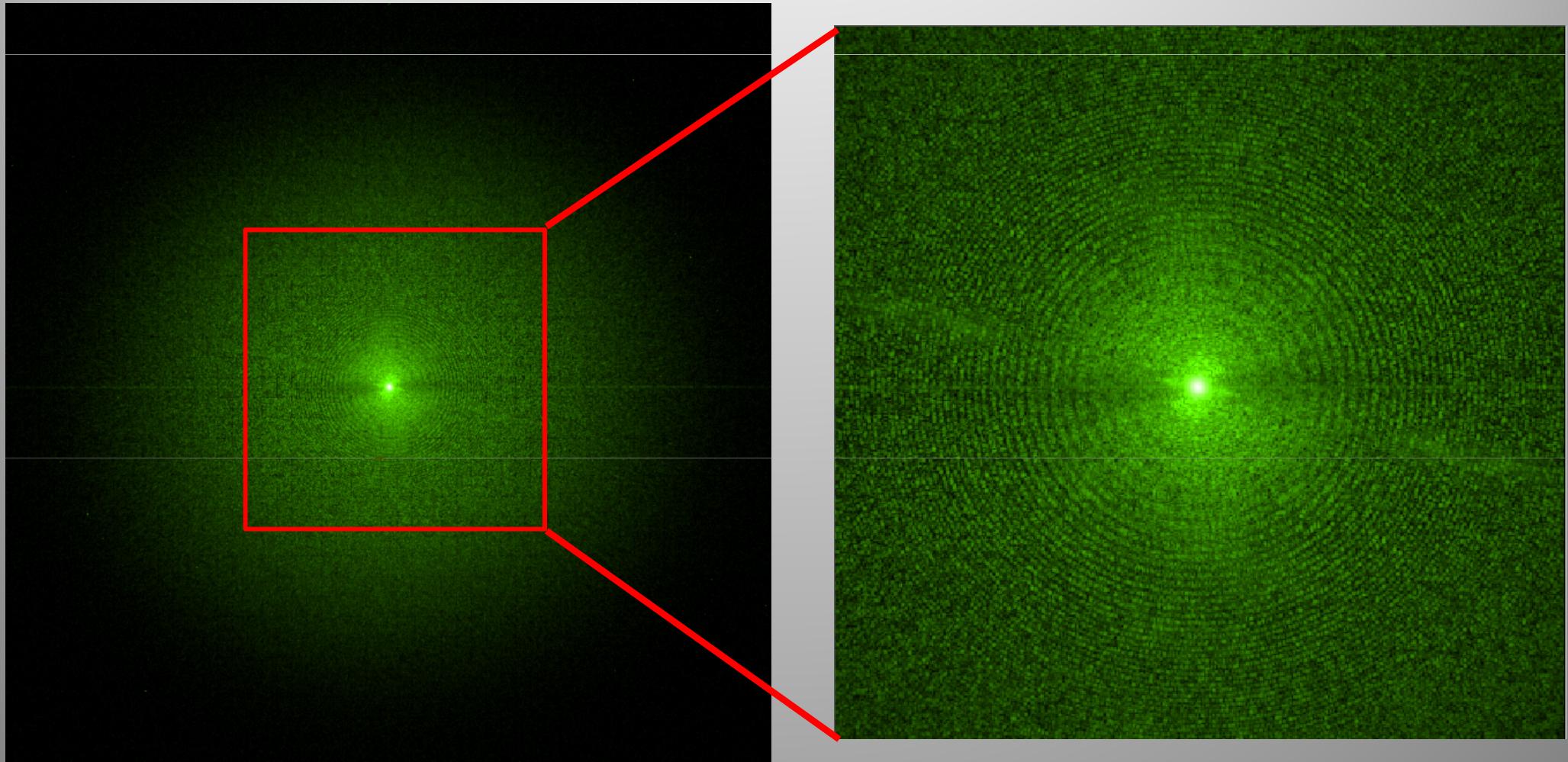
$$\Delta r = \vartheta z$$

$$\vartheta = \frac{q}{k}$$

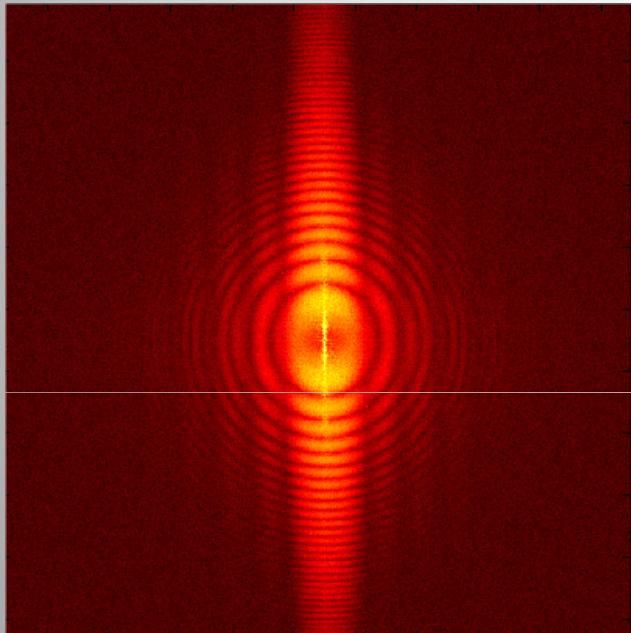


# Synchrotron - ID 06 Setup



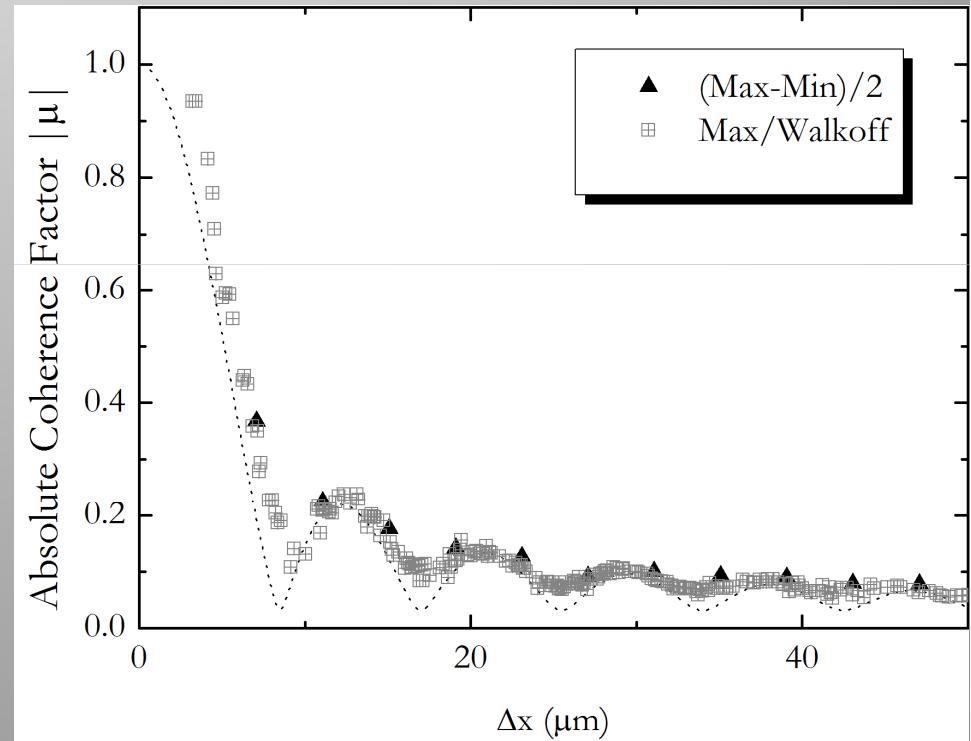
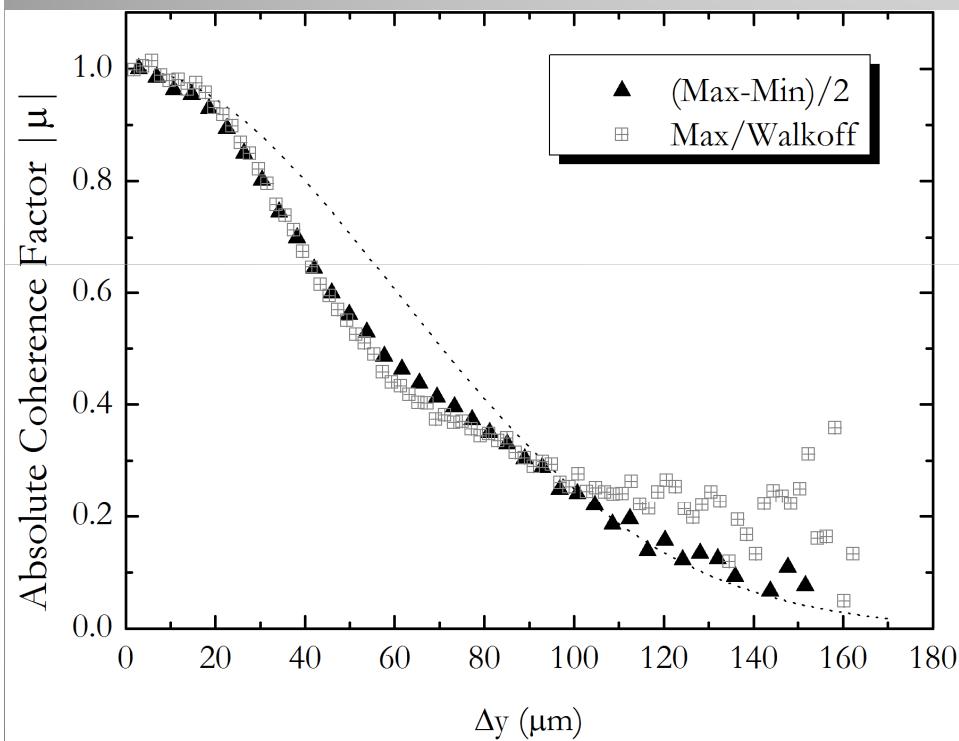
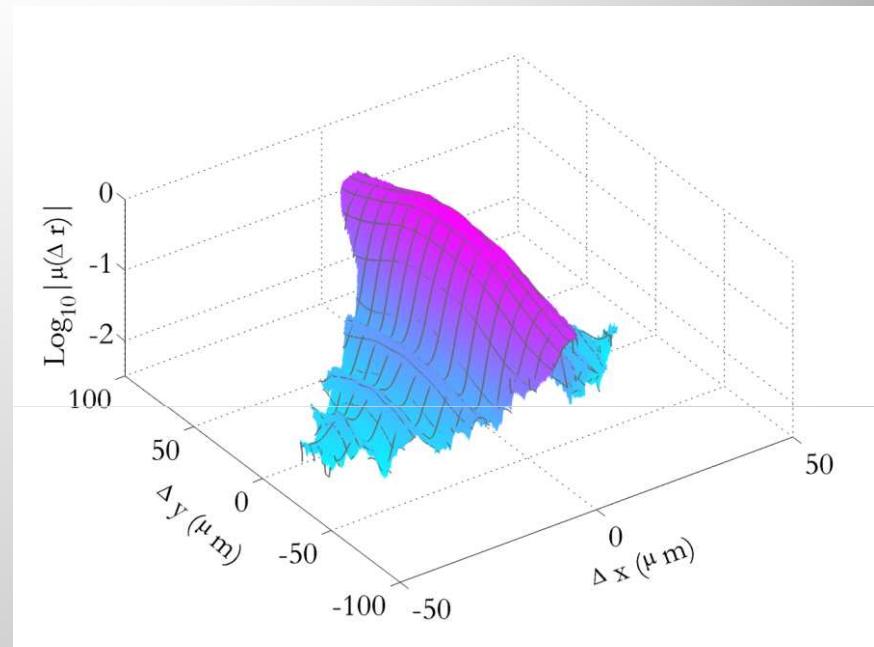
**Full Coherence Factor****Typical 2D Power Spectrum**

## ESRF ID06 - Coherence Factor



Average of 10  
Power Spectra

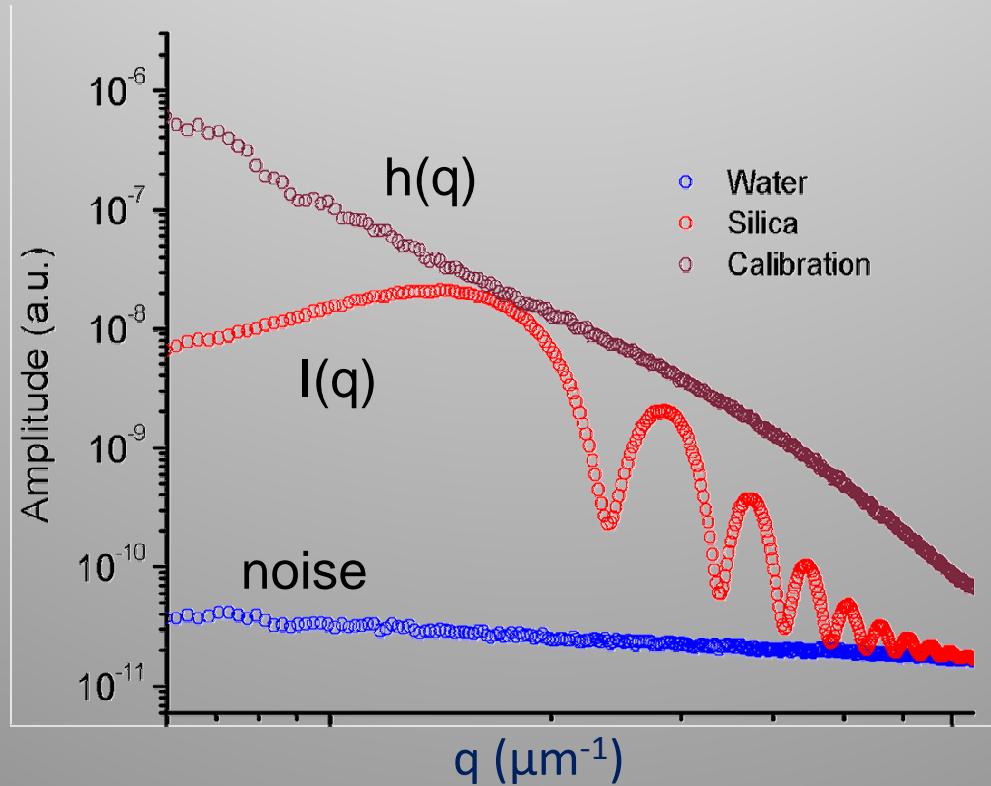
Z = 800mm



# The Signal

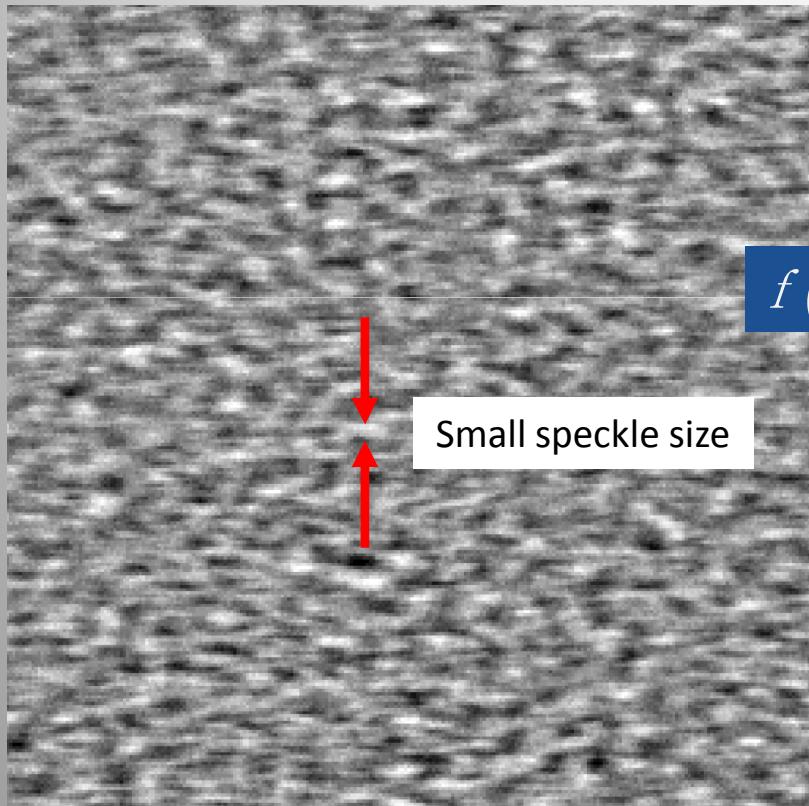
$$I(q) = C(q, z) T(q, z) F(q) H(q) + \text{noise}(q)$$

- $T(q)$  : Talbot Transfer Function
- $C(q)$ : Coherence Factor
- $F(q)$ : Particle form factor (const)
- $H(q)$ : Instrumental Transfer Function  
(scintillator)
- Shot noise



# Heterodyne Speckles

Speckles (ID06 - ESRF)

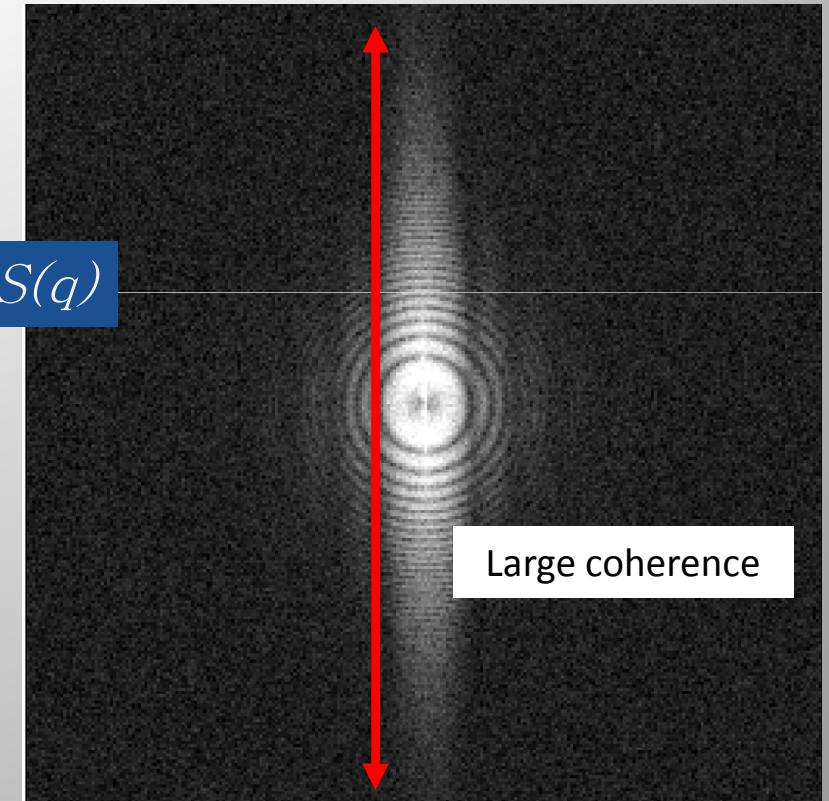


$f(r)$

$\mathcal{F}$

$S(q)$

Spatial power spectrum



$I(q)$ : particle form factor (flat)

$T(q)$ : Talbot trasfer function [  $\sin^2(q^2z/2 k)$  ]

$H(q)$  = Sensor transfer function

$$S(q) = I(q) T(q) H(q) C(q)$$

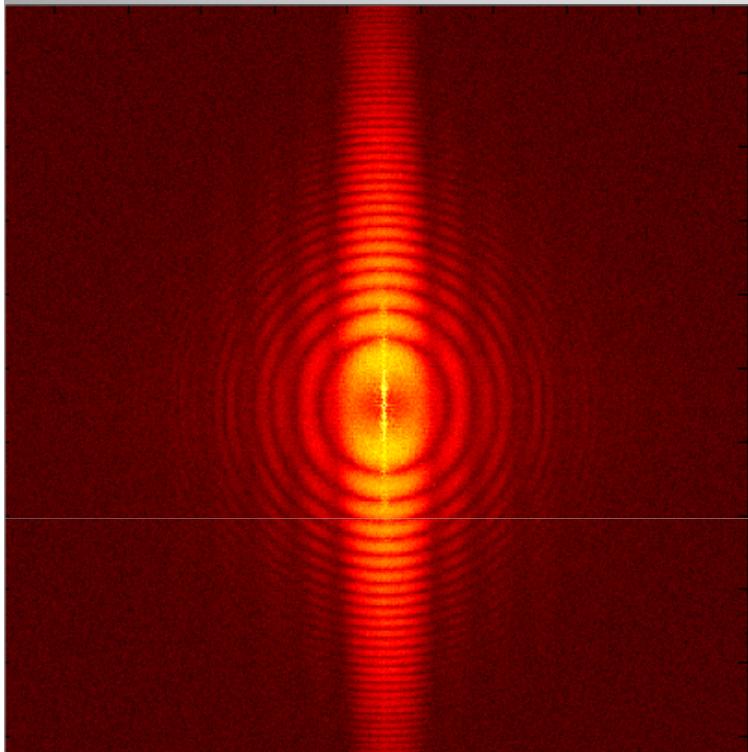
$$r = \frac{q}{k} z$$

$$C(r(q)) = |\mu|^2$$

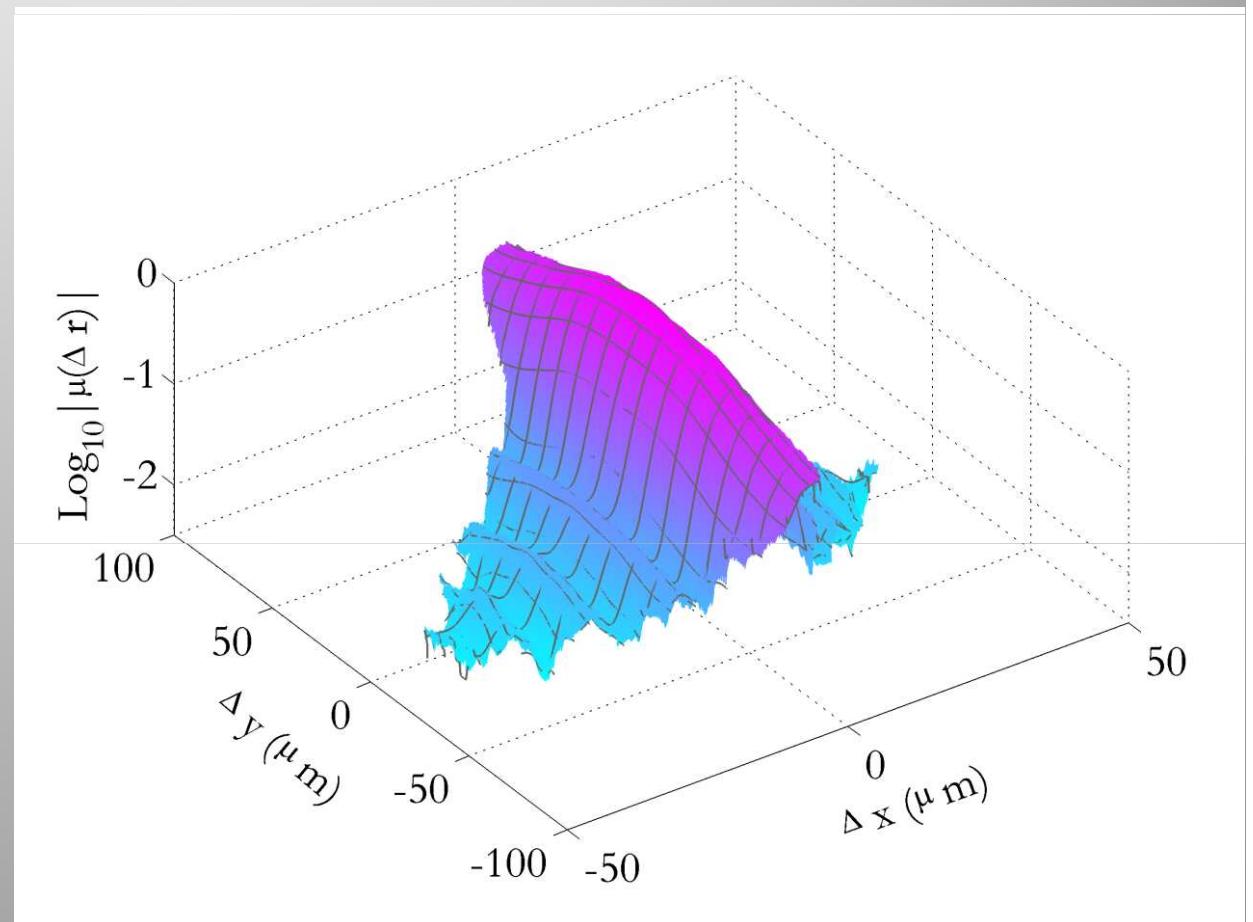
( $\mu$ : Complex Coherence Factor)

# 2D MAP

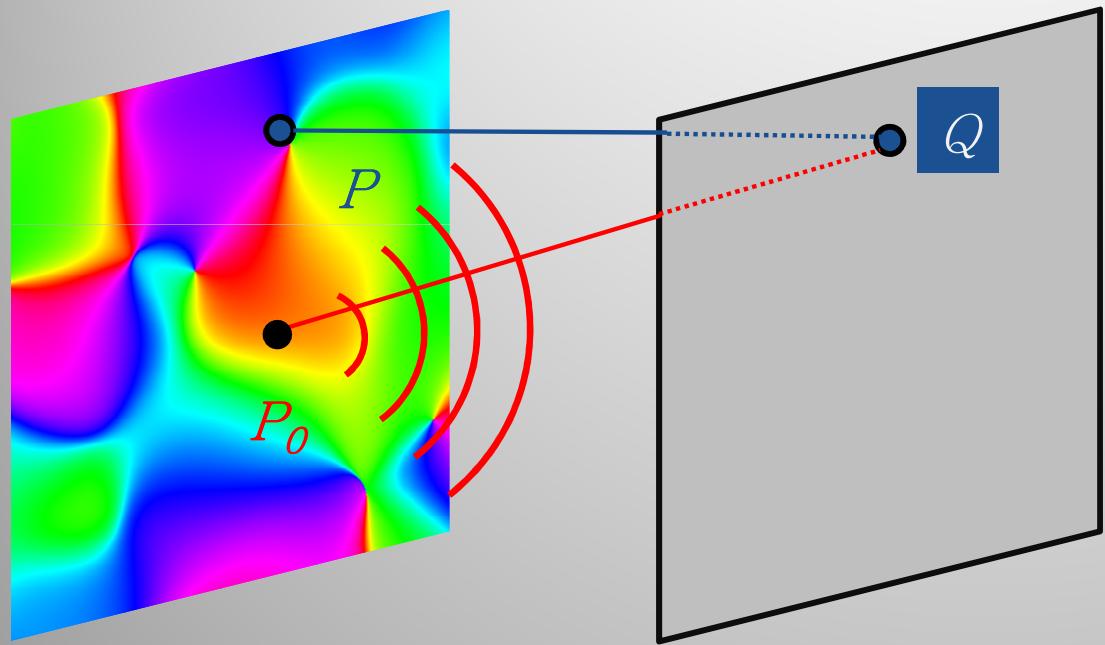
$Z = 838\text{mm}$   
10 images acquired



$H(q)$  known



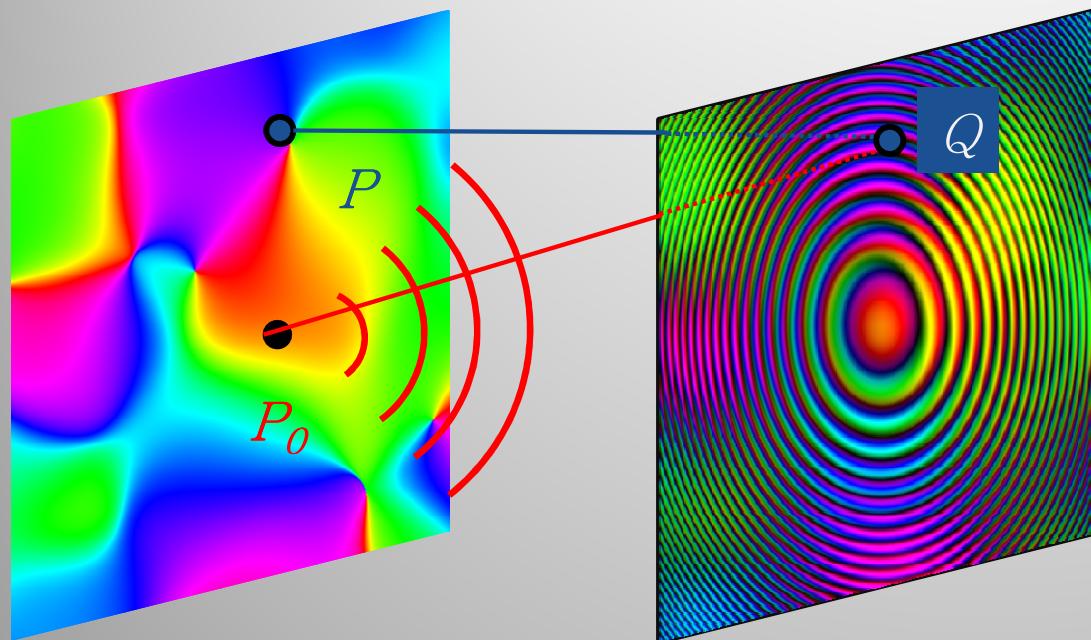
# In-Line Holography



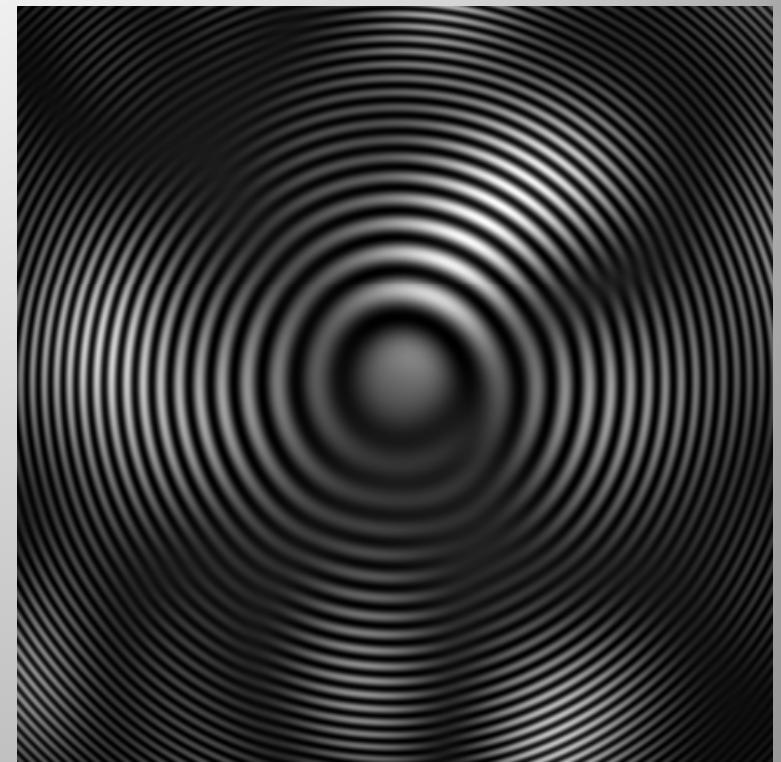
## Resulting Intensity

$$I(Q) = \text{const} + \frac{e^{ikr_0}}{r_0} e^{ikz} E(P_0)^* E(P)$$

# In-Line Holography



Simulated Instantaneous intensity



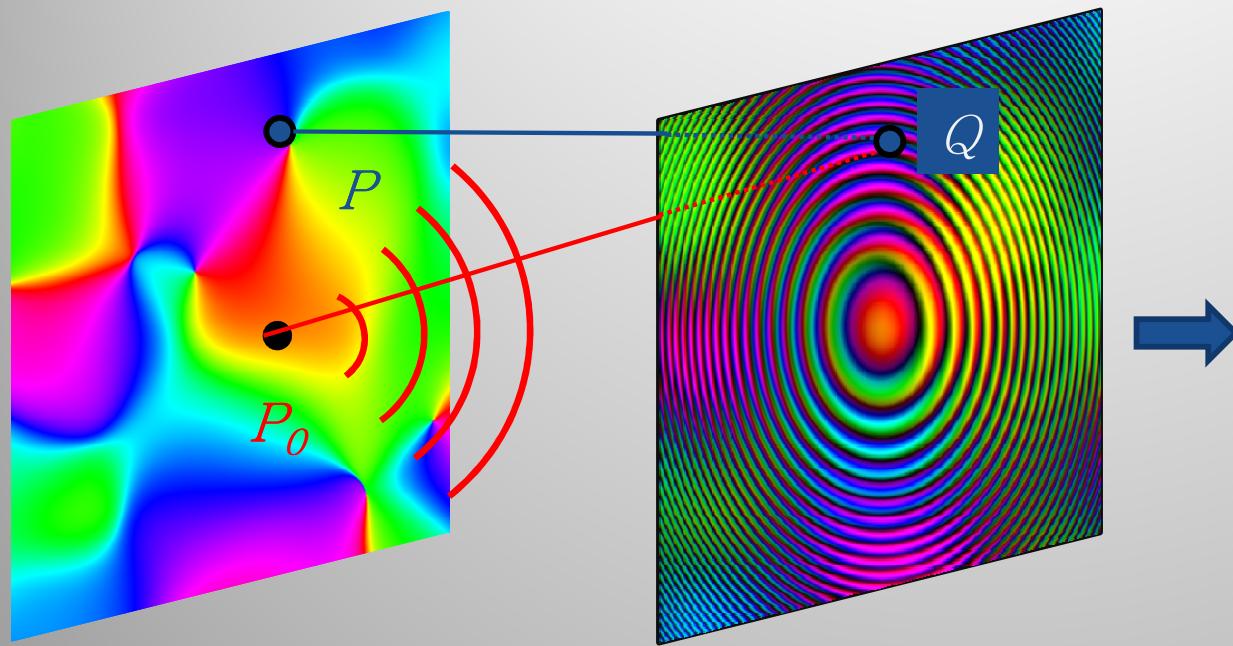
**Resulting Intensity**

$$I(Q) = \text{const} + \frac{e^{ikr_0}}{r_0} e^{ikz} E(P_0)^* E(P)$$

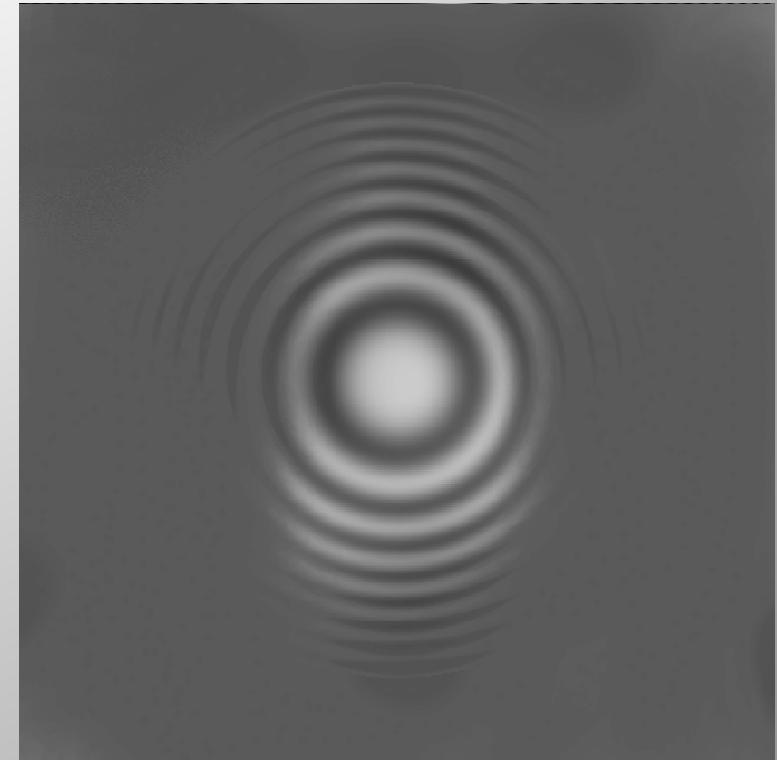
**Intensity**

$$I \sim E_S(P_0)^* E_T(P)$$

# In-Line Holography



Simulated Summation of many shots



**Resulting Intensity**

$$I(Q) = \text{const} + \frac{e^{ikr_0}}{r_0} e^{ikz} E(P_0)^* E(P)$$

**FEL:** all the spikes in the pulse

**Synchrotron:** in 1 ms  $\rightarrow 10^{14}$  longitudinal modes

**Intensity**

$$I \sim \langle E_s(P_0)^* E_T(P) \rangle$$

**Visibility:**

$$V(d) = \left( \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} \right)(d)$$