

# MEASURING THE DIRECTION OF PERMANENT MAGNET EASY AXIS BY HELMHOLTZ COIL\*

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## Abstract

The permanent magnet quadrupole (PMQ) was used in drift-tube linac (DTL) of the Compact Pulsed Hadron Source (CPHS) in Tsinghua University. In order to ensure the accuracy of the quadrupole field can meet the design requirement, we need measure the strength and direction of remanence and choose the suit magnet.

This paper proposed an easy way to get the direction of permanent magnet easy axis by Helmholtz coil without knowing the angle between magnet and the axis of the coil: the magnet rotational angle data was measured by rotary encoder and encoder would send trigger signal every turn at the same position. First, we start to record data when trigger signal was appeared. Then measure the magnet in three perpendicular directions (x, y, z). Last, calculate the remanence in three directions. We had measured some magnet by the new method and obtained satisfactory results.

## INTRODUCTION

Compared to electromagnet, the most important advantage of permanent magnet quadrupole (PMQ) is the fact that it can be made very small with the same magnetic field strength.[1] Then, there is no power supply and water cold system, so little maintenance is required after installation. On the basis of the above considerations, PMQ is selected for the DTL in CPHS.

Our PMQ consists of 16 magnets, as Fig. 1 shows. To assemble quadrupole with high quality, we should measure the strength and magnetization direction of the magnet.

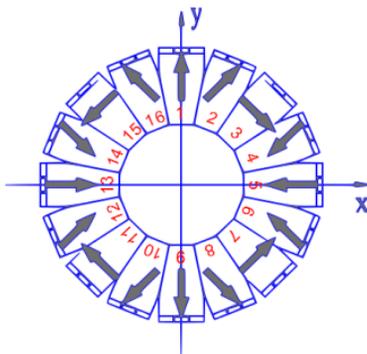


Figure 1: Composition of the PMQ.

Helmholtz coil is a pair of coils whose radius is equal to their distance, as Fig. 2 shows. While the magnet rotating, magnetic flux through the coil changed and we

can get magnetic torque by measuring the voltage of the coil. In general, we should know the angle between coil axis and magnet, but in this paper, we try to calculate this angle directly by using the trigger signal of an rotary encoder and measuring the magnet in different directions.

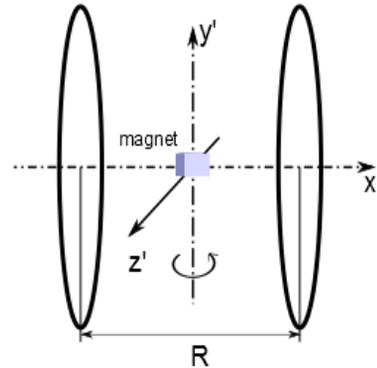


Figure 2: Structure of Helmholtz coil and its coordinates.

## PRINCEPLE OF THE MAGNETIC TORQUE MEASUREMENT

The magnet is much smaller than the coil and it can be regard as a magnetic dipole  $\mathbf{m}$ . The magnetic field is:

$$\mathbf{B} = -\mu_0 \nabla \frac{\mathbf{m} \cdot \mathbf{r}}{4\pi r^3} \quad (1)$$

The magnetic flux of the coils is:

$$\Phi = \frac{\mu_0 N m_{x'}}{GR} \quad (2)$$

Here N is turns of one coil, R is the radius,  $G=1.398$  is constant after integration and  $m_{x'}$  is magnetic torque in axis  $x'$ , as Fig.2 shows [2] While the magnet rotates at speed  $\omega$ , flux and induced emf of the coils are:

$$\Phi(t) = \frac{\mu_0 N}{GR_c} [m_{x'} \cos(\omega t) + m_z \sin(\omega t)] \quad (3)$$

$$E(t) = \frac{\mu_0 N \omega}{GR_c} [m_{x'} \sin \omega t - m_z \cos \omega t] \quad (4)$$

$E(t)$  will be measured directly. In general, the rotational speed  $\omega$  is not stable, then we can get  $m_{x'}$  and  $m_z$  after the Fourier transform of the signal  $\Phi(\varphi)$ . Here  $\Phi = -\int E(t)dt$  and  $\varphi = \int \omega(t)dt$ .  $\varphi$  is the angle the magnet has rotated.

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The  $m_x$  and  $m_z$  are known, next we should try to calculate the magnetic torque in directions x and z. Here  $(x', y', z')$  is coil coordination, as Fig. 2 shows, and  $(x, y, z)$  is magnet coordination, as Fig. 3 shows. We should try to measure  $\theta$ , the angle between magnet coordination and coil coordination.

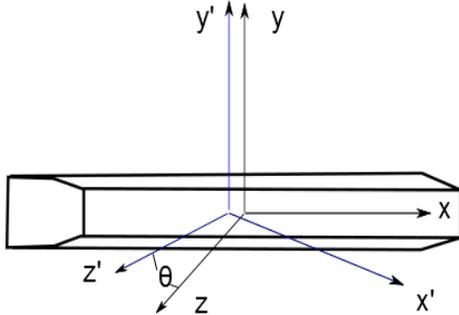


Figure 3: Magnet coordination(x, y, z).

Actually, it is not easy to measuring  $\theta$  directly, and then we put forward a method to bypass  $\theta$ . Three measurements will be taken with magnet in three mutually perpendicular directions, which is shown in Fig. 4. Three pictures from left to right in Fig. 4 are corresponding to the situation that magnet rotates around the axis of z, y and x.

In order to keep  $\theta$  same in 3 measurements, the induced emf of the coil is recorded after receiving the trigger signal from encoder.

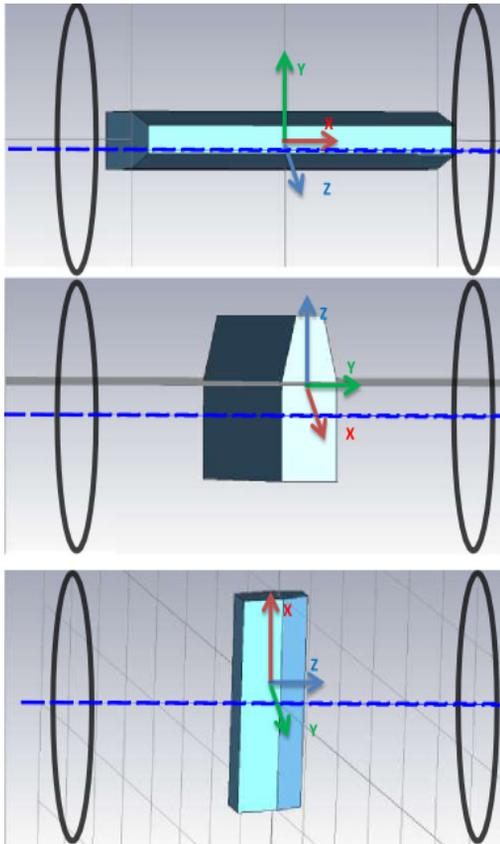


Figure 4: Measure magnet in 3 directions.

While magnet rotates around axis of y, we can get

$$m_x = m_{x'}^y \cos \theta - m_{z'}^y \sin \theta . \quad (5-a)$$

$$m_z = m_{z'}^y \cos \theta + m_{x'}^y \sin \theta . \quad (5-b)$$

Superscripts y means magnet rotates around axis of y. There are 3 unknowns ( $m_x, m_z, \theta$ ) in Equ. (5) and only 2 equations. Then we can write the same equations for the situation that magnet rotates around axis of z and x.

$$m_x = m_{x'}^z \sin \theta + m_{z'}^z \cos \theta . \quad (6-a)$$

$$m_y = m_{x'}^z \cos \theta - m_{z'}^z \sin \theta . \quad (6-b)$$

$$m_y = m_{x'}^x \sin \theta + m_{z'}^x \cos \theta . \quad (7-a)$$

$$m_z = m_{x'}^x \cos \theta - m_{z'}^x \sin \theta . \quad (7-b)$$

Now There are 4 unknowns ( $m_x, m_y, m_z, \theta$ ) and 6 equations. We get the least square solution of  $\tan \theta$  firstly, and then  $m_x, m_y$  and  $m_z$  can be get easily. Define:

$$a_x = m_{x'}^z + m_{z'}^y, a_y = m_{x'}^x + m_{z'}^z, a_z = m_{x'}^y + m_{z'}^x . \quad (8-a)$$

$$b_x = m_{z'}^z - m_{x'}^y, b_y = m_{z'}^x - m_{x'}^z, b_z = m_{z'}^y - m_{x'}^x . \quad (8-b)$$

And we can get  $\theta$  and  $m_x, m_y, m_z$ .

$$\tan \theta = -\frac{a_x b_x + a_y b_y + a_z b_z}{a_x^2 + a_y^2 + a_z^2} . \quad (9)$$

$$m_x = \frac{1}{2} [(m_{x'}^y + m_{z'}^z) \cos \theta - (m_{z'}^y - m_{x'}^z) \sin \theta] . \quad (10-a)$$

$$m_y = \frac{1}{2} [(m_{x'}^z + m_{z'}^x) \cos \theta - (m_{z'}^z - m_{x'}^x) \sin \theta] . \quad (10-b)$$

$$m_z = \frac{1}{2} [(m_{x'}^x + m_{z'}^y) \cos \theta - (m_{z'}^z - m_{x'}^y) \sin \theta] . \quad (10-c)$$

### RESULT OF THE MEASUREMENT

We acquire induced emf of the coil by a USB-6210 data acquisition devices (DAQ) and angle  $\varphi$  by an encoder. Main parameter is show in Tab. 1.

80 magnets for 5 quadrupole were measured and then assembled 5 quadrupoles. Here the magnets were divided five groups and the magnets in the same group should be same, in other words, they can be interchanged, such as

1# and 9# magnet in quadrupole, as Fig. 1 shows. The group is show in Tab. 2.

Table 1: Parameter of the measurement

Items	Parameter
Turns of Helmholtz coil	3264
Radius of Helmholtz coil\mm	240.0
sample rate of DAQ\kHz	250
number of distinct positions of encoder	720/turn
Motor speed\turns/min	120

Table 2: Group of the magnet

Type	Number of the magnet(in Fig. 1)
1	1#, 9#
2	5#, 13#
3	3#, 7#, 11#, 13 #
4	2#, 8#, 10#, 16#
5	4#, 6#, 12#, 14#

In fact, magnetic torque and the magnetization direction errors were significantly different among different magnet groups, which may be caused by different manufacture batches. The maximum deviation of the magnetic torque is reached 1.7% and the angle of easy axis error is reached 2.5°.

Each magnet was measured 5 times and the Root-mean-square (RMS) deviation is shown in Tab. 3. Then the random error of angle measurement is only about 0.08°, much smaller than the angle of easy axis error. And the random error of magnetic torque measurement is 0.004Am<sup>2</sup>, only 10% compared with the manufacturing errors of the magnet.

Table 3: RMS deviation of the meaurment

Item	m <sub>x</sub>	m <sub>y</sub>	m <sub>z</sub>	Δθ
RMS deviation	0.14%	0.11%	0.12%	0.079°

Then we selected the similar magnet and assembled quadrupoles with them. The maximum deviation of the magnetic torque in one quadrupole is smaller.

Table 4: max deviation of the magnetic torque

Quad	1	2	3	4	5
max deviation/ %	0.86	0.65	0.16	0.12	0.24

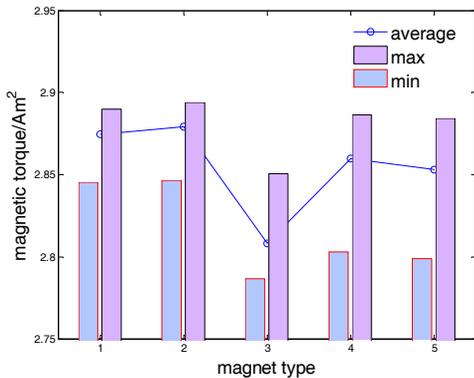


Figure 5: Magnetic torque of 5 types magnet.

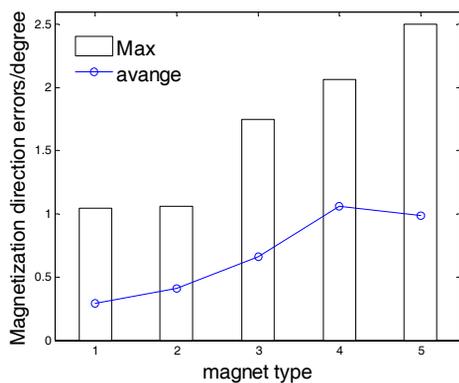


Figure 6: Angle Errors for 5 types magnet.

## CONCLUSION

In this paper, we proposed an easy way to get the direction of permanent magnet easy axis by Helmholtz coil without measuring angle. By measuring the magnet in three directions, we can calculate the angle between magnet and the axis of the coil. Then actual measurement was carried out and accuracy met the requirement.

## ACKNOWLEDGMENT

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## REFERENCES

- [1] Halbach K. Design of permanent multipole magnets with oriented rare earth cobalt material [J]. Nuclear Instruments and Methods, 1980, 169(1): 1-10.
- [2] Trout S R. Use of Helmholtz coils for magnetic measurements [J]. Magnetics, IEEE Transactions on, 1988, 24(4): 2108-2111.