X-ray Optics-Free FEL Oscillator X-OFFELO*

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** 10 years old idea: X-OFFELO was introduced in July 2002 at ICFA workshop in Chia Laguna, Sardinia and later at FEL 2005 as FEL prize talk.*

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Dedication

to abused (*mechanically, thermally, verbally… and also by radiation*) , stressed, damages, over-exploited, pushed to the limits, sworn-on

MIRRORS

Pushing the FEL oscillator power will require – at some moment- removing the optics and relying on the e-beam

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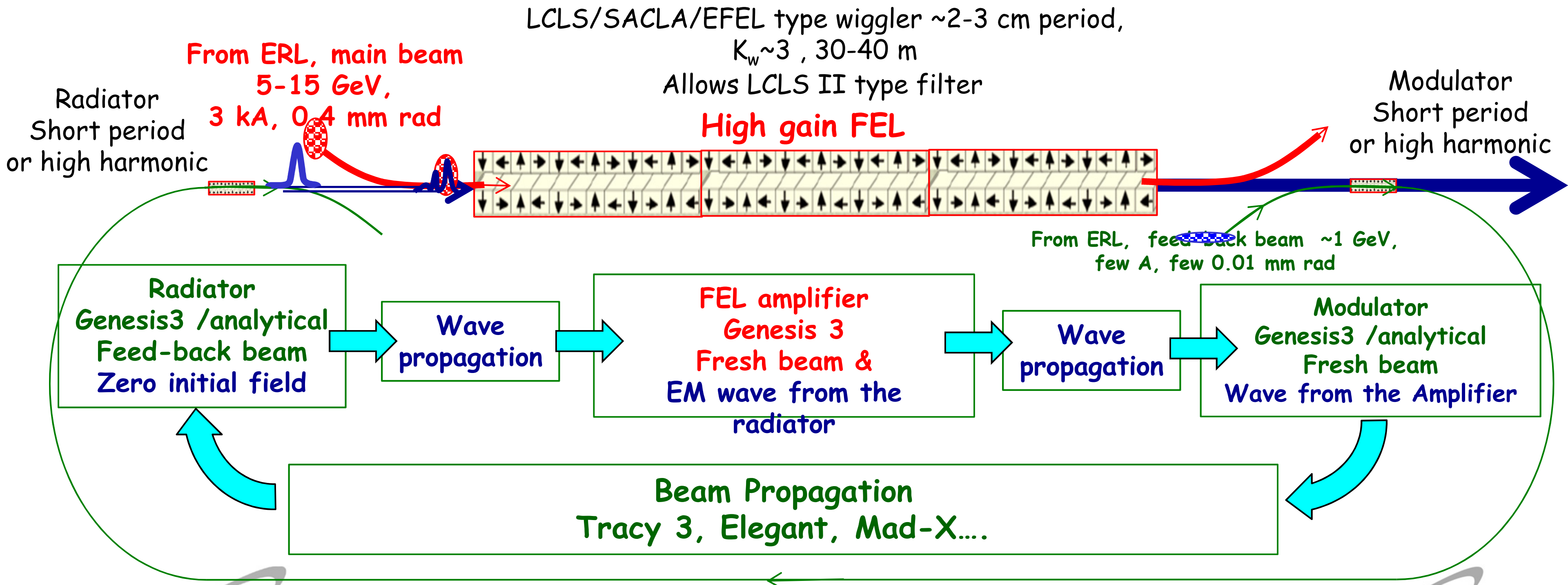
Content

- What is OFFELO?
- Main challenges
- Problems we addressed
- Simulations & results
- Conclusions/Plans

OFFELO

1. High gain amplifier/ main e-beam (from ERL or CW linac)
2. Feed-back is provided by a low-current e-beam
3. Feed-back e-beam picks the energy modulation from the FEL laser beam in modulator, preserves the correlations at 1/10th of the FEL wavelength in the long transport line, radiates coherently in the radiator.
4. The later serves as the input into the high gain FEL & compeates wit the spontaneous radiation



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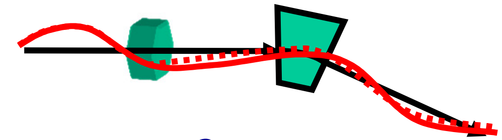
Main challenges



- Preserving correlation between particles at sub-Å level
  - Highly isochronous lattice $\boxed{\delta S_{turn} = c\delta(\tau_{exit} - \tau_{input}) < \lambda_{FEL}}$
  - Canceling time-of-flight dependence on transverse motion
  - Fundamental effects of quantum nature of synchrotron radiation

$$\sigma_{ct}[m] \cong 1.61 \cdot 10^{-5} \frac{E[GeV]^{5/2}}{\rho[m]} \sqrt{\left\langle R_{56}^2(s|C)\right\rangle_{mag}}[m] \Rightarrow \boxed{E < 1\,GeV}$$

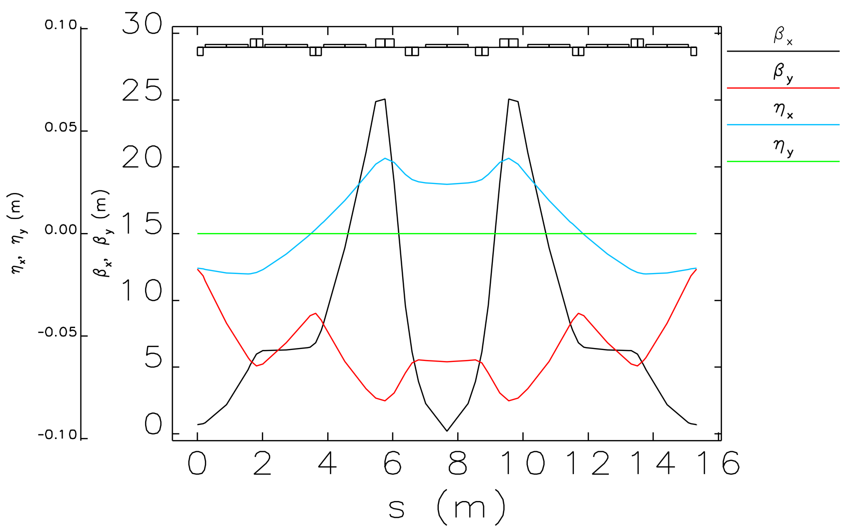
- Collective effects ⇒ Low current, long bunches
- Modulator/Radiator: Using very high harmonics or sub-mm FEL

Problems we addressed

- We developed a concept of high-order isochronous lattice comprised of a multiple cells with the total integer tunes in both directions
- We created 3km long lattice based on this concept, which preserves correlations at sub-Å scale for 1.5 GeV e-beam, including quantum effects of synchrotron radiation
- We considered the CSR wake-fields for the e-beam and found a solution for compensating the effect
- We included the high order map and random effects resulting from quantum nature of synchrotron radiation into the self-consistent simulation of this FEL oscillator
- We made first attempt of simulating the generation e-beam with required quality for the feed-back....

Lattice

- Concept*
  - use a periodic isochronous lattice** with N cell and total integer tunes in both directions
  - cell tune advances avoiding low order resonances $N\Delta\nu_x = K;\ N\Delta\nu_y = M$
  - such lattice is a natural (Brown) achromat and compensating chromaticities automatically kills second order terms in time of flight dependence on x,x',y,y' $n\Delta\nu_x + m\Delta\nu_y \neq l;\ n,m = 1,2,3,4...$
  - use additional sectupole (multipole) families to reduce higher order terms (Tracy 3)
  - Example is below: N=11x19=209, $\Delta\nu_x = 18/19; \Delta\nu_y = 5/11$ lowest order resonance - 30



Beam Optics along Beamline

½ BeamLine; S1, QD2,O,B2H,B2H,O,QF2,S2,QF2,O,B2H,B2H,O,QD2,S3, QD2,O,B2H,B2H,OFW,QF3,QF3,S4,O1F,QD3,QD3,S5,O2F,B2H
+ bilateral part

Radius of curvature= 302.7 m
O, OFW, O1F, O2F are drifts with lengths: O: L = 0.075; OFW: L = 0.3; O1F: L = 0.35; O2F: L = 0.221;
B2H is the dipole: B2H: L=0.65,ANGLE=0.002147363399583

The quadrupole settings are:
QF2: L = 0.19, K2 = 1.294; QD2: L = 0.175, K2= -1.296;
QF3: L = 0.28, K2 = 1.777; QD3: L = 0.2, K2 = -1.348.

S1: K3L=-87.6, K4L=-2.06E5; K5L=-5.16E9;
S2: K3L= 267. 9, K4L= 3.24E5, K5L=-1.90E9;
S3: K3L=-223.2, K4L= 2.43E5. K5L=-5.33E9;
S4: K3L= 61.9, K4L=-8.35E4, K5L=-3.40 E6;
S5: K3L= 23.3, K4L= 2.48E5, K5L=-4.85E8.

nu: 1.0000000000, 0.0000000000
m_11 - 1: 1.534e-13, -9.093e-14
m_12: -9.059e-13, 2.287e-11
ksi: -1.155e-03, -2.001e-03
R_56: 2.476e-19; R_566: -1.184e-11
R_5666: 3.445e-12; R_56666: 7.312e-13

Cell	$\otimes_x$	$\otimes_y$		$\otimes_x$	$2\otimes\nu_x$	$3\otimes_x$	$2\otimes_y$	$\otimes_x-2\otimes_y$	$\otimes_x+2\otimes_y$	$4\otimes_x$	$4\otimes_y$	$2\otimes_x-2\otimes_y$	$2\otimes_x+2\otimes_y$	$5\otimes_x$	$\otimes_x-4\otimes_y$	$\otimes+4\otimes_y$	$3\otimes_x-2\otimes_y$	$3\otimes_x+2\otimes_y$
1	0.947	0.455		0.95	1.89	2.84	0.91	0.04	1.86	3.79	1.82	0.99	2.80	4.74	-0.87	2.77	1.93	3.75

*VL, Chromaticity and Isochronous lattices

**D.Trbojevic et al, AIP CONFERENCE PROCEEDINGS, V. 530, (2000) p. 333

Beam transport

- Exercise $\Delta\nu_x = 18/19; \Delta\nu_y = 5/11$
  - N= 209; path length – 3221.323 m
  - lowest regular resonance order – 41

    $$19\Delta\nu_x + 22\Delta\nu_y = 28$$
  - Test beam parameters:
    - relative RMS energy spread 10^{-4};
    - Normalized emittance, x & y – 0.02 μm
  - Fifth order map
  - Synchrotron radiation at 1.5 GeV is OK

    $$\left\langle R_{56}^2(s|C)\right\rangle = 8.06\cdot 10^{-8}.$$
  - 50% of the modulation at 1 Å is preserved
  - CSR wake – manageable

I		1 x	2 x'	3 y	4 y'	5 δ	6 ct
1	1.7487905669206507e-19	1	0	0	0	0	0
1	2.0673820289257054e-20	0	1	0	0	0	0
1	2.4761280265767074e-19	0	0	0	0	1	0
1	1.0000000000000000e+00	0	0	0	0	0	1
2	-7.5525693400348191e-13	2	0	0	0	0	0
2	9.4684272921669303e-23	1	1	0	0	0	0
2	-7.5497742554840793e-13	0	2	0	0	0	0
2	1.0452067194273214e-12	0	0	2	0	0	0
2	2.4658675101818518e-21	0	0	1	1	0	0
2	1.0459284822532135e-12	0	0	0	2	0	0
2	-2.7175462537259362e-12	1	0	0	0	1	0
2	-4.1906530772670468e-22	0	1	0	0	1	0
2	4.0933549122163592e-12	0	0	0	0	2	0
3	4.2627661981805402e-12	3	0	0	0	0	0
3	-4.7920366859653766e-24	2	1	0	0	0	0
3	4.2769026098856557e-12	1	2	0	0	0	0
3	5.2144677818663399e-24	0	3	0	0	0	0
3	3.7727932849036214e-12	1	0	2	0	0	0
3	-2.1284786277441330e-22	0	1	2	0	0	0
3	-2.0111682438652990e-22	1	0	1	1	0	0
3	-4.2521617243397188e-13	0	1	1	1	0	0
3	4.1028146230153669e-12	1	0	0	2	0	0
3	-4.3815196522505170e-22	0	1	0	2	0	0
3	5.1338910746389662e-12	2	0	0	0	1	0
3	-1.4184367553033096e-23	1	1	0	0	1	0
3	-2.4921081665149797e-12	0	2	0	0	1	0
3	-7.8097237258347493e-12	0	0	2	0	1	0
3	-2.3468817289540312e-22	0	0	1	1	1	0
3	-7.1327970245969211e-12	0	0	0	2	1	0
3	-6.8345476626196068e-12	1	0	0	0	2	0
3	-6.8389553483116789e-18	0	1	0	0	2	0
3	-4.0489736363830835e-16	0	0	0	0	3	0
4	-6.0925615934169556e-13	4	0	0	0	0	0
4	-1.3634583698549667e-12	2	2	0	0	0	0

$\lvert I \rvert$	a_I	i_1	i_2	i_3	i_4	i_5	i_6	i_7
1	1.7487905669206507e-19	1	0	0	0	0	0	0
1	2.0673820289257054e-20	0	1	0	0	0	0	0
1	2.4761280265767074e-19	0	0	0	0	1	0	0
1	1.0000000000000000e+00	0	0	0	0	0	1	0
2	-7.5525693400348191e-13	2	0	0	0	0	0	0
2	9.4684272921669303e-23	1	1	0	0	0	0	0
2	-7.5497742554840793e-13	0	2	0	0	0	0	0
2	1.0452067194273214e-12	0	0	2	0	0	0	0
2	2.4658675101818518e-21	0	0	1	1	0	0	0
2	1.0459284822532135e-12	0	0	0	2	0	0	0
2	-2.7175462537259362e-12	1	0	0	0	1	0	0
2	-4.1906530772670468e-22	0	1	0	0	1	0	0
2	4.0933549122163592e-12	0	0	0	0	2	0	0
3	4.2627661981805402e-12	3	0	0	0	0	0	0
3	-4.7920366859653766e-24	2	1	0	0	0	0	0
3	4.2769026098856557e-12	1	2	0	0	0	0	0
3	5.2144677818663399e-24	0	3	0	0	0	0	0
3	3.7727932849036214e-12	1	0	2	0	0	0	0
3	-2.1284786277441330e-22	0	1	2	0	0	0	0
3	-2.0111682438652990e-22	1	0	1	1	0	0	0
3	-4.2521617243397188e-13	0	1	1	1	0	0	0
3	4.1028146230153669e-12	1	0	0	2	0	0	0
3	-4.3815196522505170e-22	0	1	0	2	0	0	0
3	5.1338910746389662e-12	2	0	0	0	1	0	0
3	-1.4184367553033096e-23	1	1	0	0	1	0	0
3	-2.4921081665149797e-12	0	2	0	0	1	0	0
3	-7.8097237258347493e-12	0	0	2	0	1	0	0
3	-2.3468817289540312e-22	0	0	1	1	1	0	0
3	-7.1327970245969211e-12	0	0	0	2	1	0	0
3	-6.8345476626196068e-12	1	0	0	0	2	0	0
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3	-4.0489736363830835e-16	0	0	0	0	3	0	0
4	-6.0925615934169556e-13	4	0	0	0	0	0	0
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4	-3.5328230951830668e-12	0	2	0	0	2	0	0
4	-1.9114924005443654e-12	0	0	2	0	2	0	0
4	2.8285490015434341e-18	0	0	1	1	2	0	0
4	-3.8167550015267932e-12	0	0	0	2	2	0	0
4	-8.7122594198860168e-12	1	0	0	0	3	0	0
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5	-9.2222599378940371e-09	2	0	2	0	1	0	0
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5	5.1539973882010306e-09	0	0	2	2	1	0	0
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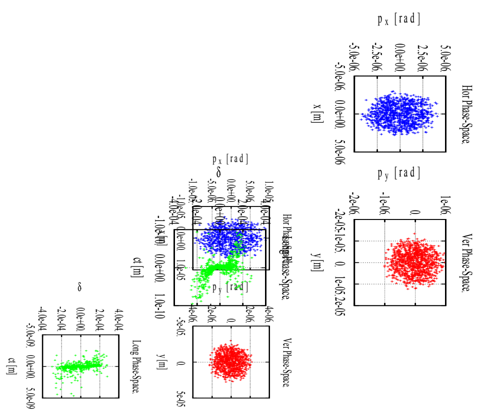

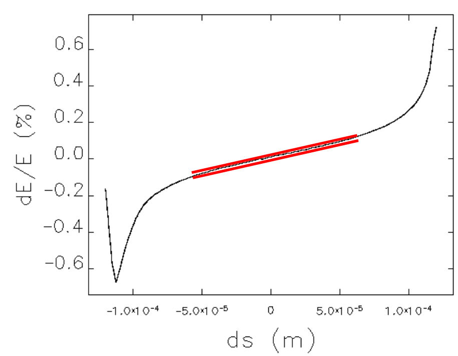

Final longitudinal phase space

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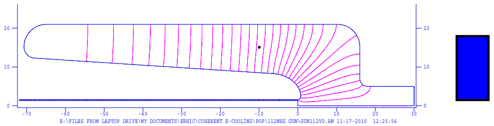
We also explored 300 m path length.

Injector simulations

preliminary

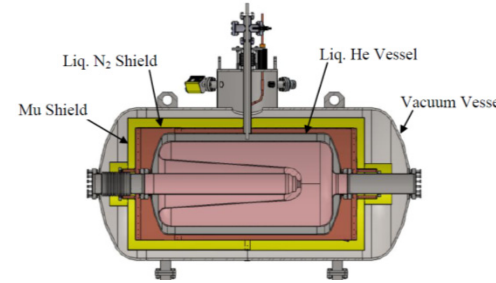
112 MHz
SRF Gun

Solenoid



Cathode E peak field 20 MV/m

e^- @ 2.5 MeV

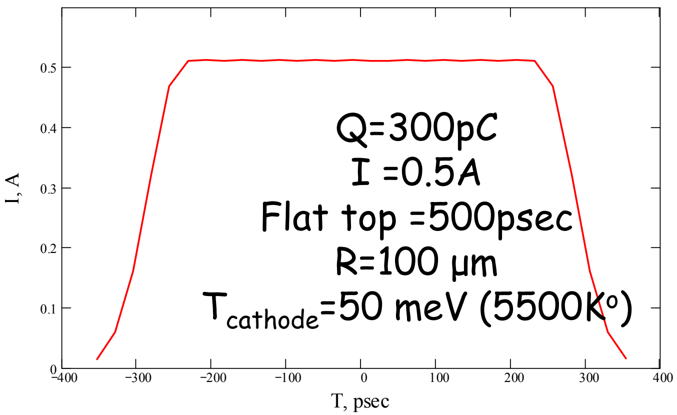


112 MHz SRF Gun

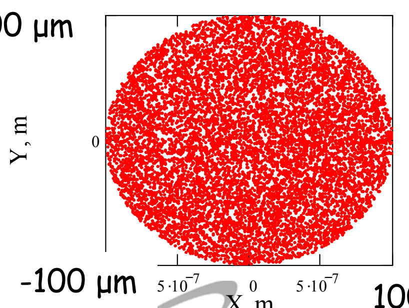
Under construction NIOWAVE

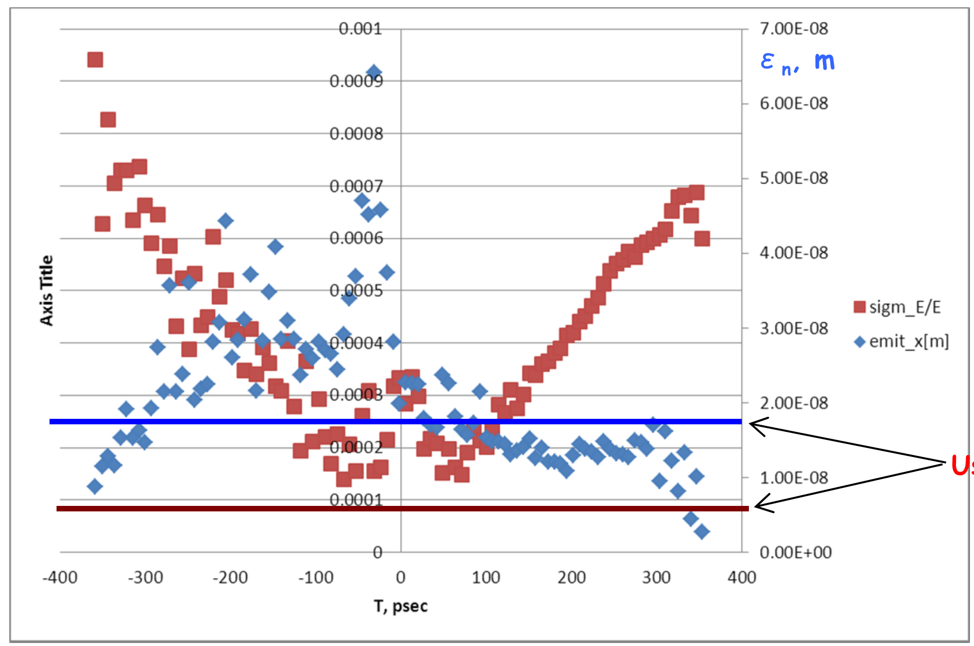
Tests – December 2012

Initial pulse shape



Transverse distribution





Slice emittance and energy spread
after acceleration to 2.5 MeV
as a result of PARMELA simulation

During first 100 psec
Effective charge Qeff=50 pC
Rms energy spread 1.5-3e-4
Rms emittance 15-20 nm rad

Undulator/Wiggler for Modulator/Radiator

- It is very desirable to use low energy < 1 GeV for feed-back beam to avoid the most fundamental limitation by quantum nature of synchrotron radiation
  - Unless we use accelerator/decelerator scheme (later slide)…
- This results in two potential solutions:
  - Using very high harmonic, N ~ 25; $JJ_N \sim 10^{-3} - 10^{-4}$
  - Using an TEM wiggler with Kw ~10^{-1}

High Harmonic

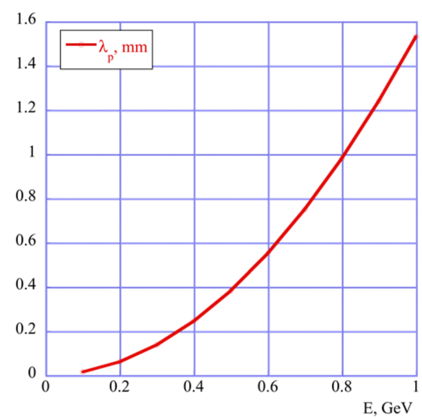
Energy ~ 1.5 GeV
Wiggler period ~ 3 cm
Kw ~ 3

$$\lambda_{FEL} = \frac{\lambda_w}{2\gamma^2(2N-1)}\left(1+\frac{K_w^2}{2}\right)$$

For 1Å FEL it yields
N ~ 25-50
JJ ~ 10^{-3}

FEL driven TEM undulator
Efb ~ 250 MeV, FEL pump- at 0.1 mm

$$\lambda_p = 4\gamma^2\lambda_{FEL}; \quad K_w^2 << 1$$



Well within achievable parameters

$$N_u K^2 \sim 0.3 \cdot \hat{p}[GW]$$

Rep-rate ~ 1 MHz
Pulse length ~ 10 psec
Intra-cavity:
Peak power ~ 1 GW
Energy in pulse ~ 10 mJ
Average power ~10 kW
FEL
Q ~ 10^3
Average power ~ 10 W
N=1;
JJ=0.996;
Kw ~ 0.17

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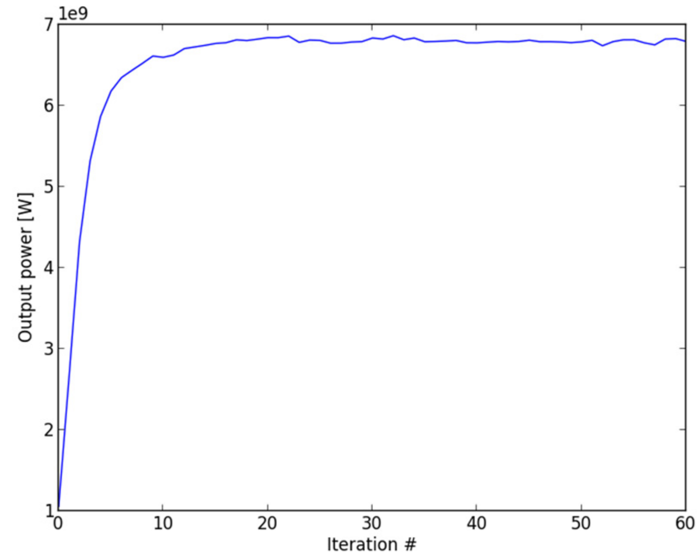
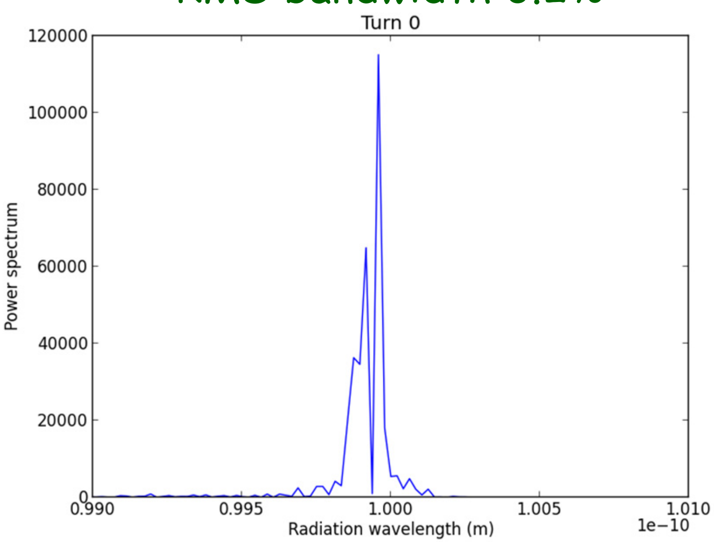
Preliminary Simulation Results

Parameter	HG FEL	Feed-back I	Feed-back II	Units
Wavelength	1	1	1	Å
Energy	10	0.75	1.5	GeV
Wiggler period	3	0.0426	0.01	cm
a_w	1.24	0.1	2.95	
N_w	1600	28	28	
Wiggler length	48	0.01	1	m
Peak current	3000	50	400	
Norm emittance	0.5	0.02	0.02	µm rad
RMS energy spread	$5\ 10^{-5}$ 500	10^{-5} 7.5	10^{-5} 15	 KeV

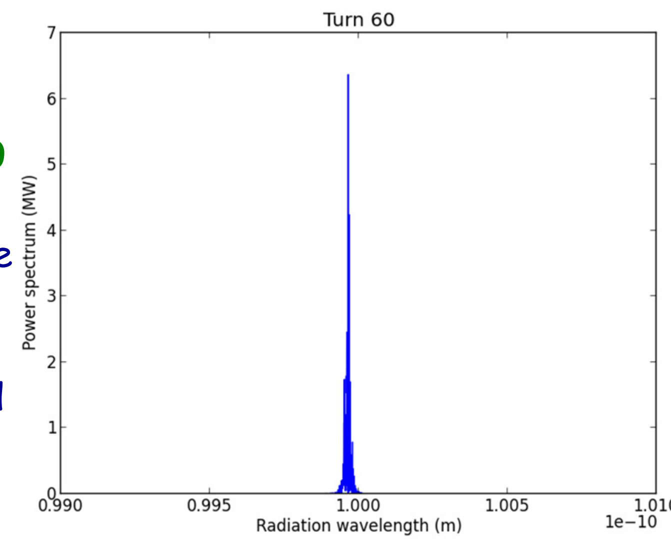
Peak power evolution in OFFELO



First pass SASE spectrum
RMS bandwidth 0.2%

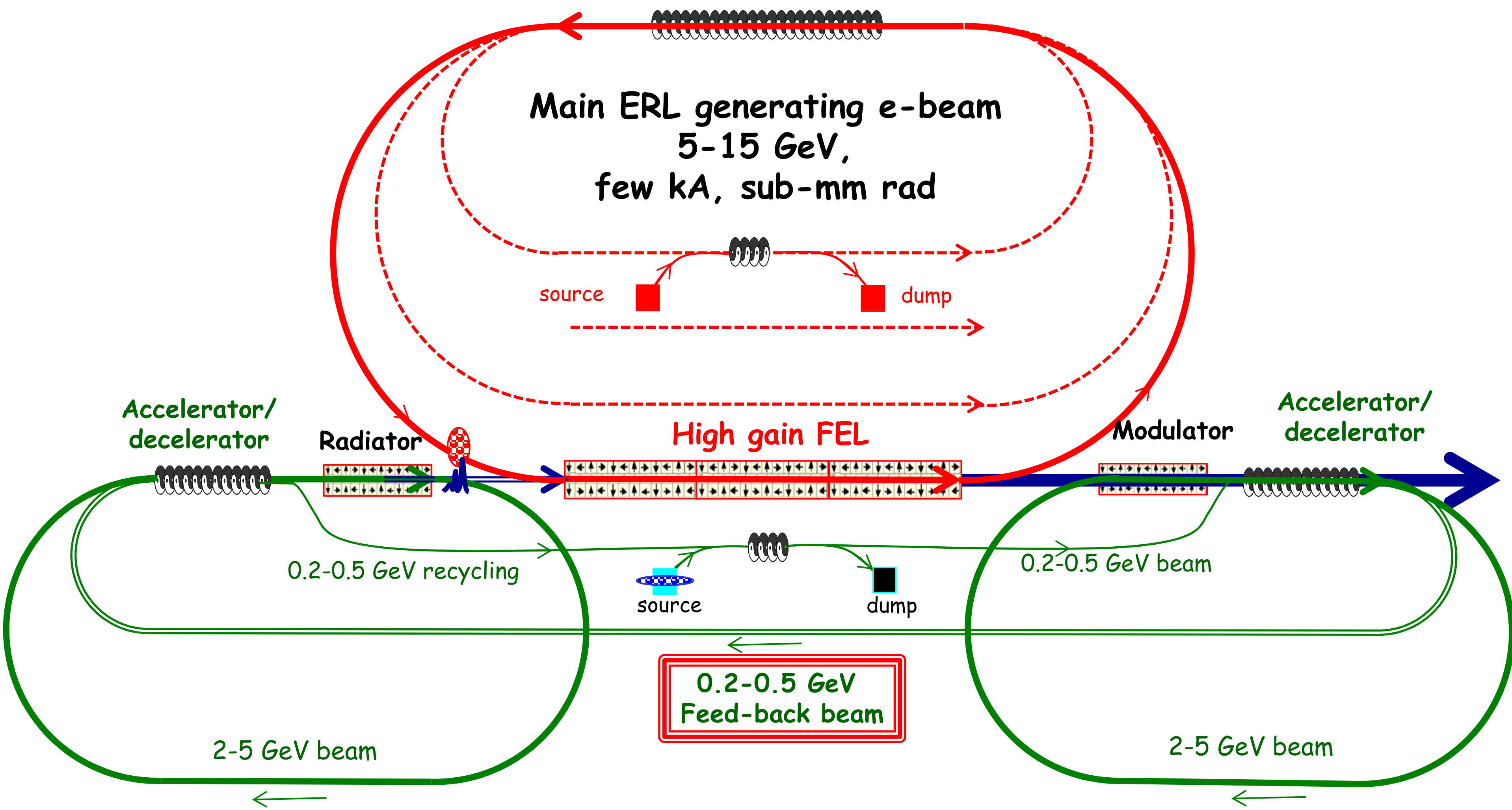


Pass #60 – OFFELO spectrum
RMS bandwidth 0.01%



- **20-fold narrowing of the spectrum after 60 passes**
- Using LCLS II technique should bring it to 10 ppm level
- System is not optimized & improvements are expected..

Alternative Feed-back scheme



- Use beam with necessary energy of few GeV for effective energy modulation (i.e. use of a typical wiggler)
- Decelerate the feed-back beam to much lower energy (*let's say few 100s MeV)* where synchrotron radiation is mitigated
- Turn the beam around, accelerate it to radiate in the radiator, decelerate it and dump it

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Conclusions

- FEL oscillator without optics seems to be feasible
  - No show-stoppers had been found
  - sub-mm FEL works the best for modulator and the radiator
  - An arc lattice can be designed to meet the challenge
- Using intra-cavity power of sub-mm FEL for modulator and the radiator works best for the presented scheme

Feed-Back e-beam

- Normalized slice emittance $\varepsilon_n \sim 0.02\ \mu$m rad is a serious challenge and we are considering lattices capable of tolerating $\varepsilon_n \sim 0.2\ \mu$m with 250 MeV e-beam
- Our test-studies of 300-m feed-back beam-line showed very high tolerance to the larger emittance and energy spread
- Possible additional (and expensive) technical technical improvement - the Accelerator/Decelerator scheme the feed-back beam

Laundry List

- Sensitivity to the errors, Ripples in the power supplies
- **Locking-in the feed-back using long wavelength laser system**
- Space charge effects in the feed-back loop
- Intra-beam scattering
- Wake-fields
- Optimization of the system
- ……….
- Starting R&D with sun-μm before going to Å scale is worth considering
- Technical details – such as electron-beam mirror, can be studied using existing ATFs

Back-up slides

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Preserving the phase correlations

$$H=-\frac{(1+K_o(s))}{c}\left\{p_o{}^2c^2+2E_o\delta E+\delta E^2+P_x^2+P_y^2\right\}-\frac{e}{c}A_s+\frac{\delta E}{v_o}$$

$$\{x,P_x\},\ \{y,P_y\},\ \{\tau=(t_o(s)-t),\delta E\}$$

$$\frac{d\tau}{ds}=-\frac{\partial H}{\partial(\delta E)};\quad \frac{dt_o}{ds}=\frac{1}{v_o}.$$

$$\tau_{exit}-\tau_{input}=M_5(x,P_x,y,P_y,\delta E)$$

$$\delta S_{turn}=c\,\delta(\tau_{exit}-\tau_{input})<\lambda\!\!\!^{-}{}_{FEL}$$

$$\left|\delta S_{turn}\right|\le \lambda\!\!\!^{-}{}_{FEL};$$

$$\delta S_{turn}=\delta S_{turn}(\delta E)+\delta S_{turn}(\varepsilon_{x,y})+\delta S_{HO}(\delta E,\varepsilon)+\delta S_{random};$$

$$Example:\ L=100m;\quad \lambda\!\!\!^{-}=10^{-10}m;\ \varepsilon=10^{-10}m\cdot rad;\ \sigma_E=0.01\%$$

1. Energy spread and compaction factors

$$\delta S_{turn}(\delta E)=L\cdot\left\{R_{56}\left(\frac{\delta E}{E}\right)+R_{566}\left(\frac{\delta E}{E}\right)^2+R_{5666}\left(\frac{\delta E}{E}\right)^3+\ldots\right\};$$

$$\Rightarrow\ \left|\alpha_c\right|=\left|R_{56}(0,L)\right|<10^{-8};\left\|R_{566}(0,L)\right\|<10^{-4};\left\|R_{5666}(0,L)\right\|<1\ldots$$

-> second order isochronous system

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$\lambda_{FEL} \sim 1\mathring{A}$

2. Emittance effects

Linear term: comes from symplectic conditions

$$M^T S M = S;$$
$$S = \begin{bmatrix} \sigma & 0 & 0 \\ 0 & \sigma & 0 \\ 0 & 0 & \sigma \end{bmatrix}; \quad \sigma = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$\delta S_{turn}(\varepsilon_{x,y}) = [\eta \quad \eta'] \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ x' \end{bmatrix} + f(\varepsilon) + O(\varepsilon^2)$$

$$|\eta| < 10^{-5} m \Big/ \sqrt{\beta[m]}; \quad |\eta'| < 10^{-5} \sqrt{\beta[m]};$$

It is not a problem to make the turn achromatic with η=0 and η'=0
It is a bit more complicated to make the condition energy independent.
An elegant solution - sextupoles combined with quadrupoles with $K_2=K_1/2\eta$:

$$x'' = -\frac{K_1 x + K_2 \cdot \left((x + \eta \cdot \delta)^2 - y^2 \right)}{1+\delta} = -K_1 \cdot x + O(x^2, y^2)$$
$$y'' = \frac{K_1 y + 2K_2 y \cdot (\eta \cdot \delta + x)}{1+\delta} = K_1 \cdot y + O(xy)$$

$$\int_0^L O(x^2, y^2, xy, \eta^2) \Rightarrow 0$$

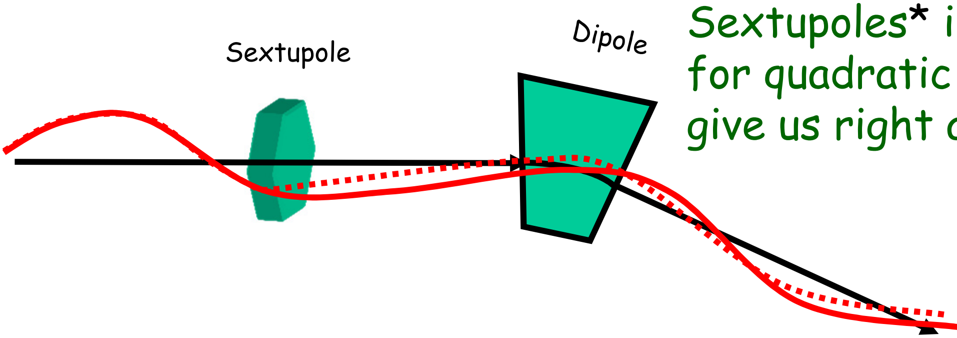
Solution is a second order achromat (N cell with phase advance 2π M, M/N is not integer, etc.) with second order geometrical aberration cancellation

2. Emittance effects

Quadratic term

$$\delta S_2 \propto \int_o^L \frac{x'^2 + y'^2}{2} ds$$

$$x = a_x \sqrt{\beta_x(s)} \cos(\psi_x(s) + \varphi_x) + \eta(s)\frac{\delta E}{E_o}; \; y = a_y \sqrt{\beta_y(s)} \cos(\psi_y(s) + \varphi_y).$$



Sextupoles* in the arcs are required to compensate for quadratic effect sextupole kick + symplectic conditions give us right away:

$$\Delta x'_{sext} = K_2 l \cdot (x^2 - y^2) \;\; \Rightarrow \;\; \delta S = -\eta(s) \cdot \Delta x'_{sext} = -\eta(s) K_2 l \cdot (x^2 - y^2)$$

$$\frac{1}{2}\int_o^L (x'^2 + y'^2) ds - \sum_n \eta(s_n)(K_2 l)_n \cdot (x^2(s_n) - y^2(s_n)) ds \Rightarrow 0$$

Sextupoles located in dispersion area give a kick ~ x^2-y^2 which affect the length of trajectory. Two sextupoles placed 90° apart the phase of vertical betatron oscillations are sufficient to compensate for quadratic term with arbitrary phase of the oscillation

Four sextupoles located in the arcs where dispersion are sufficient to satisfy the cancellation of the quadratic term in the non-isochronism caused by the emittances. Fortunately, the second order achromat compensates the chromaticity and the quadratic term simultaneously. *In short it is the consequence of Hamiltonian term*:

$$h \propto -g(s) \cdot \delta \cdot \left(\frac{x^2 - y^2}{2} \right) \Rightarrow C_x \cdot \delta \cdot \frac{a_x^2}{2} + C_y \cdot \delta \cdot \frac{a_y^2}{2}$$

•This scheme is similar to that proposed by Zolotarev and Zholetz. (PRE 71, 1993, p. 4146) for optical cooling beam-line and tested using COSY INFINITY. It is also implemented for the ring FEL: A.N. Matveenko et al. / Proceedings 2004 FEL Conference, 629-632

Synchrotron Radiation

$\lambda_{FEL} \sim 1\mathring{A}$

- Energy of the radiated quanta

$$\varepsilon_c[keV] = 0.665 \cdot B[T] \cdot E_e^2[GeV]$$

- Number of radiated quanta per turn

$$N_c \cong 2\pi\alpha\gamma \cong 89.7 \cdot E[GeV]$$

- Radiation is random -> the path time will vary
- The lattice should be designed to minimize the random effects

$$(\delta S_{rand})^2 \approx N_c \left(\frac{\varepsilon_c}{E_e}\right)^2 \left\langle R^2{}_{56}(s,L) \right\rangle$$

$R_{56}(s,L)$ is the *longtudinal dispersion* from azimuth s to L

$$\Rightarrow \quad \sqrt{\left\langle R^2{}_{56}(s,L) \right\rangle} < \sqrt{\frac{2}{N_c}} \frac{E_e}{\varepsilon_c} \lambdabar$$

It looks as the toughest requirement for the scheme to be feasible

$$\sqrt{\left\langle R^2{}_{56}(s,L) \right\rangle} < 2.25 \cdot 10^{-5} m \cdot E_e^{-3/2}[GeV] \cdot B^{-1}[T]$$

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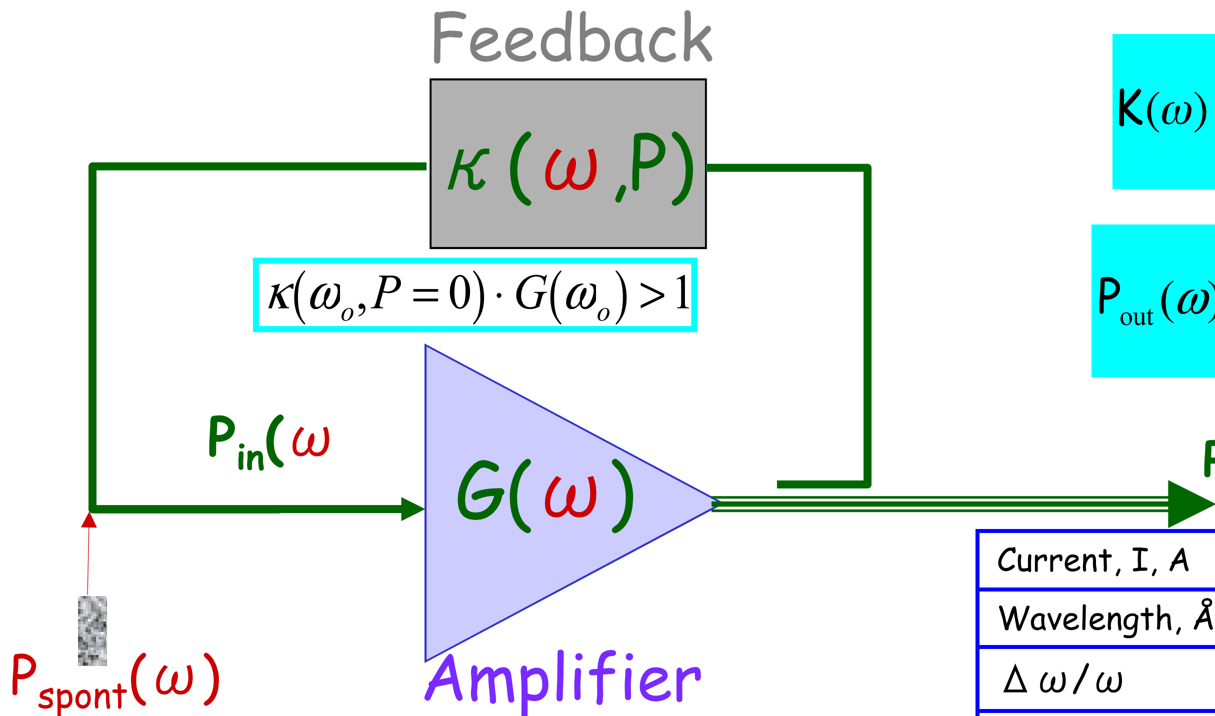
Oscillator

$$K(\omega) = G(\omega)\ \kappa(\omega, P)$$

$$K(\omega) \approx K(\omega_o) \cdot \left(1 - \frac{(\omega - \omega_o)^2}{\Delta\omega^2}\right)$$

$$P_{out}(\omega) \approx P_{out}(\omega_o) \cdot \left(1 - \frac{(\omega - \omega_o)^2}{\delta\omega^2}\right)$$



Current, I, A	~ 100 A
Wavelength, Å	~ 1
$\Delta\omega/\omega$	~ 10^{-3}
Ko	~ 1.2
$\delta\omega/\omega$ Correlations length	~ 3 10^{-8} ~ 3 mm !!!

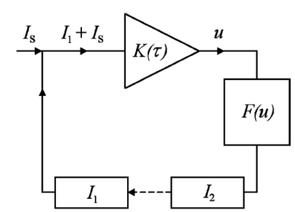
Detailed analysis of the lasing linewidth of saturated oscillator is in:

High-gain ring FEL as a master oscillator for X-ray generation

Nikolay A. Vinokurov*, Oleg A. Shevchenko

Nuclear Instruments and Methods in Physics Research A 528 (2004) 491–496

$$\boxed{\frac{\delta\omega}{\omega} \sim \frac{e\omega}{I}\left(\frac{\Delta\omega}{\omega}\right)^2 |K(\omega)|^2}$$



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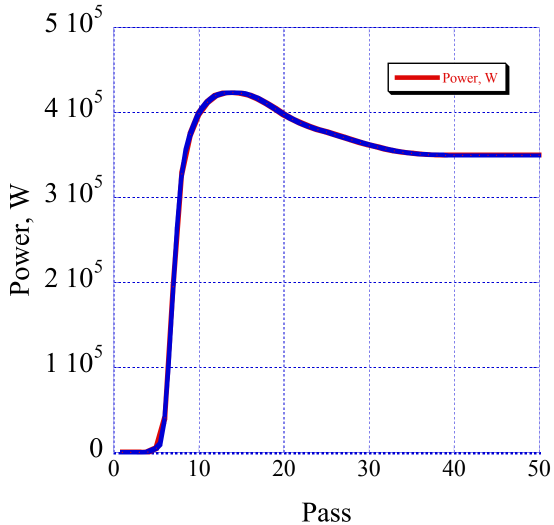
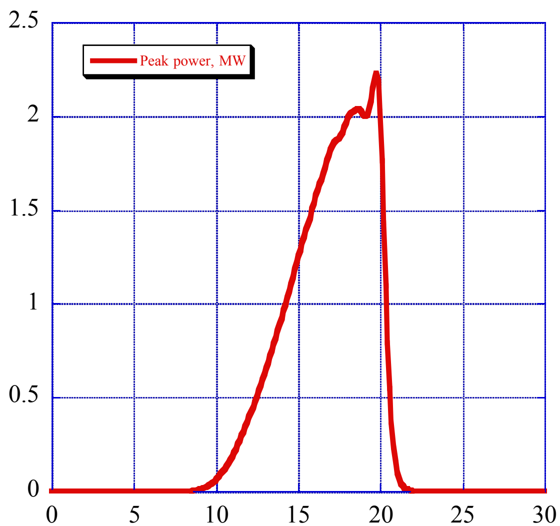
FEL simulation results for OFFELO at BNL R&D ERL GENISIS simulations



5 cm undulators period and 0.7 nC electron beam at rep. frequency 9.38 MHz the GENESIS simulation gives: wavelength **29 microns**, peak power **2 MW** and average power 400 W. For full current mode operation rep. rate 703.75 MHz we obtain **30 kW** far infrared in CW mode.

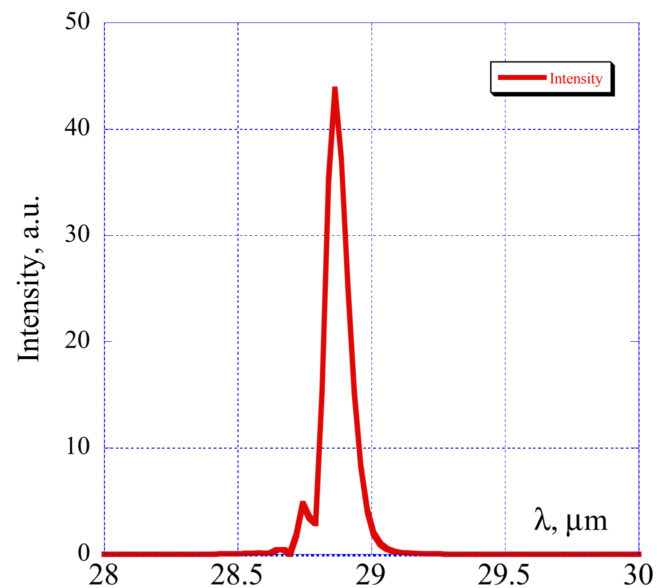
Peak power reaches 2 MW

The central wavelength 29 μm and FWHM 0.35%.



Optical pulse

Close to the Fourier limited spectrum



Spectrum