CALCULATION OF WAKEFIELD IN PLASMA-FILLED DIELECTRIC CAPILLARIES GENERATED BY A RELATIVISTIC ELECTRON BEAM*

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Abstract

In this paper we give an analytical solution of TM_{0n} mode for wakefields generated by a relativistic electron beam passing through plasma-filled capillary waveguides. The numerical solution shows that the fields of TM_{0n} modes could not be ignored when the plasma wave length is comparable with the effective radius of the capillary tube, which means that the boundaries are not shielded completely by plasma. Numerical examples are given in several typical cases.

MOTIVATION

Acceleration gradient in plasma wakefields accelerators can be much higher than those of conventional RF accelerators [1-3], which is attractive for the reason of reducing the size and cost of high energy electron accelerators. In recent studies and experiments [4-6], one promising way in driving a plasma wave with strong longitudinal electric field, is using a capillary discharge waveguide with an intense electron beam passing through. In these studies, the regime of plasma waves, known as blowout or bubble regime, is highly nonlinear, where the drive pulse is intense and short enough to drive the first wake into breaking. In this regime, only one wakefield bubble is loaded with electrons, which means nonlinear positron wakes are much smaller than those of the corresponding electron case [7]. However, for the possible applications of plasma wakefields accelerators for future high energy electron-positron colliders, not only electrons but also injected positrons need to be accelerated to high energy. In order to accelerate positrons by plasma wakefields, plasma waves could be switched from blowout regime to linear regime by decreasing the plasma density. In this case, there will be accelerating fields for positions as the same gradient as for electrons.

Under the linear responses and no boundaries assumptions, wakefields in the plasma generated by a charged beam were completely studied in 1980s [8]. However the cylindrical capillary tube is a slow wave structure similar to a conventional dielectric wakefields accelerator. Therefore, eigen RF waves of Cherenkov radiations will be excited when a charged beam passing through [9-11], as well as the plasma waves. For the blowout regime, electrons density in the plasma is usually 10^{18}~10^{19} cm^{-3}, and the corresponding plasma wavelength is much smaller than the radius of capillary tube (~100um). Because of the shielding effects by the plasma, it approaches to no boundaries case with nearly zero eigen RF fields, which can also be seen in our numerical results. When the plasma electron density is down to some value in the linear regimes, the eigen RF fields will be several percent of those of plasma waves or even more, which means eigen RF modes could not be ignored in this case. Some numerical examples are given as a reference.

ANALYTICAL SOLUTIONS FOR THE WAKE FIELDS

The plasma filled dielectric capillary tube is a cylindrical structure which consists of a plasma channel with radius *a* surrounded by an isotropic dielectric with radius *b*, and outmost metallic waveguide.

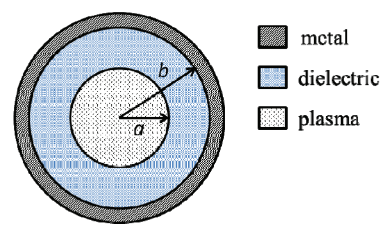

Figure 1: Cross-section of a plasma filled dielectric tube.

The plasma can be seen as a collective dielectric in the approximation of linear regime with a dielectric constant,

$$\varepsilon_p = 1 - \omega_p^2 / \omega^2 . \quad (1)$$

Considering a source particle traveling with velocity $v=\beta c$ along the cylindrical channel at a offset r_0 to its axis, the particular and general solutions of the longitudinal electric fields in frequency domain can be found analytically as [12],

$$\begin{aligned} \tilde{E}_z^{part} &= -\frac{jq\omega\left(1-\varepsilon_p\beta^2\right)}{4\pi^2\varepsilon_0\varepsilon_p v^2}\begin{cases} I_m(k_1 r)K_m(k_1 r_0), & r \le r_0, \\ K_m(k_1 r)I_m(k_1 r_0), & r_0 \le r \le a, \end{cases} \\ \tilde{E}_z^{gen} &= \begin{cases} A_m I_m(k_1 r), & 0 \le r \le a, \\ B_m\left[J_m(k_2 b)Y_m(k_2 r) - Y_m(k_2 b)J_m(k_2 r)\right], & a \le r \le b. \end{cases} \end{aligned} \quad (2)$$

Using the conditions of continuity for E_z and D_r at inner boundary $r = a$, the coefficients A_0 and B_0 can be solved for TM_{0n} modes,

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$$A_0 = \frac{jq}{4\pi^2\varepsilon_0 v^2} I_0(k_1 r_0) \frac{\omega(1-\varepsilon_p\beta^2)\left(\frac{\varepsilon_p}{k_1}K_0' + \frac{\varepsilon_r}{k_2}\frac{p_0'}{p_0}K_0\right)}{\varepsilon_p\left(\frac{\varepsilon_p}{k_1}I_0' + \frac{\varepsilon_r}{k_2}\frac{p_0'}{p_0}I_0\right)}, \qquad (3)$$

$$B_0 = A_0 I_0(k_1 a) - \frac{jq\omega}{4\pi^2\varepsilon_0\varepsilon_p v^2}(1-\varepsilon_p\beta^2) I_m(k_1 r_0)\frac{K_0(k_1 a)}{p_0}.$$

In the above the following abbreviations have been used,

$$\begin{aligned} k_1 &= \omega/v\sqrt{1-\varepsilon_p\beta^2}, k_2 = \omega/v\sqrt{\varepsilon_r\beta^2-1}, \\ p_0 &= J_0(k_2 b)Y_0(k_2 a) - Y_0(k_2 b)J_0(k_2 a), \\ p_0' &= J_0(k_2 b)Y_0'(k_2 a) - Y_0(k_2 b)J_0'(k_2 a). \end{aligned} \qquad (4)$$

Then we can define an eigen function by using the denominator of the coefficient A_0,

$$EF(\omega) = \varepsilon_p k_2\left(\varepsilon_p k_2 p_0 I_0' + \varepsilon_r k_1 p_0' I_0\right) \qquad (5)$$

Synchronized frequency ω and dispersion relation ω-v_p can be found by solving EF(ω)=0. Finally, in order to obtain the time and space dependent wakefields in the plasma channel, one needs to take the inverse Fourier transforms of the complete solution,

$$\begin{aligned} E_z(r,z,t) &= \int_{-\infty}^{\infty} d\omega e^{-j(z-vt)\omega/v}\tilde{E}_z(r,\omega) \\ &= 2\pi j e^{j(z-vt)\omega/v}\sum \mathrm{Res}\left[\tilde{E}_z(r,\omega)\right]\Big|_{\omega=\omega_i} \end{aligned} \qquad (6)$$

Note that ε_p in plasma region is not a constant but a function of synchronized frequency ω, thus the solutions of eigen RF wakefields in the plasma-filled dielectric structure are much different with those for conventional dielectric wakefields structure with no plasma case.

NUMERICAL CALCULATIONS

Based on the equations above, we calculate the monopole modes excited by a Gaussian drive beam inside the plasma channel with a = 100 μm, b = 200 μm, ε_r = 2.5, plasma density n_e = 10^{16} cm^{-3} (then plasma frequency $f_p \approx 8980\times n_e^{0.5}$ = 898 GHz), beam length and size $\sigma_r = \sigma_z$ = 20 μm. Since the wakefields excited by the beam have to be synchronized to the relativistic beam, the phase velocity has to be matched to the beam velocity ($v_p \approx c$).

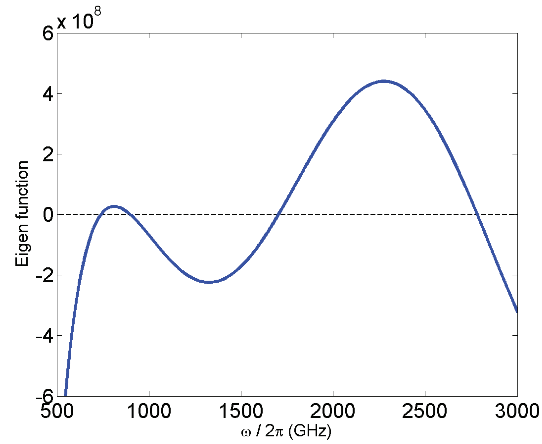

Figure 2: Eigen function as a function of frequency.

Shown in Fig. 2, the prime 4 poles are located at 740 *GHz*, 898 *GHz*, 1.703 *THz* and 2.782 *THz*. The first, third and fourth poles are corresponding to TM_{01}, TM_{02} and TM_{03} RF modes. However, as the wave with f = 898 *GHz* is quite special, the dielectric constant $\varepsilon_p = 1-\omega_p^2/\omega^2$ turns to be zero in the plasma. Then the longitudinal wakefields in time and space domain at this frequency of the plasma wave can be analytically obtained,

$$E_z(r,z,t) = 2\pi j e^{j\frac{\omega}{v}(z-vt)}\mathrm{Res}\left[\tilde{E}_z(r,\omega)\right]\Big|_{\omega=\omega_p} = \frac{q}{2\pi\varepsilon_0}\frac{\omega_p^2}{c^2}\cos\left[\frac{\omega_p}{c}(z-vt)\right]\left[\frac{K_0\left(\frac{\omega_p a}{c}\right)}{I_0\left(\frac{\omega_p a}{c}\right)}I_0\left(\frac{\omega_p r}{c}\right) - \begin{cases} K_0\left(\frac{\omega_p r_0}{c}\right)I_0\left(\frac{\omega_p r}{c}\right), 0\le r\le r_0 \\ I_0\left(\frac{\omega_p r_0}{c}\right)K_0\left(\frac{\omega_p r}{c}\right), r_0\le r\le a \end{cases}\right] \qquad (7)$$

which is just the wakefields of Langmuir wave excited by the plasma oscillation. If we consider a structure with an infinite radius a and a drive particle with an offset r_0=0, Eq. (7) can be simplified as,

$$E_z(r,z,t) = -\frac{q}{2\pi\varepsilon_0 c^2}\omega_p^2 K_0\left(\frac{\omega_p}{c}r\right)\cos\left[\frac{\omega_p}{c}(z-vt)\right] \qquad (8)$$

which is exactly the same as the results by considering the plasma as a fluid without boundaries [8].

There are two differences between plasma wave and TM waves. One is the group velocity of plasma wave $v_g = d\omega/dk$ = 0, the other is that the the I_0 term of plasma wave in Eq. (7) perform an accelerating field for the source particle itself. Even so, the total wakefields of all the waves perform decelerating fields.

By taking some calculation, it appears that the longitudinal components of the eigen RF waves reach several percent of the longitudinal fields of the plasma wave (shown in Table. 1).

Table. 1. Parameters of the Prime 4 Waves Excited by a Relativistic Gaussian Beam with $\sigma_r=\sigma_z=20\mu m$

Modes	frequency (GHz)	v_g/c	Amplitude of E_z (MV/m/nC)
TM01	740	0.324	189.4
Plasma wave	898	0	6459
TM02	1703	0.434	188.8
TM03	2782	0.495	86.7

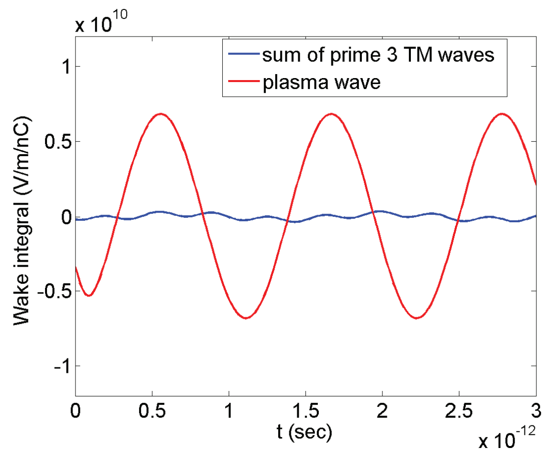

Figure 3: Time profile of the longitudinal wakefields of the prime 3 TM waves (blue) and the plasma wave (red).

While keeping other parameters, the amplitudes of the TM wave and the plasma wave as a functions of the plasma density are shown in Fig. 4.

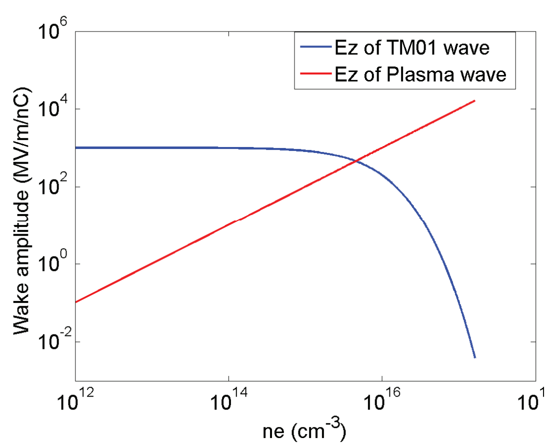

Figure 4: Wake amplitude of the TM_{01} wave (blue) and the plasma wave (red) as functions of plasma density.

For the high plasma density cases ($n_e > 10^{16}$ cm^{-3}), it shows that the strength of TM_{01} mode turns down rapidly while increasing the plasma density, and the plasma wave strength is proportional to the plasma density n_e. Note that the strongest TM_{0n} mode may be not the TM_{01} mode but rather a higher mode of which the synchronized frequency is more close to ω_p in case of $\omega_{TM01} \ll \omega_p$. Even so, all the TM waves reach nearly zero in high n_e case.

For the low plasma density cases ($n_e < 8\times10^{15}$ cm^{-3}), the plasma wavelength is smaller than the effective structure radius ($r_{eff} = a+\varepsilon_r\times(b-a) = 350\ \mu m$, the corresponding plasma density to this wavelength is just 8×10^{15} cm^{-3}). In this case the boundaries are not shielded completely by plasma. Then eigen TM waves can be excited and cannot be ignored since they're much stronger than the plasma wave.

SUMMARY

We derived analytical formulas for calculating the wakefields in plasma filled dielectric capillaries excited by a relativistic electron beam and checked some numerical examples for some typical cases. If the plasma density is down to a low value so that plasma wavelength is larger than structure radius, both eigen RF waves and plasma waves can be excited. Further studies, such as beam break up caused by dipole modes due to beam offset to the axis at low plasma density, are needed for controlling the beam instability.

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