

# DESIGN OF LOW MOMENTUM COMPACTION LATTICES FOR THE TPS STORAGE RING

C.C. Kuo and H.J. Tsai

National Synchrotron Radiation Research Center, Hsinchu 30076, Taiwan

## Abstract

The nominal bunch length is around 10 ps rms in the Taiwan Photon Source (TPS). To further reduce bunch length to a few ps rms range, low momentum compaction factor configurations (low alpha), i.e., quasi-isochronous machines, are designed. The beam dynamics issues of the TPS low alpha lattice are reported.

## INTRODUCTION

Short pulse beam in an electron storage ring can generate coherent IR/THz sources and provide time-resolved experiments in the x-ray regime. To generate short pulse beam in a storage ring, there are several approaches, e.g. lower beam energy, low alpha lattice configurations, laser sliced beam, crab-cavity deflected beam, high rf frequency and potential, and different rf frequency and potential combinations, etc.

Low alpha mode operation is an economical way to generate short pulse beam. Several light sources are in routine operations. However, bunch current is limited due to low current threshold of bunch lengthening, hence resulting in significant reduced photon flux and brightness. Nevertheless, such an operation mode is still of much interest for a number of potential users. In this paper, we focus on the lattice design and beam dynamics issues of positive and negative low alpha configurations in TPS.

## MOMENTUM COMPACTION FACTOR

TPS is a 3-GeV, 518-m synchrotron light source in Taiwan and currently under construction. It is expected to start the beam commissioning in the latter part of 2014. TPS lattice consists of 24 DBA cells and the natural emittance is 1.6 nm-rad. The nominal momentum compaction factor  $\alpha_1$ ,  $\alpha_2$  are  $2.4 \times 10^{-4}$  and  $2.1 \times 10^{-3}$ , respectively. With 3.5 MV, 500 MHz radiofrequency system, the nominal bunch length is 10 ps rms and rf energy aperture is between -6% to 4% [1].

The electron bunch length can be expressed as

$$\sigma_t = \frac{\sigma_E / E}{f_{rev}} \left( \frac{\alpha E}{2\pi h} \right)^{1/2} \frac{1}{(e^2 V_{rf}^2 - U_0^2)^{1/4}},$$

$$\frac{\sigma_E}{E} = \gamma \left( C_q \frac{I_3}{2I_2 + I_4} \right)^{1/2} \quad (1)$$

where  $h$  is the harmonic number;  $U_0$  is the radiation loss per turn;  $I_2$ ,  $I_3$ ,  $I_4$  are synchrotron integrals;  $\gamma$  is the

Lorentz factor and  $C_q = 3.83 \times 10^{-13}$  m. Keeping same rf system and beam energy, bunch length is proportional to the square root of alpha. Alpha is defined as the relative change in electron path length in one revolution with respect to momentum deviation as shown in Eq. 2.

$$\alpha = \frac{\Delta C}{C} / \delta, \quad \alpha(\delta) = \alpha_1 + \alpha_2 \delta + \alpha_3 \delta^2 + O(\delta^3) \quad (2)$$

where

$$\alpha_1 = \frac{1}{C} \oint_C \frac{\eta_1(s)}{\rho} ds$$

$$\alpha_2 = \frac{1}{C} \oint_C \left( \frac{\eta_1'(s)^2}{2} + \frac{\eta_2(s)}{\rho} \right) ds \quad (3)$$

$$\alpha_3 = \frac{1}{C} \oint_C \left( \eta_1'(s)\eta_2'(s) - \frac{\eta_1(s)\eta_1'(s)^2}{2\rho} + \frac{\eta_3(s)}{\rho} \right) ds,$$

$C$  is the circumference;  $\delta$  is the momentum deviation with respect to nominal particle momentum;  $\eta_{1,2,3}$  and  $\eta'_{1,2}$  are dispersion and dispersion derivative in terms of momentum expansion [2]. The first order alpha can be reduced by creating positive and negative dispersion inside dipoles so that the integrated dispersion is near zero in the dipole magnets. Another way is to have inverted bends in the lattice such as in a ring, NewsUBARU. In TPS, we change the distribution of dispersion function so that it crosses zero in the dipoles with equal positive and negative integrated dispersions in dipoles. The second order alpha can be minimized by sextupole optimization. This procedure is essential to guarantee that the nonlinear beam dynamics is acceptable in both transverse and longitudinal planes. The third order alpha can be controlled mainly by octupole field.

## LONGITUDINAL DYNAMICS

For a highly relativistic particle without synchrotron radiation loss, longitudinal motion can be written as:

$$\begin{cases} \delta'(\phi) = \frac{eV_{rf}}{T_0 E} (\sin \phi - \sin \phi_s) \\ \phi'(\delta) = \frac{2\pi h}{T_0} (\alpha_1 \delta + \alpha_2 \delta^2 + \alpha_3 \delta^3), \end{cases} \quad (4)$$

where  $T_0$  is the revolution time;  $\phi_s$  is the synchronous phase. The path length change due to momentum-independent term is ignored and turn by turn mapping equations can be generated accordingly. Neglecting the damping term, the Hamiltonian of the synchrotron motion can be expressed as:

$$H(\phi, \delta) = h\left(\frac{1}{2}\alpha_1\delta^2 + \frac{1}{3}\alpha_2\delta^3 + \frac{1}{4}\alpha_3\delta^4\right) + \frac{eV_{rf}}{2\pi E}(\cos\phi - \cos\phi_s + (\phi - \phi_s)\sin\phi_s) \quad (5)$$

The fixed points are with  $\mathcal{D}'(\phi) = \phi'(\delta) = 0$ . The condition that  $\alpha_2^2 > 4\alpha_1\alpha_3$  will have three stable fixed points, otherwise there are only one stable and one unstable fixed points.

## LOW ALPHA LATTICE FUNCTION

### High Emittance, Low Alpha Lattice

By changing the dispersions to negative values in the straight sections and crossing zero in dipoles, we can obtain low alpha lattices with high emittance. We use lattice design code MAD to match the optical functions, lattice tunes and low alphas [3]. A number of lattices with very small positive and negative alphas down to  $1 \times 10^{-6}$  or less can be matched. Figure 1 shows optical functions of one high emittance, low alpha lattice.

### Low Emittance, Low Alpha Lattice

By keeping dispersion positive in the straights and getting both positive and negative dispersions inside dipoles, we can get low alphas with low emittance lattices. One family of quadrupoles needs to be reversed in polarity. The working tunes are higher than the high emittance lattice in the horizontal plan but smaller in vertical. Figure 2 depicts the optical functions of one low emittance, low alpha lattice.

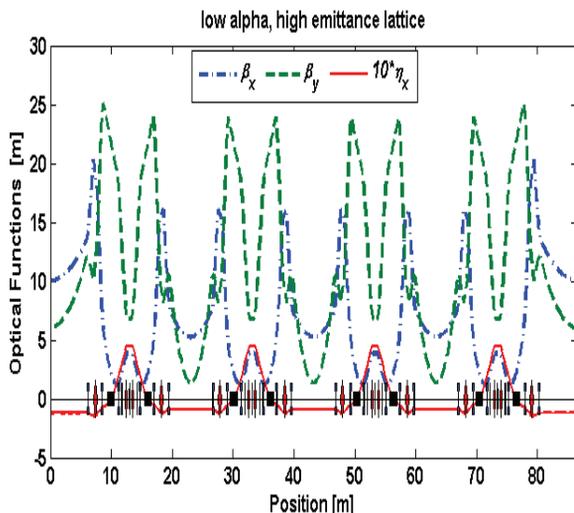


Figure 1: Optical functions for high emittance lattice.

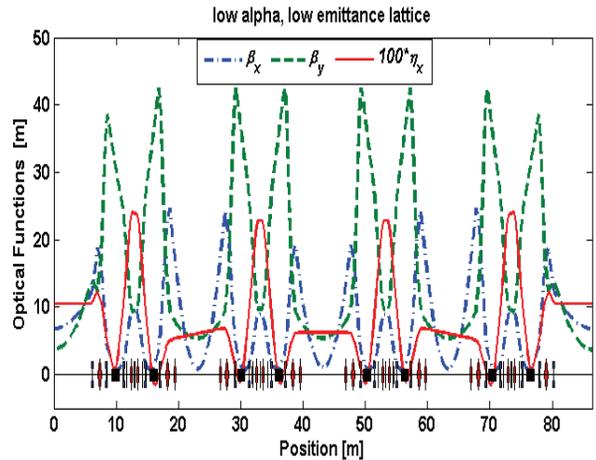


Figure 2: Optical functions for low emittance lattice.

## SEXTUPOLE OPTIMIZATION

The second order momentum compaction factor can be minimized by sextupoles and chromaticity corrections need to be done simultaneously with these sextupoles too.

$$\begin{cases} \Delta\alpha_2 = \frac{-1}{2C} \oint \eta_{1,x}^3 \Delta K_2 ds \\ \Delta\xi_x = \frac{1}{4\pi} \oint \eta_{1,x} \beta_x \Delta K_2 ds \\ \Delta\xi_y = \frac{1}{4\pi} \oint \eta_{1,x} \beta_y \Delta K_2 ds \end{cases} \quad (6)$$

Furthermore, the nonlinear driving terms also need to be minimized using OPA code [4]. The maximum sextupole strengths in the high emittance modes are smaller than  $6.0 \text{ m}^{-2}$  and three sextupole families are with reversed polarities. On the other hand, for the low emittance, it is constrained by the hardware limit in the focusing sextupoles in the arcs. Some major parameters of two low alpha lattices are listed in Table 1.

Table 1: Major Parameters of Low Alpha Lattices

Parameter	High emittance 32.5 nm-rad	Low emittance 3.2 nm-rad
$\alpha_1$	$-1.95 \times 10^{-6}$	$2.55 \times 10^{-5}$
$\alpha_2$ (with/without sextupoles)	$-3.66 \times 10^{-4}$ / $1.08 \times 10^{-2}$	$2.66 \times 10^{-4}$ / $5.02 \times 10^{-3}$
$\alpha_3$ (with/without sextupoles)	$-2.49 \times 10^{-2}$ / $-1.70 \times 10^{-1}$	$-3.61 \times 10^{-3}$ / $6.16 \times 10^{-3}$
$v_x/v_y$	21.220/12.360	29.386/8.265
Bunch length $\sigma_1$ (3.5MV)	0.86 ps	3.12 ps
Nat. Chromaticity $\xi_x/\xi_y$	-35.47/-32.35	-50.02/-53.42

Both transverse dynamics and longitudinal beam behavior are investigated along with nonlinear optimization. For high emittance lattice, Fig. 3~5 show the dynamic aperture, corresponding frequency map and longitudinal phase space up to third order of alpha. Similar results for low emittance cases are depicted in Fig. 6~8.

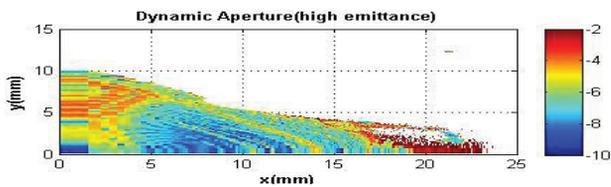


Figure 3: Dynamic aperture for low alpha, high emittance lattice. Tune diffusion rate is indicated.

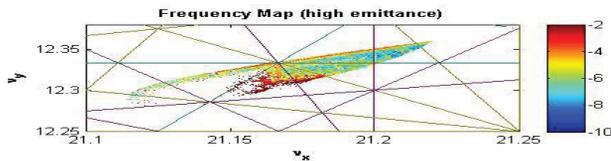


Figure 4: Frequency map for low alpha, high emittance lattice.

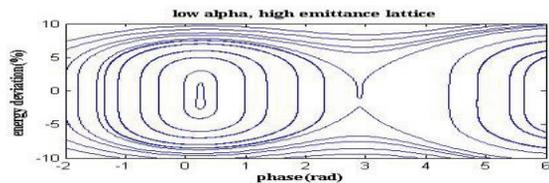


Figure 5: Longitudinal phase space up to third order alpha for low alpha, high emittance lattice.

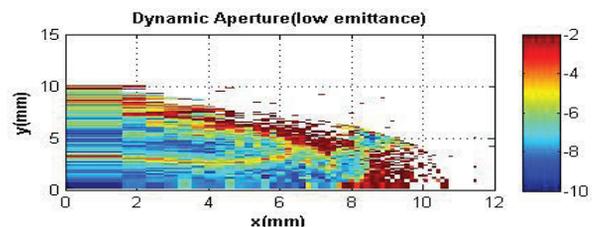


Figure 6: Dynamic aperture for low alpha, low emittance lattice.

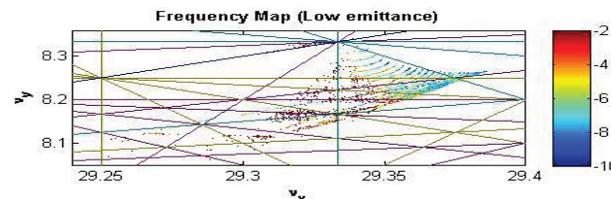


Figure 7: Frequency map for low alpha, low emittance lattice.

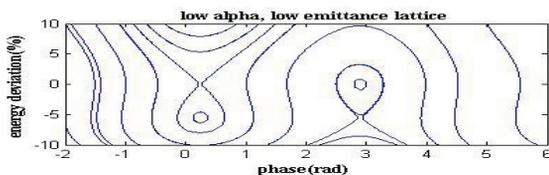


Figure 8: Longitudinal phase space up to third order alpha for low alpha, low emittance lattice.

### INJECTION EFFICIENCY

For these low alpha lattices with small transverse acceptance and/or small longitudinal acceptance, injection efficiency is simulated using Tracy II code with 6-D tracking and radiation damping is turned on [5]. Figure 9a and Fig. 9b show the beam distribution before injection

and after 10000 turns for a Gaussian distributed beam of 2000 particles for low emittance lattice. Injection errors and magnetic field errors are included, but no chamber limits, and injection efficiency can be 86% for low emittance lattice, while for high emittance lattice it is 36%. In the 6-D tracking, alpha up to fifth order are included, and for the low emittance case, it shows only one stable fixed point and energy acceptance is more than 6%.

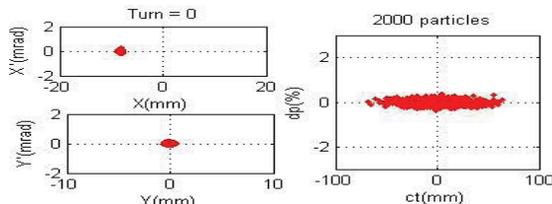


Figure 9a: Beam distribution before injection. Coordinate centers corresponding to design orbit and longitudinal phase of  $\pi$ .

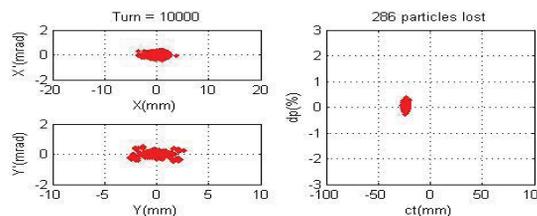


Figure 9b: Beam distribution with 10000 turns after injection for low emittance lattice. 86% injection efficiency is obtained. Longitudinal phase of  $\pi$  corresponding to  $ct=0$  mm.

### ENERGY APERTURE

Using Tracy II 6-D tracking, we also obtain energy aperture along the ring path. Figure 10 gives the simulated results for the low emittance lattice, where 1% coupling is assumed and chamber size limits are included.

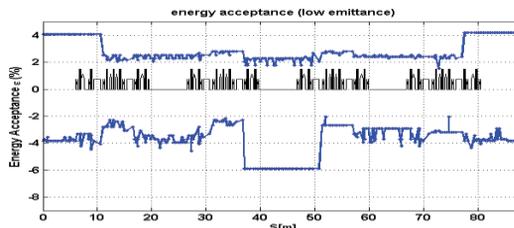


Figure 10: Energy aperture using Tracy II 6-D tracking for the low emittance lattice. 1% coupling is assumed. RF voltage is 3.5MV and vacuum chamber limits are included.

### REFERENCES

- [1] C.C. Kuo et al., "Accelerator Physics Issues for TPS" IPAC'10, Kyoto, May 2010, MOOCMH01, p.36 (2010); <http://www.JACoW.org>
- [2] I.P.S. Martin et al., "Experience with low alpha lattices at the Diamond Light Source", PRSTAB **14**, 040705 (2011)
- [3] MAD, <http://mad.web.cern.ch/mad/>
- [4] A. Streun, <http://people.web.psi.ch/streun/opa/>
- [5] Tracy II (L. Nadolski--SOLEIL, M. Boege--SLS, J. Bengtsson--NSLS-II)