TRANSVERSE INSTABILITIES OF TWO TWISTED BEAMS IN A STORAGE RING

B.C. Jiang, M.Z. Zhang, Z.T. Zhao, SINAP/SSRF, Shanghai, 201800, China
G.X. Xia, School of Physics and Astronomy, University of Manchester, Manchester, UK and Cockcroft Institute, Daresbury, UK

Abstract

Two twisted beams (two beams run on the different closed orbits) in a storage ring which are produced by fast kickers can potentially deliver two bunches of radiation through one insertion device or one bending magnet, in this way the beam line stations can be potentially doubled. To operate in such a mode, higher beam current is required to keep the brightness comparable to that of the single beam operation mode and more RF buckets need to be filled mitigating single bunch instabilities. In this paper, we analyse coupled bunch instabilities such as the resistive wall instability, the ion trapping instability and fast ion instability to address the higher current operation possibility. The results show that two twisted beams operation mode can significantly weaken some of these beam instabilities.

INTRODUCTION

The never stopping demands of synchrotron beam line stations push accelerator scientists to build a storage ring containing more straight sections for insertion device or more effectively using the existing ones such as canted the beam lines [1]. In this paper, we propose using fast kickers to kick parts of the beam to a different orbit, in this way there will be two closed orbits in one storage ring. Two bunch trains running on the two separated orbits may deliver two bunches of radiations which potentially can support two beam line stations with one source point such as an insertion device or a bending magnet. We call this operation mode Two Twisted Beams (TTB) operation mode as sketched in Fig 1.

Fast kicker has already been developed such as at ALS to kicker the camshaft bunch [2] and at KEK-ATF to test ILC damping ring inject/extract scheme [3]. In ALS the kicker angle is 73 μrad in vertical plane which is about 20 times of the vertical beam divergence and the camshaft beam is totally separated from the bunch train. Though two separated beams can be produced, to keep the brightness of each TTB comparable to the single beam operation mode, more beam current needs to be stored in the ring. As one pushes to the higher current region, the limitations should to be considered. The limitations of the highest beam current differ from facilities, which can be limited by the RF power, heat loading of the vacuum chamber, front end of the beam line, or the beam instabilities.

In this paper we limited our discussion on the beam instabilities of the TTB.

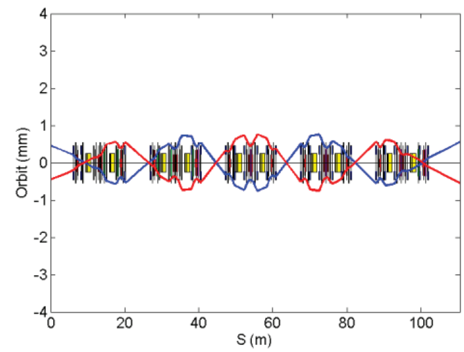

Figure 1: Sketch of the closed orbits for Two Twisted Beam operation mode in SSRF storage ring (a quarter of the full ring lattice is shown here).

RESISTIVE WALL INSTABILITY

Tune Separation Effects

Two bunch trains running on different orbits will introduce amplitude dependence tune shift. In general conditions the tune split for the different orbit is much larger than the tune shift induced by the resistive wall impedance. To avoid analytical treatment complexity of the instability for the unsymmetrical filling pattern [4], we assume here the first bunch train only contains one bunch and the second bunch train contains uniform filled bunches. The equation of the betatron oscillations with driving force of transverse wake field is written by

$$\ddot{y}_1 + 2\alpha\dot{y}_1 + \omega_{\beta 1}^2 y_1 = -\frac{c r_e N}{T_0\gamma}\sum_k \sum_{m=0}^{M-1} W_\perp\left(-kC - \frac{m-1}{M}C\right)\left[y_m\left(t - kT_0 - \frac{m-1}{M}T_0\right)\right. \quad (1)$$

Here we assume that the wake field generated by the first bunch is neglected. And α is the radiation damping rate, M is the harmonic number, c is the speed of light, N is the number of electrons in one bunch, T_0 revolution period, γ relative energy, $W_\perp$ transverse wake field, r_e classical electron radius, C circumference of the storage ring. Assume the second bunch train has the oscillation pattern:

$$y_m^\mu(t) = A \times Exp\left(2\pi \frac{m\mu}{M} i\right) Exp(-i\Omega_\mu t). \quad (2)$$

Where $\mu = 0,1,2 \ldots. M-1$. By substituting Eq.(2) to Eq.(1) we get:

$$\ddot{y}_1 + 2\alpha\dot{y}_1 + \omega_{\beta 1}^2 y_1 = iA\frac{r_e N}{\gamma T_0}\frac{4\pi M}{Z_0 T_0}\sum_{p=-\infty}^{\infty} Z_\perp[(pM + \mu)\omega + \omega_\beta)]\ Exp(-i\Omega_\mu t). \quad (3)$$

ISBN 978-3-95450-122-9

Where Z_0 is the impedance of vacuum. Eq.(3) is a forced vibration equation. The solution can be written under here:

$$y_1 = F * Exp(-i\Omega_\mu t - \psi). \quad (4)$$

$$F = A\frac{i\frac{r_eN\,4\pi M}{\gamma T_0 Z_0 T_0}\sum_{p=-\infty}^{\infty} Z_\perp[(pM+\mu)\omega+\omega_\beta)]}{[\left(\Omega_\mu^2-\omega_{\beta 1}^2\right)^2+2\alpha^2\Omega_\mu^2]^{1/2}} \approx A\frac{i\frac{r_eNM}{\gamma T_0 Z_0 \nu_\beta}\sum_{p=-\infty}^{\infty} Z_\perp[(pM+\mu)\omega+\omega_\beta)]}{\alpha/\sqrt{2}}. \quad (5)$$

$$\psi = atan\frac{2\alpha\Omega_\mu}{\omega_{\beta 1}^2-\Omega_\mu^2}. \quad (6)$$

Eq.(4) shows that for a forced vibration, the bunch will experience a steady state oscillation, the amplitude will not growth exponentially. The real part of numerator in Eq.(5) is the instability growth rate of the second bunch train. The amplifying factor of initial amplitude A is the ratio of instability growth rate of second bunch train to damping rate multiplied by $\sqrt{2}$.

Offset effects

In counting of transverse offset of two bunch trains, the equation of the betatron oscillations neglecting radiation damping is rewritten as:

$$\ddot{y}_n + \omega_\beta^2 y_n = -\frac{cr_eN}{T_0\gamma}\sum_k\{\sum_{m=0}^{K} W_\perp\left(-kC-\frac{m-n}{M}C\right)[y_m\left(t-kT_0-\frac{m-n}{M}T_0\right)-\Delta y_1] + \sum_{m=K+1}^{M-1} W_\perp\left(-kC-\frac{m-n}{M}C\right)[y_m\left(t-kT_0-\frac{m-n}{M}T_0\right)-\Delta y_2]\}. \quad (7)$$

Where K is bunch number of the first bunch train. $\Delta y_{1,2}$ are the offsets of the orbits of the first and the second bunch train. After rearranging, the equation can be written in the following form.

$$\ddot{y}_n + \omega_\beta^2 y_n = -\frac{cr_eN}{T_0\gamma}\{\sum_k\sum_{m=0}^{M-1} W_\perp\left(-kC-\frac{m-n}{M}C\right)y_m\left(t-kT_0-\frac{m-n}{M}T_0\right)-\Delta F\}. \quad (8)$$

$$\Delta F = \sum_k[\sum_{m=0}^{K} W_\perp\left(-kC-\frac{m-n}{M}C\right)\Delta y_1 + \sum_{m=K+1}^{M-1} W_\perp\left(-kC-\frac{m-n}{M}C\right)\Delta y_2]. \quad (9)$$

Here some assumptions are made:

$$y_n^\mu = A\times Exp\left(2\pi\frac{n\mu}{M}i\right)Exp(-i\Omega_\mu t) + f(t). \quad (10)$$

$$\omega_{\beta 1} \approx \omega_{\beta 2} \approx \Omega_\mu. \quad (11)$$

The solution of Eq.(7) can be written in the following:

$$\Omega_\mu - \omega_\beta = -i\frac{r_eNM}{\gamma T_0 Z_0 \nu_\beta}\sum_{p=-\infty}^{\infty} Z_\perp[(pM+\mu)\omega+\omega_\beta)]. \quad (12)$$

$$f(t) = -\frac{\Delta F}{\omega_\beta^2}. \quad (13)$$

The result shows that the resistive wall instability of TTB is the same as that for a single bunch train except with an additional static term, which is created by the orbit offset. The static term do not contribute to instability and could be compensated by correctors.

ION TRAPPING

Almost all of the working storage rings leaves one or more RF buckets unfilled as clearing gaps to avoid ions trapped by the electron bunches [5]. The ions trapped by the electron bunches are studied in this section. To decrease the single bunch current, more RF buckets needs to be filled when pursuing high current. The more RF buckets to be filled, the more possibilities of the ions are trapped.

For a Gaussian beam, potential well produced by the electron bunch can be written as [6]

$$V(x,y) = -\frac{N}{\pi\epsilon_0}\int_0^\infty \frac{1-e^{-\frac{x^2}{2\sigma_x^2+4t}-\frac{y^2}{2\sigma_y^2+4t}}}{\sqrt{(2\sigma_x^2+4t)(2\sigma_y^2+4t)}}dt. \quad (14)$$

Here σ_x and σ_y are RMS transverse beam sizes, $\varepsilon 0$ is the permittivity of free space. For 2D treatment we derive $E(y)$ at $x=0$:

$$E(y) = \frac{\partial V(x,y)}{\partial y}|_{x=0} = -\frac{N}{\pi\epsilon_0}\int_0^\infty \frac{2e^{-\frac{y^2}{2\sigma_y^2+4t}}y}{\sqrt{(2\sigma_x^2+4t)}(2\sigma_y^2+4t)^{3/2}}dt. \quad (15)$$

Neglecting longitudinal motion of the ion, when an electron bunch passes by, the transverse kick to the ions is:

$$\Delta\dot{y} = -\frac{2r_pcN}{A}\int_0^\infty \frac{2e^{-\frac{(y-y_e)^2}{2\sigma_y^2+4t}}(y-y_e)}{\sqrt{(2\sigma_x^2+4t)}(2\sigma_y^2+4t)^{3/2}}dt. \quad (16)$$

Here A is the mass number of the ion.

A simulation code is developed to investigate motions of the ions. The kick to the ion produced by the electron bunch is calculated by the numerically evaluate integral. The result of integral term in Eq.(16) is shown in Fig. 2. In the region $y<<\sigma_y$, it is likely linear and well matches the expansion formula $\Delta\dot{y} \propto \frac{2y}{\sigma_y(\sigma_y+\sigma_x)}$ [7].

In the code, the kick strength is calculated using interpolation method according to the instant y value. The interpolation table is that for plotting Fig. 2. Between two adjacent kicks is a 2ns free space drifting. In our simulation, 500 macro ions are created randomly in one σ_y in Gaussian distribution. Electron beam cycles 5 turns. The vacuum chamber is assumed to be 40mm in diameter

ISBN 978-3-95450-122-9

and the other beam parameters are listed in Table 1. For different kinds of modes the simulation results are shown in Fig. 3.

Table 1: Basic Beam Parameters of SSRF Storage Ring

Parameters	Value
Beam size σ_x/σ_y	158 μm /10 μm
Number of electrons per bunch	0.54e10
RF frequency	500 MHz
Harmonic Number	720
Ion type	CO^+
Orbits separation(y plane)	200 μm

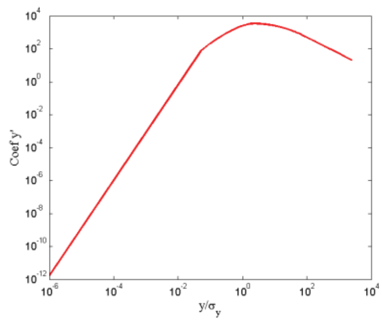

Figure 2: The integral term in Eq(16) as a function of y/σ_y.

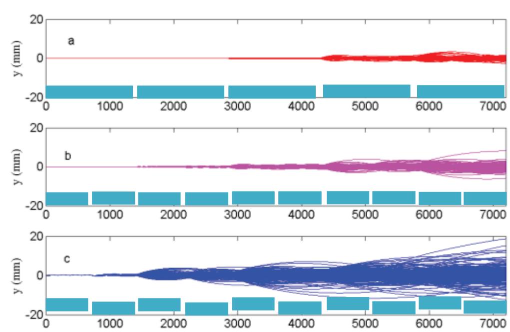

Figure 3: The oscillation amplitude of ions with respect to the circulating time in SSRF storage ring. a) 1 bunch train with 700 bunches followed by 20 empty gap, b) 2 bunch trains with 350 bunches each, separated by 10 empty gap and c) 2 bunch trains with 350 bunches each, separated by 10 empty gap, run at different closed orbit depart with distance in between about 200μm.

At different fixed points (different orbit departure, beam size), we gets different results. All of the results show that TTB helps to drive the ions to bigger amplitudes in the finite turns. This means TTB can reduce ion trapping.

FAST ION INSTABILITY

Fast ion instability (FII) is observed in many electron storage rings [8], which is caused by the linearly increased density of the ions along the bunch train. It's a single passage phenomena, the number of ions produced by the long bunch train is large enough to disturb the motion of the tail bunches thus increase the oscillation amplitudes and the beam size. After passage of the bunch train most of the ions lost during the empty gap.

The FII is simulated by a weak-strong simulation code. In this simulation, the beam is regarded as the rigid Gaussian beam and the ions produced by the collisional ionization are represented by the macro-particles. The beam parameter used is the same as that used in ion trapping simulation.

The vertical oscillation amplitude of the bunch centroid is charactered by the half of the Courant-Synder invariant $J_y = [\gamma y^2 + 2\alpha y y' + \beta y'^2]/2$, where α, β and γ are the Twiss parameters of the ring. For case c) in Fig. 3, the growth of J_y is about two orders smaller than case b) as shown in Fig. 4.

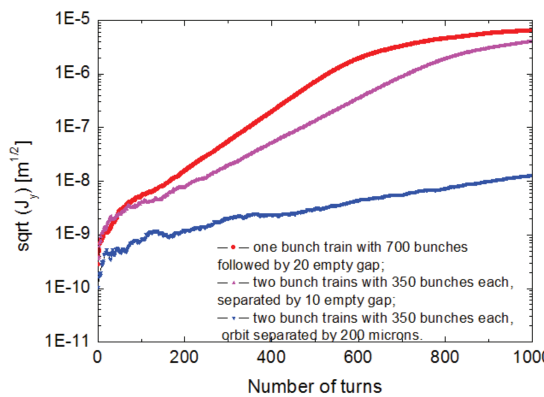

Figure 4: Growth of maximum oscillation amplitude for cases as for Fig. 3.

DISCUSSION

For the tune split effects on the resistive wall instability, the analytical procedure is simplified. The real situation will be more complex because two trains will interact each other. A simulation is underway to find more detailed information.

This TTB operation mode can also be applied to colliders for which the kick angle requirement can be released. And for the positron ring, TTB may alleviate the build-up of electron clouds as well. This needs to be further studied.

REFERENCES

[1] Patric K., et al., Proceedings of the 2003 Particle Accelerator Conference, Portland, Oregon U.S.A., 833-835, (2003).
[2] C. Sun, et al., Phys. Rev. Lett. **109**, 264801 (2012).
[3] T. Naito, et al., Proceedings of IPAC'10, Kyoto, Japan, 2386-2388, (2010).
[4] Zhao Zhen-Tang, Jiang Bo-cheng, High Energy Physics and Nuclear Physics, **28**, (2004)1209-1213.
[5] Bocheng Jiang, et al., Nuclear Instruments and Methods in Physics Research A **614** (2010) 331–334.
[6] Y.C. Lee, EPAC 96 Barcelona 1996.
[7] D.Villevald, S. Heifets, SLAC-TN-06-032.
[8] Guoxing Xia, et al., Proceeding of IPAC'10, Kyoto, Japan, 1991-1993 (2010).