

A TRANSVERSE FEEDBACK SYSTEM USING MULTIPLE PICKUPS FOR NOISE MINIMIZATION

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Abstract

A New concept for using multiple pickups for estimating beam angle at the kicker is addressed. The estimated signal should be the driving feedback signal. The signals from the different pickups are delayed, such that they correspond to the same bunch. Consequently a weighted sum of the delayed signals is suggested as an estimator of the beam angle at the kicker. The weighting coefficients are calculated such that the estimator is unbiased, i.e., the output corresponds to the actual beam angle at the kicker for non-noisy pickup signals. Furthermore, the estimator must give the minimal noise power at the output among all linear unbiased estimators. Finally results for the heavy ions synchrotron SIS 18 at the GSI are shown.

INTRODUCTION

Transversal beam oscillations can occur in synchrotrons directly after injection. Furthermore, Higher beam intensities can excite coherent transversal instabilities, which lead to beam oscillations, when the natural damping becomes not enough to attenuate the oscillations generated by the interaction between the travelling beam and the different objects of the accelerator.

A powerful way to mitigate coherent instabilities is to use feedback system. Transversal Feedback System (TFS) senses instabilities of the beam by means of Pickups (PUs), and acts back on the beam by means of actuators called Kickers. In [1] an approach has been addressed for calculating the horizontal and vertical beam directions at the position of the Kicker along the accelerator ring using PUs at two different positions along the accelerator ring for each of the horizontal and vertical directions. The reason we need PUs at two different positions is that only beam displacements from the ideal trajectory but not the directions can be measured by PUs.

In general the signals at the PUs are disturbed by noise. The signal to noise ratio can be unacceptably low or not high enough, especially for lower currents, where the beam is getting corrected by big noise portion during the feedback. That will worsen the feedback correction because it will lead to beam heating [2].

In this work we address a new approach to mitigate noise at the PUs by using more than two PUs at different positions to estimate beam direction at the Kicker, this is done by calculating a weighted sum of the signals after synchronization. The idea behind that is to have more degrees of freedom by using more PUs to adjust the weights in a way

to minimize the noise part at the estimated signal, while keeping a correct formula with absence of noise. This is the so called Minimum-variance unbiased estimator (MVUE).

SYSTEM MODEL

For each position along the synchrotron ring three coordinate axes are defined, which determine the different beam displacements from the ideal trajectory. Figure 1 shows the transversal directions: x for horizontal displacement and y for vertical displacement. The longitudinal direction axis is marked as s.

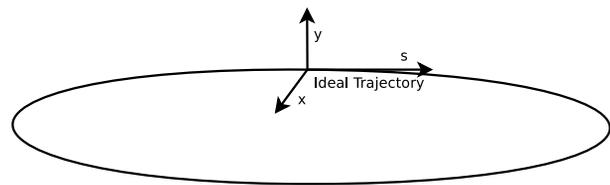


Figure 1: The coordinate system.

The TFS is composed of multiple PUs at different positions and one Kicker for each transversal direction. The signals from the PUs, which correspond to the transversal beam displacements from the ideal trajectory, are delayed differently, such that they correspond to the same bunch. The driving signal at the kicker is an amplification of the weighted sum of the delayed signals. Figure 2 shows a block diagram of the TFS.

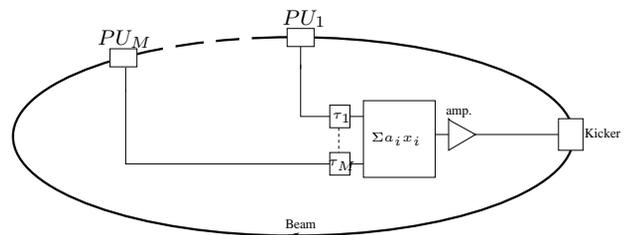


Figure 2: Block diagram of the TFS.

Let x_i be the signal at the pickup PU_i , which is located at the position s_i along the accelerator ring. This signal corresponds to the beam transversal (horizontal or vertical) beam displacement \tilde{x}_i perturbed by noise z .

$$x_i = \tilde{x}_i + z_i. \tag{1}$$

As vector notation one can write

$$\mathbf{x} = \tilde{\mathbf{x}} + \mathbf{z} \tag{2}$$

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where $\mathbf{x} = [x_1, x_2, \dots, x_M]^T$ denotes the vector of the signals from the M PUs, $\mathbf{z} = [z_1, z_2, \dots, z_M]^T$ denotes the noise vector from the PUs and $\tilde{\mathbf{x}} = [\tilde{x}_1, \tilde{x}_2, \dots, \tilde{x}_M]^T$ denotes the actual beam displacements at the PUs.

The derivative of the beam transversal displacement at the Kicker at position s along the accelerator ring, which corresponds to beam direction at that position, can be estimated using the pickup signals x_{i_1} and x_{i_2} , $i \in \{1, 2, \dots, M\}$ according to the vector summation approach introduced in [1] as

$$\bar{x} = \alpha_{i_1} x_{i_1} + \alpha_{i_2} x_{i_2} \quad (3)$$

$$= \alpha_{i_1} \tilde{x}_{i_1} + \alpha_{i_2} \tilde{x}_{i_2} + \alpha_{i_1} z_{i_1} + \alpha_{i_2} z_{i_2} \quad (4)$$

$$= x' + z \quad (5)$$

where x' is the actual beam direction at kicker position s , α_{i_1} and α_{i_2} are constants, which depend on the lattice functions of the accelerator, the positions s_{i_1} , s_{i_2} and s of PU $_{i_1}$, PU $_{i_2}$ and the Kicker respectively. z denotes the noise part in the estimation of beam direction at the Kicker.

MATHEMATICAL DERIVATION

In order to mitigate the noise part in the estimation of beam direction at the Kicker, we address here a new approach for calculating optimal weighted sum of the signals from multiple pickups.

The idea of this approach is to average out the noise by estimating the beam direction at the Kicker position s using the signals from M PUs, i.e., three and more, in an optimal way, such that the noise power at the estimated signal is minimized and the the weighted sum of the actual beam displacements at the PUS without noise corresponds to the actual beam direction at the Kicker.

The optimization problem can be formulated as

$$[a_1, \dots, a_M] = \underset{a_1, \dots, a_M}{\operatorname{argmin}} E \left| \sum_{i=1}^M a_i z_i \right|^2 \quad (6a)$$

$$s.t. \quad \sum_{i=0}^M a_i \tilde{x}_i = x' \quad (6b)$$

In order to find a solution for this optimization problem, we first reformulate it using vector summations of PU pairs signals like in the following: The beam direction at the position s can be estimated using any two of the M PUs signals according to vector summation. Let's take as pairs x_1 with x_2 , x_2 with x_3 and so on and so forth till x_{M-1} with x_M . This can be written in matrix notation as follows

$$\begin{pmatrix} \tilde{x}_i \\ \vdots \\ \tilde{x}_{M-1} \end{pmatrix} = \mathbf{\Lambda} \begin{pmatrix} x_1 \\ \vdots \\ x_M \end{pmatrix} \quad (7)$$

$$= \mathbf{\Lambda} \tilde{\mathbf{x}} + \mathbf{\Lambda} \mathbf{z} \quad (8)$$

$$= \begin{pmatrix} x' \\ \vdots \\ x' \end{pmatrix} + \mathbf{\Lambda} \mathbf{z} \quad (9)$$

where the matrix $\mathbf{\Lambda}$ is given through the vector summation of the above mentioned PUs pairs as

$$\mathbf{\Lambda} = \begin{pmatrix} \alpha_{11} & \alpha_{12} & 0 & \dots & 0 \\ 0 & \alpha_{22} & \alpha_{23} & 0 & \dots & 0 \\ 0 & 0 & \ddots & \ddots & 0 & \dots \\ \vdots & \vdots & \dots & 0 & \ddots & \dots \\ 0 & \dots & 0 & \dots & \alpha_{M-1M-1} & \alpha_{M-1M} \end{pmatrix} \quad (10)$$

Let $\mathbf{w} = [w_1, w_2, \dots, w_{M-2}, 1 - \sum_{i=1}^{M-2} w_i]^T$ then the following holds

$$\mathbf{w}^T \begin{pmatrix} \tilde{x}_i \\ \vdots \\ \tilde{x}_{M-1} \end{pmatrix} = x' + \mathbf{w}^T \mathbf{\Lambda} \mathbf{z} \quad \forall w_1, \dots, w_{M-2} \in \mathcal{R}^{M-2} \quad (11)$$

The optimization problem given in Eq. 6 is equivalent to finding the optimal vector \mathbf{w}_{opt} which minimizes the noise power in Eq. 11, i.e., $E|\mathbf{w}^T \mathbf{\Lambda} \mathbf{z}|^2$, Where

$$[a_1, \dots, a_M]_{opt} = \mathbf{w}_{opt}^T \mathbf{\Lambda}. \quad (12)$$

The reason for this equivalence is that both problems have the same number of dimensions in addition to describing the same unbiased estimator with minimum noise in both .

The vector \mathbf{w} can be written in the following form

$$\mathbf{w} = \mathbf{D} \hat{\mathbf{w}} + \mathbf{e}_{M-1} \quad (13)$$

where

$$\hat{\mathbf{w}} = [w_1, w_2, \dots, w_{M-2}]^T, \quad (14)$$

$$\mathbf{e}_{M-1} = [0, 0, \dots, 0, 1]^T \in \mathcal{R}^{M-1 \times 1} \quad (15)$$

and $\mathbf{D} \in \mathcal{R}^{M-1 \times M-2}$ with all-ones on the main diagonal, all -1 on the last row and zeros elsewhere.

Therefore, the noise power is given by

$$\begin{aligned} P_N &= E|\mathbf{w}^T \mathbf{\Lambda} \mathbf{z}|^2 \\ &= \hat{\mathbf{w}}^T \mathbf{D}^T \mathbf{\Lambda} \mathbf{R}_{zz} \mathbf{\Lambda}^T \mathbf{D} \hat{\mathbf{w}} \\ &\quad + 2 \hat{\mathbf{w}}^T \mathbf{D}^T \mathbf{\Lambda} \mathbf{R}_{zz} \mathbf{\Lambda}^T \mathbf{e}_{M-1} + \mathbf{e}_{M-1}^T \mathbf{D}^T \mathbf{\Lambda} \mathbf{R}_{zz} \mathbf{\Lambda}^T \mathbf{e}_{M-1} \end{aligned}$$

An optimal solution $\hat{\mathbf{w}}_{opt}$ can be found by setting the derivative of P_N with respect to \mathbf{w} to zero and solving

$$\left. \frac{\partial P_N}{\partial \hat{\mathbf{w}}} \right|_{\hat{\mathbf{w}}_{opt}} = 0 \quad (16)$$

which leads to

$$\hat{\mathbf{w}}_{opt} = -(\mathbf{D}^T \mathbf{\Lambda} \mathbf{R}_{zz} \mathbf{\Lambda}^T \mathbf{D})^{-1} \mathbf{D}^T \mathbf{\Lambda} \mathbf{R}_{zz} \mathbf{\Lambda}^T \mathbf{e}_{M-1} \quad (17)$$

Finally the optimal weights $[a_1, \dots, a_M]_{opt}$ can be calculated using Eq. 12 and Eq. 17.

RESULTS

In this section we show simulation results of the above addressed approach for the Synchrotron SIS 18 at the GSI. In the SIS 18 there are 12 beam position PUs for the horizontal and the vertical directions, which are located periodically along the synchrotron ring. There is also one feedback kicker for each transversal direction. The phase difference between each two neighbour pickups corresponds to the machine tune divided by 12. For the horizontal direction we have phase difference between each two neighbour pickups of 129.3° , and 99.2° for the vertical direction. During acceleration focusing changes continuously from so called doublet mode to triplet mode, which changes the betatron functions during operation. The simulation results for the doublet mode, i.e., at the beginning of acceleration directly after injection, for both of the transversal and vertical directions will be shown. The technical parameters for these two scenarios are shown in Table 1. β_{pu} and α_{pu} are the values of the betatron functions at the PUs positions. β_k and α_k are the values of the betatron functions at the kicker position. $\Delta\phi_1^\circ$ denotes the phase difference between the kicker position and the position of the closest PU.

Table 1: Technical Parameters for Doublet Mode

mode	β_k	β_{pu}	α_k	α_{pu}	$\Delta\phi_1^\circ$
Doublet x	26.44	6.67	-2.22	0.67	105.7
Doublet y	6.69	20.06	-0.54	1.04	74.5

The results are depicted in Fig. 3 and Fig. 4 for horizontal and vertical directions of doublet mode respectively. As a reference we take the noise power for using the closest two PUs to the kicker, which are the currently used PUs for the TFS in the SIS 18. For each direction of doublet mode two curves are depicted, i.e. the noise power reduction by using increasing number of closest PUs to the kicker and the noise power reduction by using the best combinations of increasing number of PUs. The figures show, that the noise power can be reduced by about 6.5 db for horizontal direction and about 3.5 db for vertical direction just by using the best combination of two PUs rather than using the closest two pickups to the kicker. Furthermore, one can notice from the figures that noise power can be reduced by about 11.5 db for horizontal direction and about 8.5 db for vertical direction by using the whole 12 PUs in the SIS 18. It is also interesting to notice, that using more than the best 8 PUs for horizontal direction and 9 PUs for vertical direction doesn't bring any noticeable enhancement.

CONCLUSION

A new technique for reducing noise power by using multiple PUs in TFS has been addressed in this work. Simulation results has shown enhancement by using this technique. However we should have in mind the implementation challenges of this technique, where multiple PUs sig-

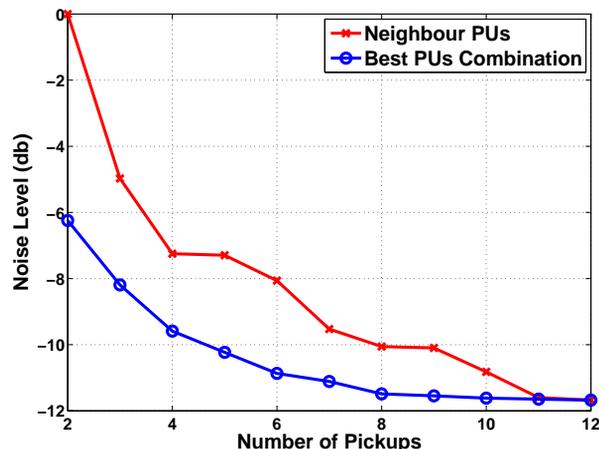


Figure 3: Noise reduction for horizontal direction of doublet mode.

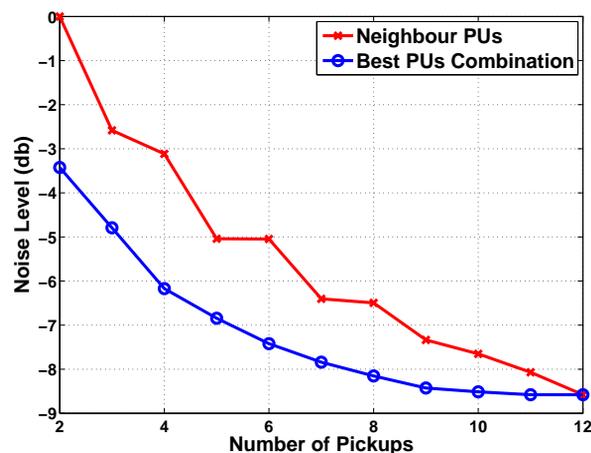


Figure 4: Noise reduction for vertical direction of doublet mode.

nals must be synchronized.

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