RADIATION OF A CHARGED PARTICLE BUNCH MOVING ALONG BOUNDARY OF WIRE METAMATERIAL *

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Abstract

The material under consideration represents a periodical volume structure of long parallel conductive wires. If wavelengths are much greater than periods, the structure can be described as some anisotropic medium possessing both frequency and spatial dispersion [1] (so-called wire metamaterial). Earlier we considered the radiation of bunches moving in boundless wire metamaterial. It has been discovered that this radiation is nondivergent, and it is perspective for diagnostics of bunches [2]. Now we consider the case when the bunch moves in vacuum along the boundary of the semi-infinite metamaterial perpendicularly to the wires. Analytical and numerical analysis of the problem is performed. It is shown that radiation from a point charge concentrates in some vicinity of certain planes and propagates along the wires with speed of light. Series of computations show that the radiation under consideration can be useful for determination of sizes and shape of bunch.

INTRODUCTION

Cherenkov radiation (CR) is actively used for detection of charged particles [3]. As well, this phenomenon can be applied for diagnostics of particle bunches. One of the problems for such devices is essential attenuation of radiation with increase of distance. As was shown in previous papers [2, 4], using a structure called "wire metamaterial" can help to solve this problem. This medium is a set of infinitely long wires with radius r_0 arranged in a square lattice with period d (Fig. 1).

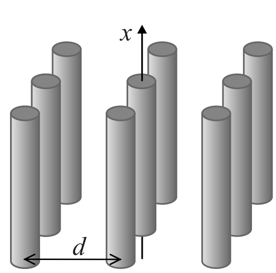

Figure 1: Scheme of the structure.

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Properties of such structure were investigated in many papers [1, 5, 6, 7]. Its effective permittivity tensor (for the case when the wires are in vacuum and do not possess any coating) has the following form [1]:

$$\hat{\varepsilon} = \begin{pmatrix} \varepsilon_{\parallel} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \varepsilon_{\parallel} = 1 - \frac{\omega_p^2}{\omega^2 - k_x^2 c^2 + 2i\,\omega_d\,\omega}, \tag{1}$$

where $\omega_p^2 = 2\pi c^2 d^{-2} \left[\ln\left(d/r_0\right) - C\right]^{-1}$; ω_d is a small parameter that is responsible for energy losses ($\omega_d \ll \omega_p$) and C is some constant (as a rule, $C \sim 1$). Note, that the "meta-medium" under consideration possesses both frequency and spatial dispersion.

As was shown [2, 4], the charge moving perpendicularly to the wires in unbounded metamaterial generates non-divergent radiation which propagates along the conductors and is concentrated in the small vicinity of determined lines. Diagrams for field distribution show, that such radiation can help to estimate sizes and shape of the bunch. However there are certain difficulties for realization of devices on base of the wire metamaterial. One of them is that wires will essentially affect the bunch moving inside the structure. Therefore it is more attractive to use a bounded wire structure under condition that the bunch moves in vacuum along the structure boundary.

ANALYTICAL INVESTIGATION

We suppose that the "lower" semi-space $x<0$ is vacuum and "upper" one $x>0$ is a metamaterial with permittivity tensor (1). Lets a point charge moves perpendicularly to the wires along the line $x=-a_0$, $y=0$ with velocity $\vec{V}=V\vec{e}_z=c\beta\vec{e}_z$ (Fig. 2).

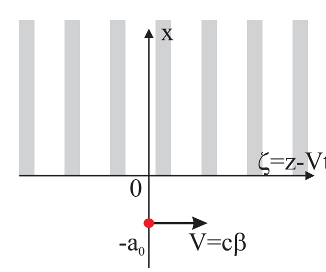

Figure 2: Geometry of the problem.

To solve the problem, we have to meet certain boundary conditions. There are four classic conditions (continuity of the tangential components of the electric and magnetic fields). However they are insufficient in the case

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under consideration, because two waves of different polarizations can be excited in the vacuum and three waves can be excited in the wire metamaterial [1]. Therefore we must have five boundary conditions to obtain the only solution. As additional condition we can use the requirement $E_x|_{x=-0} = E_x|_{x=+0}$, which is explained by vanishing of the electric current at the wire extremities [8].

On the base of these conditions, we can obtain expressions for all types of waves in vacuum and metamaterial. The result can be presented as two-hold integrals for the three waves excited in the metamaterial. Two of them are of evanescent type and rapidly decrease with the distance from the charge. They represent "quasi-Coulomb" field that is carried by the charge.

The third wave, called "extraordinary anisotropic", concentrates at the vicinity of the plane $z = \beta(ct - x)$ and propagates along the wires. It has two electrical components E_y and E_z; magnetic part can be expressed as $H_z = E_y \mathrm{sgn} x$, $H_y = -E_z \mathrm{sgn} x$. Analysis shows that for ultra-relativistic charge ($\beta \to 1$) we can write the wave part of the field as one-hold integrals:

$$E_y = q \int_0^{+\infty} k_y \sin(k_y y) e^{-a_0 k_y} dk_y \times \left\{ \Theta(\hat{\xi}) \left[e^{-k_y \hat{\xi}} - \frac{4 k_y k_{yp}}{(k_y + k_{yp})^2} e^{-\hat{\xi} k_{yp}} \right] + \Theta(-\hat{\xi}) \frac{2k_y^2 - 2k_y k_{yp} + (2k_y \hat{\xi} + 1)\,\omega_p^2/c^2}{(k_y + k_{yp})^2} e^{k_y \hat{\xi}} \right\}, \quad (2)$$

$$E_z = q \int_0^{+\infty} k_y \cos(k_y y) e^{-a_0 k_y} dk_y \times \left\{ \Theta(\hat{\xi}) \left[e^{-k_y \hat{\xi}} - \frac{4 k_{yp}^2}{(k_y + k_{yp})^2} e^{-\hat{\xi} k_{yp}} \right] + \Theta(-\hat{\xi}) \frac{-2k_y^2 + 2k_y k_{yp} - (2k_y \hat{\xi} + 3)\,\omega_p^2/c^2}{(k_y + k_{yp})^2} e^{k_y \hat{\xi}} \right\}, \quad (3)$$

where $k_{yp} = \sqrt{\omega_p^2/c^2 + k_y^2}$, $\hat{\xi} = \zeta + x$ ($\zeta = z - Vt \approx z - ct$), and $\Theta(\hat{\xi})$ is Heaviside function.

Wave part of the field is similar to the radiation in unbounded metamaterial, but it is concentrated near the plane $\zeta + x = 0$ (not lines, as for unbounded structure). Moreover, it is asymmetric with respect to this plane. If the observation point is shifted along the line $(\zeta + x = \mathit{const}) \cap (y = \mathit{const})$ inside the metamaterial, the wave field doesn't vary, and the evanescent waves become negligible when $x \gg c/\omega_p$. Thus, in the considered problem, the propagating field is non-divergent, like in the unbounded wire metamaterial.

Note that some difficulty consists in presence of "quasi-Coulomb" part of the field. Evaluation shows that the role of this part is negligable if the observation point is situated at the distance $> c/\omega_p$ from the metamaterial boundary.

TYPICAL NUMERICAL RESULTS

Results of numerical calculations are presented in Fig. 3. They show "snapshots" of the wave field distribution in some plane x=const>0. As well, we present the Poynting vector $\vec{S} = c/(4\pi)[\vec{E} \times \vec{H}]$, which has the only component along x-axis (note that $\vec{S}$ doesn't describe the full energy flux in media with spatial dispersion [9]).

As one can see (Fig. 3), the "shape" of the different components depend on the distance from the charge to the metamaterial boundary. It is especially notable for E_z and S_x, which are extended along y-axis proportionally to this distance. Moreover, when $a_0 = 0$, these distributions are almost the same as for the charge moving in unbounded medium [2]. We performed analytical calculations for the integral Eq. (3) with $a_0 = 0$ and $y = 0$ and showed that the component E_z has the same singularity at $\zeta + x = 0$ as in the case of unbounded medium: $\sim \ln[\omega_p |\zeta + x|/(2c)]$.

Further, obtained results (for arbitrary value of a_0) are used as the Green function in order to calculate the wave field of the finite linear bunch with the following charge dencity:

$\rho = \delta(y)\delta(x + a_0)/(2\sigma)$ for $|\zeta| < \sigma$, and $\rho = 0$ for $|\zeta| > \sigma$.

As one can see (Fig. 4), the bunch length can be easily determined from such pictures. Precision of such estimations will decrease with increase of distance from the bunch to the boundary (the pictures in the third row looks fuzzier than the ones in the second row). Nevertheless, the structure of the field of the bunch moving along the wire metamaterial can be used for the estimation of the bunch parameters.

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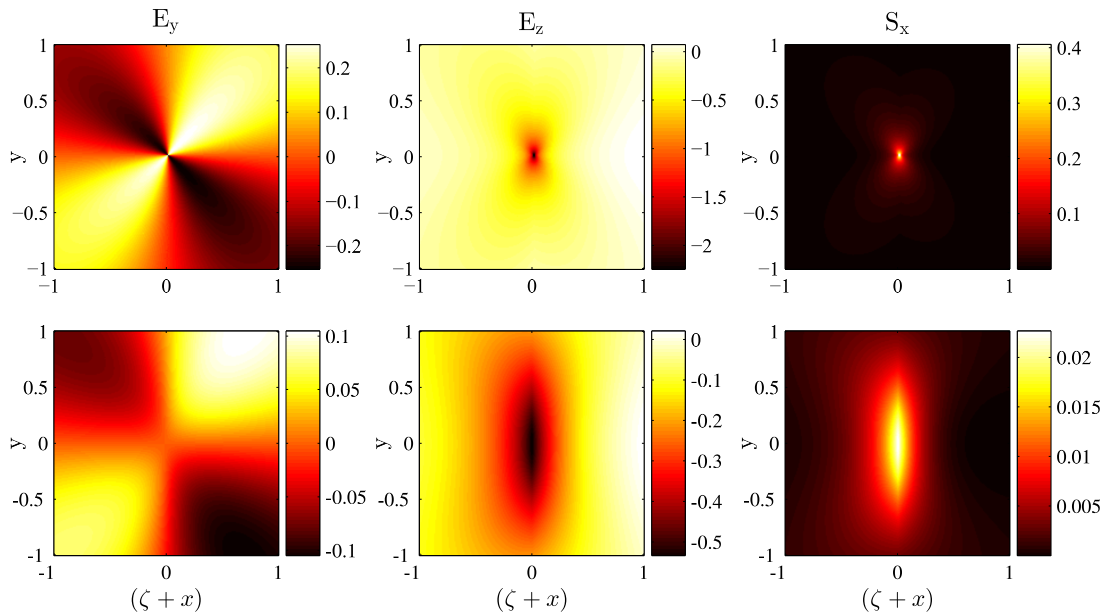

Figure 3: "Snapshots" of the wave field and the component S_x of the Poynting vector in the plane x=const>0 for the point charge; $a_0 = 0$ (first row) and $a_0 = 0.5c/\omega_p$ (second row); $\beta = 1$; electric field is in $q\,\omega_p^2/c^2$ units; S_x is in $q^2\,\omega_p^4/c^3$ units; coordinates in c/ω_p units.

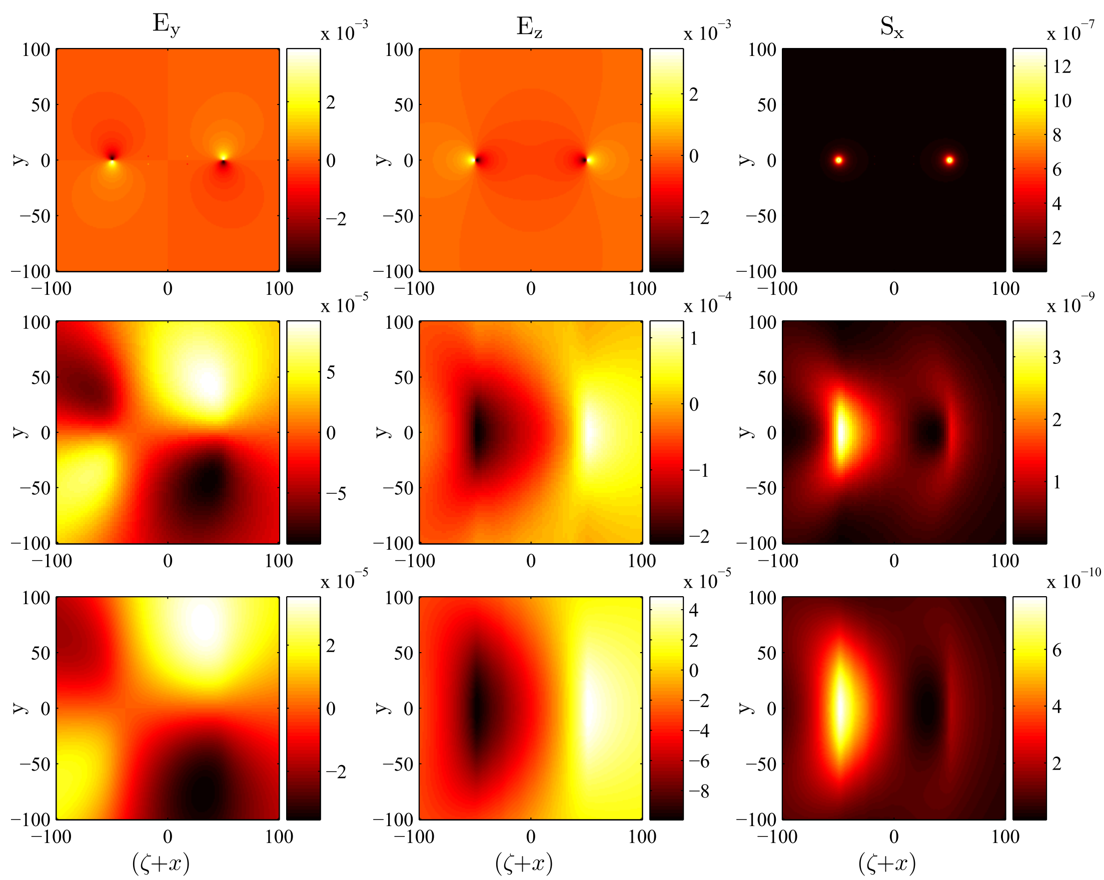

Figure 4: "Snapshots" of the wave field and the component S_x of the Poynting vector in the plane x=const>0 for the linear bunch in the unbounded metamaterial (first row) as well as in the semi-bounded one for $a_0 = 50$ (second row) and $a_0 = 100c/\omega_p$ (third row); $\sigma = 50c/\omega_p$; $\beta = 1$; electric field is in $q\,\omega_p^2/c^2$ units; S_x is in $q^2\,\omega_p^4/c^3$ units; coordinates in c/ω_p units.