WAKEFIELDS OF ULTRARELATIVISTIC BUNCHES IN COLD MAGNETIZED PLASMA*

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Abstract

We deal with electromagnetic field of various bunches moving in a cold magnetized plasma along the external magnetic field. The main attention is paid to the case of ultrarelativistic motion. First, for the case of point charge, we obtain the approximate formulas which are valid in the far-field zone and in the vicinity of the charge trajectory. These expressions predict the beating behavior of the far field and the harmonic behavior of the near field. Moreover, the magnitude of the longitudinal components of both electric and magnetic field as well as the transversal electric field possess singularity on the charge trajectory. Second, using formulas for the point charge field as Green function, we develop an effective algorithm for calculation of the bunch wakefield. Plots of wakefields produced by typical bunches are given. Prospects of using the bunch field properties for further development of the plasma wakefield acceleration technique are discussed.

GENERAL RESULTS

Radiation of charged bunches moving in plasma was studied over a period of several decades [1]. In the last years, the essential interest to these problems is connected with potentials of the plasma wakefield acceleration (PWFA) method [2], where an accelerating field of 40 GeV/m has recently been achieved [3]. In this paper, we present an effective technique for calculation of wakefields of various bunches, which can be utilized for development of the PWFA scheme.

We consider a cold electron plasma under the external magnetic field H_{ext} described by permittivity tensor [4]

$$\hat{\varepsilon} = \begin{pmatrix} \varepsilon_1 & -i\varepsilon_2 & 0 \\ i\varepsilon_2 & \varepsilon_1 & 0 \\ 0 & 0 & \varepsilon_3 \end{pmatrix}, \qquad (1)$$

$$\varepsilon_1 = 1 - \frac{\omega_p^2}{\omega^2 - \omega_h^2}, \quad \varepsilon_2 = \frac{-\omega_p^2 \omega_h}{\omega(\omega^2 - \omega_h^2)}, \quad \varepsilon_3(\omega) = 1 - \frac{\omega_p^2}{\omega^2}, \qquad (2)$$

where $\omega_p^2 = 4\pi N e^2 / m$ is a plasma frequency (N is an electron density, e and m are an electron charge and a mass respectively), $\omega_h = |e| H_{ext} / (mc)$ is a "gyrofrequency" and c is the light speed in vacuum. In the previous papers [5-7], we have presented some results concerning the field of a point charge (or small bunch) with a charge q moving with constant velocity $\upsilon = \beta c$ along H_{ext}. In particular, it was shown that the extraordinary wave is only generated by the moving charge in the case under consideration. This fact is illustrated in Fig. 1, where the isofrequency curves for wave vectors of ordinary and extraordinary waves are shown for typical frequency of radiation. Since the curve k_o lies entirely inside the circle $k = \omega / c$, the phase velocity of the ordinary wave is larger than c, $\upsilon_{pho} = \omega / k_o > c$, therefore this fast wave cannot be excited by a charge moving slower than light. In contrast, the curve k_e lies entirely outside the circle $k = \omega / c$, the phase velocity of the extraordinary wave is less than c and this slow wave is only excited by the moving charge.

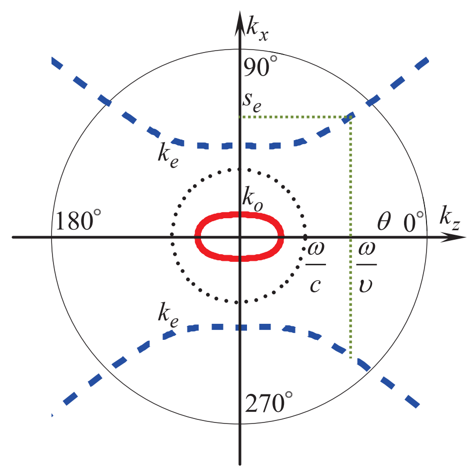

Figure 1: Typical isofrequency curves for wave vectors of the ordinary k_o and extraordinary k_e waves.

Moreover, in [5-7] we have presented two approaches for description of the field of point charge. The first one predicts the beating behavior of the field in the far-field zone. The second one is valid in certain vicinity of the charge trajectory behind the charge [7] and is more essential in the context of PWFA method. This approach gives the following formulas at $\rho \to 0$:

$$\{E_\rho, H_{z,\varphi}\} \approx \{E_{\rho 0}, H_{z0,\varphi 0}\} \sin(\omega_\Sigma \zeta / \upsilon), \qquad (3)$$

$$\{H_\rho, E_{z,\varphi}\} \approx \{H_{\rho 0}, E_{z0,\varphi 0}\} \cos(\omega_\Sigma \zeta / \upsilon), \qquad (4)$$

$$E_{\rho 0} = \frac{2q\omega_p^2}{\upsilon \omega_\Sigma \rho}, \quad H_{z0} = \frac{2q\omega_p^2 \omega_h}{\upsilon c \omega_\Sigma} \ln\left(\frac{\rho \omega_p}{c}\right), \qquad (5)$$

$$H_{\varphi 0} = \frac{q\omega_p^2 \omega_h^2}{c \upsilon^2 \omega_\Sigma} \rho \ln\left(\frac{\rho \omega_p}{c}\right), \quad H_{\rho 0} \approx -\frac{1}{\beta} E_{\varphi 0}, \qquad (6)$$

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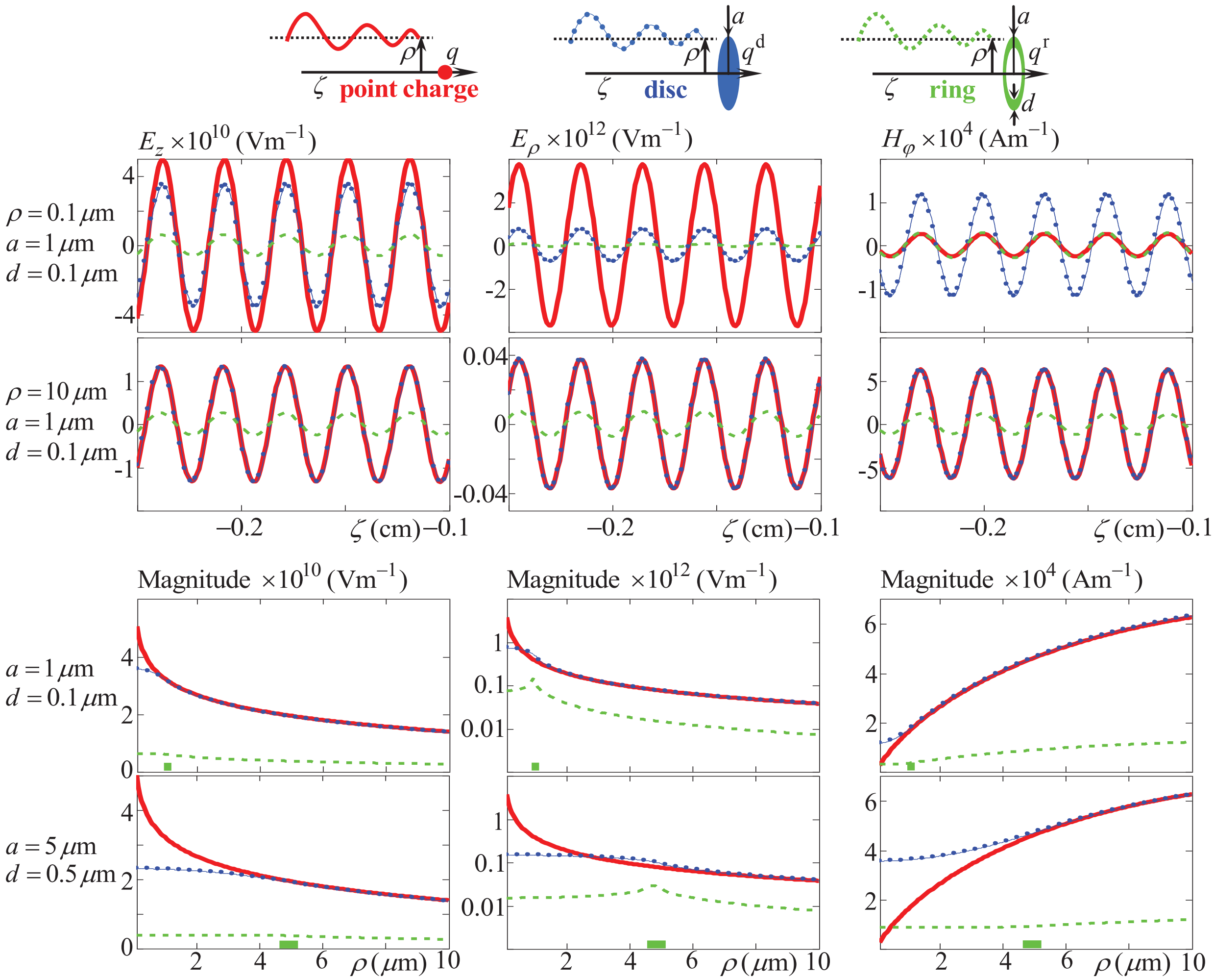

Figure 2: Wakefields of point charge, charged disc and charged ring versus ζ (top) and versus ρ (bottom). Calculation parameters are: $\omega_p = 2\pi \times 10^{12}\,\mathrm{s}^{-1}$, $\omega_h = 0.14\omega_p$ ($B_{ext} = 5\,\mathrm{T}$), $\beta = 0.999$ ($\gamma = 22$), $\lambda = 2\pi\beta c/\omega_\Sigma = 300\,\mu\mathrm{m}$; $q = q^{\mathrm{d}} = -1\mathrm{nC}$, $q^{\mathrm{r}} = q(2ad - d^2)/a^2$.

$$E_{z0} = \frac{2q\omega_p^2}{\upsilon^2}\ln\left(\frac{\rho\omega_p}{c}\right), \quad E_{\varphi 0} = \frac{q\omega_p^2\omega_h}{\upsilon c^2}\rho\ln\left(\frac{\rho\omega_p}{c}\right), \quad (7)$$

$\zeta = z - \upsilon t$. As one can see, E_ρ possesses strong inverse proportional singularity, while E_z and H_z possess weaker logarithmical ones. The rest of components vanishes $\sim \rho\ln\rho$ as $\rho \to 0$. Moreover, all components behave harmonically with longitudinal wavelength $\lambda = 2\pi\beta c/\omega_\Sigma$, where $\omega_\Sigma^2 = \omega_p^2 + \omega_h^2$. Possibilities of decreasing the orthogonal electric component and enlarging the longitudinal magnetic one by increasing the external magnetic field have been also shown in [5-7].

FIELDS OF BUNCHES

Wakefields of various bunches can be easily calculated via convolution of the charge distribution of the bunch with functions (3)–(7) treated as the Green functions. In this way, we have calculated wakefields of thin charged disc, charged ring and different trains of charged rings. Results are presented in Figs. 2 and 3.

Figure 2 compares the fields of a point charge, a charged disc and a charged ring between each other. It is supposed that $q^{\mathrm{d}} = q$ while $q^{\mathrm{r}} = q\,s_{\mathrm{r}}/s_{\mathrm{d}}$, where s_{d} and s_{r} are squares of a disc and a ring, respectively. As one can see from ζ dependencies (Fig. 2, top), for relatively large offset ρ the field of a disc practically coincides with the field of a point charge while the field of a ring is always smaller due to the smaller charge. For relatively small offset ρ the electric field of a point charge exceeds that of a disc while the field of a disc exceeds the field of a ring. However, the azimuthal magnetic component of a disc exceeds field of both point charge and ring. Similar conclusions can be made from the ρ dependencies of magnitudes of oscillations (Fig. 2, bottom). However, it

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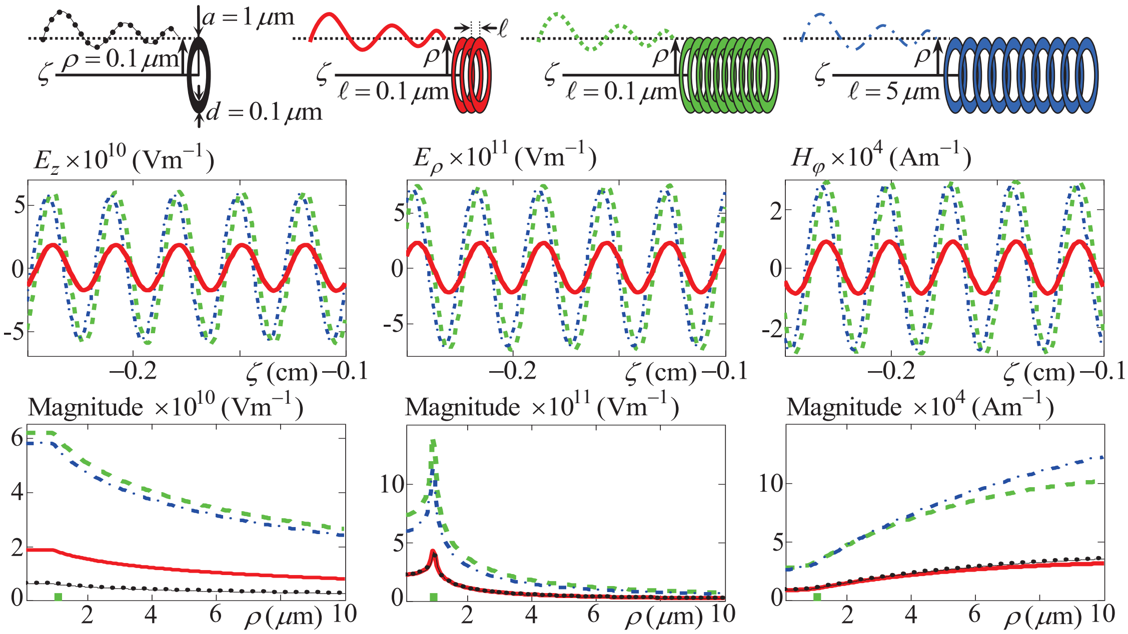

Figure 3: Wakefields of charged rings trains versus ζ and ρ. Calculation parameters are the same as in Fig. 2.

should be noted that for the transversal electric component a certain region of ρ exists ($\rho \approx a$) where the field of a disc exceeds the field of a point charge. Moreover, approximately in this region the transversal electric field of a ring possesses a typical peak

Figure 3 shows typical wakefields produced by different trains of the ring bunches. As one can see, for relatively small distance ℓ between rings, magnitude of oscillations increases with increase in number of rings while frequency of oscillations does not change. It is notable that for E_ρ and H_φ components the difference in magnitude is essential for the 10 rings train, while for the 3 rings train this difference is practically absent. From $\ell = 5a$ and on, the magnitude of oscillations decreases and the phase of oscillations undergoes a change. This is connected with noncoherency effects which arise in this case due to the fact that the total train length become comparable with the longitudinal wavelength λ. In the longitudinal electric component, the typical plateau at $\rho < a$ is expressed more brightly for long trains compared with a single ring. Similarly, the peak in the transversal electric component of wakefield of long train for $\rho \approx a$ is manifested more clearly in comparison with the case of a single ring.

CONCLUSION

In this paper, we have presented an effective approximate approach for calculation of wakefields of various bunches moving in cold magnetized plasma along the external magnetic field. We have illustrated this approach by way of example of several types of bunches. For example, for relatively short train of thin ring bunches, the longitudinal electric field possesses a plateau near the charge trajectory, while the transversal electric field exhibits an expressed peak opposite the ring. Moreover, we have shown that accelerating field increases nonmonotonically with the train length and have found the optimal length providing the maximum field. These features can be used for further development of the PWFA technique.

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