QUASISTATIC FIELD INFLUENCE ON BUNCHES FOCUSING BY WAKEFIELDS IN THE PLASMA-DIELECTRIC WAVEGUIDE*

R.R. Kniaziev#, V.N. Karazin Kharkov National University, Kharkov, Ukraine
G.V. Sotnikov##, NSC Kharkov Institute of Physics and Technology, Kharkov, Ukraine

Abstract

Acceleration of charged particles by wakefields, excited by a drive electron bunch in the dielectric waveguide, is a perspective method in accelerator physics. We have previously proposed to use plasma, filling the drift channel of the dielectric structure (DS), for focusing the accelerated bunch. The analytical expressions, obtained for the components of the electromagnetic field, consider only the propagating wake fields, and do not consider quasi-static fields of electron bunches that are important for calculating bunches dynamics. In this paper we report the results of numerical calculations of the bunches quasi-static field influence on focusing by wakefields in the plasma-dielectric accelerator. We refine analytical expressions for the electromagnetic field [1] by adding components of bunch quasi-static fields and show the correlation of total force and their quasi-static components.

INTRODUCTION

Recently we reported about the electron bunches acceleration by wakefields in the dielectric structure with their simultaneous radial focusing [1,2]. This led to the question of the limits of accelerated bunch focusing. One of limiting factor of a focusing is quasi-static self-field of bunches. Therefore we decided to study the influence of the Coulomb field on the dynamics of bunches of particles. In this paper, we examine the effect of the Coulomb field on the dynamics of bunches of particles in the plasma-dielectric wakefield accelerator. We present an analytical results as well as numerical modelling.

$$E_{z,r} = \begin{cases} I_{z,r}, r < r_b \\ II_{z,r}, r_b \le r < a \\ III_{z,r}, a \le r < b \end{cases} ; \quad H_\varphi = \begin{cases} I_\varphi, r < r_b \\ II_\varphi, r_b < r < a \\ III_\varphi, a < r < b \end{cases} ;$$

$$I_z = -\frac{4QA_{pz}}{r_b^2 L_b I_0(k_p a)}\left[\begin{array}{l} rI_0(k_p a)\Delta_1(k_p r, k_p a) - \\ -r_b I_0(k_p r)\Delta_1(k_p r_b, k_p a)\end{array}\right] - \frac{8Qv_0A_{sz}}{r_b\kappa_p(w_s) a L_b w_s^2 D'(w_s)} \frac{I_1(\kappa_p(w_s)r_b) I_0(\kappa_p(w_s)r)}{I_0^2(\kappa_p(w_s)a)} - \frac{4Qv_0A_{kz}}{r_b w_{si}^2 L_b a D_i'(w_{si})}\frac{J_1(\kappa_p(w_{si})r_b)J_0(\kappa_p(w_{si})r)}{\kappa_p(w_{si})J_0^2(\kappa_p(w_{si})a)}$$

THEORETICAL GROUNDS

The dielectric structure (the plasma waveguide with a dielectric insert element) is a homogeneous plasma cylinder with the radius a, surrounded by the ideally conducting shell with the radius b. The dielectric element fills all the space between the shell an the plasma. We shall analyze the waveguide excitation in the approaching linear isotropic plasma with the density of n_p. In the central part along the system axis there moves an axial-symmetrical relativistic electron bunch with the current density in the injection plane of $(z = 0)$

$$\mathbf{j}_0 = j_m T(t_0) R(r_0)\mathbf{e}_z$$

where j_m – maximum current density
$T(t_0)$ – function describing the longitudinal profile of the bunch; $R(r_0)$ – function describing current distribution in the cross-section; t_0 – time of the entry of the bunch into the plasma; r_0 – radial coordinate; $\mathbf{e}_z$ – unit vector in the axial direction.

For the cylinder solid bunch having the radius r_b and the length L_b, and with homogeneously distributed density of particles:

$$n(r_0, t_0) = \frac{v_0}{L_b r_b^2}\left[\Theta(t_0) - \Theta(t_0 - L_b / v_0)\right]\Theta(r_b - r)$$

the final expressions for the wake field components are as follows [2]. If we want to observe the effect of the Coulomb field, the expressions for the field components take the form:

$$II_z = -\frac{4QA_{pz}}{r_b L_b I_0(k_p a)} I_1(k_p r_b)\Delta_0(k_p a, k_p r) - \frac{8Qv_0A_{sz}}{r_b\kappa_p(w_s) a L_b w_s^2 D'(w_s)}\frac{I_1(\kappa_p(w_s)r_b)I_0(\kappa_p(w_s)r)}{I_0^2(\kappa_p(w_s)a)} - \frac{4Qv_0A_{kz}}{r_b w_{si}^2 L_b a D_i'(w_{si})}\frac{J_1(\kappa_p(w_{si})r_b)J_0(\kappa_p(w_{si})r)}{\kappa_p(w_{si})J_0^2(\kappa_p(w_{si})a)};$$

$$III_z = -\frac{8Qv_0A_{sz}}{aL_b r_b\kappa_p w_s^2}\frac{1}{D'(w_s)}\frac{I_1(\kappa_p r_b)}{I_0(\kappa_p a)}\frac{F_0(\kappa_d r, \kappa_d b)}{F_0(\kappa_d a, \kappa_d b)} + \frac{4Qv_0A_{kz}}{w_{si}^2 r_b L_b a\kappa_p(w_{si})}\frac{1}{D_i'(w_{si})}\frac{\Delta_0(\kappa_d r, \kappa_d b)}{\Delta_0(\kappa_d a, \kappa_d b)}\frac{J_1(\kappa_p(w_{si})r_b)}{J_0(\kappa_p(w_{si})a)}$$

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# RKniaziev@gmail.com
sotnikov@kipt.kharkov.ua

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$$I_r = -\frac{4QA_{pr}}{r_b L_b}\frac{I_1(k_p r)}{I_0(k_p a)}\Delta_1(k_p r_b, k_p a) + $$

$$+\frac{8QA_{sr}}{a w_s r_b \kappa_p^2(w_s) L_b}\frac{1}{D'(w_s)}\frac{I_1(\kappa_p(w_s) r_b) I_1(\kappa_p(w_s) r)}{I_0^2(\kappa_p(w_s) a)} - ;$$

$$-\frac{4QA_{kr}}{a L_b w_{si} r_b \kappa_p^2(w_{si})}\frac{1}{D_i'(w_{si})}\frac{J_1(\kappa_p(w_{si}) r_b) J_1(\kappa_p(w_{si}) r)}{J_0^2(\kappa_p(w_{si}) a)}$$

$$III_r = -\frac{4Qv_0}{a r_b L_b\sqrt{\beta_0^2\varepsilon_d - 1}} \times$$

$$\times\left[\frac{2A_{sr} I_1(\kappa_p r_b)}{w_s^2 \kappa_p(w_s) D'(w_s) I_0(\kappa_p a)}\frac{F_1(\kappa_d r, \kappa_d b)}{F_0(\kappa_d a, \kappa_d b)} + \right.$$

$$\left. + \frac{A_{kr}}{w_{si}^2 \kappa_p(w_{si}) D_i'(w_{si})}\frac{\Delta_1(\kappa_d r, \kappa_d b)}{\Delta_0(\kappa_d a, \kappa_d b)}\frac{I_1(\kappa_p(w_{si}) r_b)}{I_0(\kappa_p(w_{si}) a)}\right]$$

$$II_r = -\frac{4QA_{pr}}{r_b L_b}\frac{I_1(k_p r_b)}{I_0(k_p a)}\Delta_1(k_p r, k_p a) +$$

$$+\frac{8QA_{sr}}{a w_s r_b \kappa_p^2(w_s) L_b}\frac{1}{D'(w_s)}\frac{I_1(\kappa_p(w_s) r_b) I_1(\kappa_p(w_s) r)}{I_0^2(\kappa_p(w_s) a)} - ;$$

$$-\frac{4QA_{kr}}{a L_b w_{si} r_b \kappa_p^2(w_{si})}\frac{1}{D_i'(w_{si})}\frac{J_1(\kappa_p(w_{si}) r_b) J_1(\kappa_p(w_{si}) r)}{J_0^2(\kappa_p(w_{si}) a)}$$

$$H_{\varphi w} = \beta_0 \varepsilon(w) E_{rw}$$

$$F_0(x, y) = J_0(x) N_0(y) - N_0(x) J_0(y),$$

$$F_1(x, y) = -J_1(x) N_0(y) + N_1(x) J_0(y),$$

$J_0(x), J_1(y)$ and $N_0(x), N_1(y)$ are Bessel and Neiman functions of zero and first order, respectively. $\beta_0 = v_0 / c$; $\varepsilon(\omega) = \varepsilon_p(\omega) = 1 - \omega_p^2 / \omega^2$, if $r < a$ and $\varepsilon(\omega) = \varepsilon_d$ if $a \le r < b$; $\omega_p = \sqrt{4\pi e^2 n_p / m}$ is plasma frequency, $-e$ and m are electron charge and mass; ε_d is relative permeability of the dielectric insert, which we suppose to be independent of frequency.

$$\Delta_0(x, y) = I_0(x) K_0(y) - K_0(x) I_0(y),$$

$$\Delta_1(x, y) = I_1(x) K_0(y) + K_1(x) I_0(y),$$

$I_0(x), I_1(y)$ and $K_0(x), K_1(y)$ are Bessel and Macdonald modified functions of zero and first order, respectively.

$$A_{p,s} = Q\left(t - \frac{z}{v_0}\right)\sin w_{p,s}\left(t - \frac{z}{v_0}\right) -$$

$$-Q\left(t - \frac{z}{v_0} - \frac{L_b}{v_0}\right)\sin w_{p,s}\left(t - \frac{z}{v_0} - \frac{L_b}{v_0}\right)$$

$$A_k = \Theta(-\xi)\left[1 - e^{w_{si}\xi}\right] - \Theta\left(\frac{L_b}{v_0} - \xi\right)\left[1 - e^{w_{si}\left(\xi - \frac{L_b}{v_0}\right)}\right] +$$

$$+\Theta(\xi)\left[1 - e^{-w_{si}\xi}\right] - \Theta\left(\xi - \frac{L_b}{v_0}\right)\left[1 - e^{w_{si}\left(\frac{L_b}{v_0} - \xi\right)}\right]$$

$$A_{pr,sr} = \left[1 - \cos w_{p,s}(\xi)\right]\Theta(\xi) -$$

$$-\left[1 - \cos w_{p,s}\left(\xi - \frac{L_b}{v_0}\right)\right]\Theta\left(\xi - \frac{L_b}{v_0}\right)$$

$$A_{kr} = \Theta(-\xi)\left[1 - e^{w_{si}\xi}\right] - \Theta\left(\frac{L_b}{v_0} - \xi\right)\left[1 - e^{w_{si}\left(\xi - \frac{L_b}{v_0}\right)}\right] -$$

$$-\Theta(\xi)\left[1 - e^{-w_{si}\xi}\right] + \Theta\left(\xi - \frac{L_b}{v_0}\right)\left[1 - e^{w_{si}\left(\frac{L_b}{v_0} - \xi\right)}\right]$$

$\Theta(x)$ is Heaviside step function; $k_p = \omega_p / v_0$, $\kappa_p(w) = \frac{w}{v_0}\sqrt{1 - \beta_0^2 \varepsilon_p(w)}$, $\kappa_d(w) = \frac{w}{v_0}\sqrt{\beta_0^2 \varepsilon_d - 1}$,

$\kappa_p^s = \kappa_p(\omega = \omega_s)$, $\kappa_d^s = \kappa_d(\omega = \omega_s)$, $D'(\omega_s) = \left.\frac{dD(\omega)}{d\omega}\right|_{\omega = \omega_s}$

$$D(\omega) = \frac{\varepsilon_p(\omega)}{\sqrt{1 - \beta_0^2 \varepsilon_p(\omega)}}\frac{I_1(\kappa_p a)}{I_0(\kappa_p a)} + \gamma_d \frac{F_1(\kappa_d a, \kappa_d b)}{F_0(\kappa_d a, \kappa_d b)}$$

$$D_i(w) = \frac{\varepsilon_{pi}(w)}{\kappa_{pi}(w)}\frac{J_1(\kappa_{pi}(w) a)}{J_0(\kappa_{pi}(w) a)} - \frac{\varepsilon_d}{\kappa_d(w)}\frac{\Delta_1(\kappa_d(w) a, \kappa_d(w) b)}{\Delta_0(\kappa_d(w) a, \kappa_d(w) b)};$$

$$\varepsilon_{pi}(w) = 1 + \frac{w_p^2}{w^2}; \quad \kappa_{pi}(w) = \frac{w}{v_0}\sqrt{1 - \beta_0^2 \varepsilon_{pi}(w)}$$

$D(\omega), D_i(\omega)$ are the dispersion function, and the eigen frequencies ω_s, ω_{si} are determined when solving the dispersion equation $D(\omega_s) = 0$, $D_i(\omega_s) = 0$.

NUMERICAL CALCULATIONS

Here are the results of the wake field calculations. For our calculations we choose the dielectric waveguide with the dimensions presented in Table 1, with the dielectric tube made of fused silica. In the same table the electron bunch and plasma parameters are given.

The results of calculations for plasma with such parameters are shown in Figures 1-4.

As can be seen from Fig.1, Fig.3 and Fig.4, axial profiles of total forces higher by several orders than the axial profile of the Coulomb part of the forces. From Fig. 2, we can see that the longitudinal force varies weakly in the cross section of the transport channel, and the radial force is focusing on the entire cross section of the channel.

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Table 1: Dielectric Structure (Fused Silica)

Parameter	Value
Outer radius of dielectric tube	5.11 mm
Inner radius of dielectric tube	4 mm
Relative dielectric constant, ε	3.75
Bunch energy	14 MeV
Bunch charge	1nC
Bunch length L_b (box charge distribution)	2 mm
Bunch radius r_b (box charge distribution)	2 mm
Density of drive bunch electrons, n_b	$2.485 \cdot 10^{11}$ cm^{-3}
Plasma density	$3.871 \cdot 10^{10}$ cm^{-3}

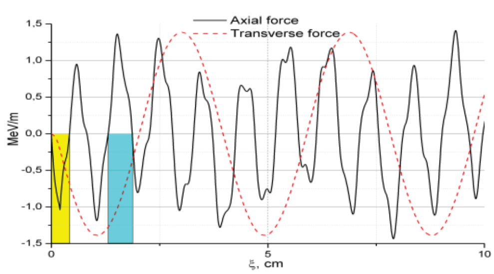

Figure 1: The axial profile of the axial force (black line) and the axial profile of the transverse force (red line) at the distance r = 2 mm from the waveguide axis. Plasma density $n_p = 3n_b$. The drive bunch (yellow rectangular) moves from right to left. The cyan rectangular shows a possible location of the test (accelerating) bunch.

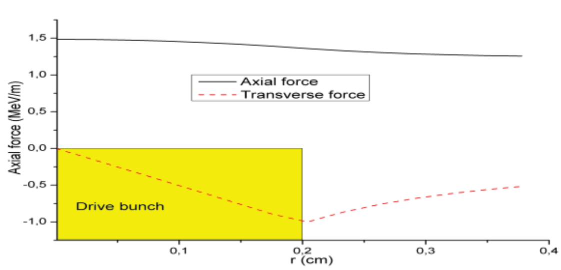

Figure 2: The transverse profile of the longitudinal (black line) and transverse forces (red line), acting upon a test particle, located at the distance of 1.523 cm from the drive bunch head. Plasma density $n_p = 3n_b$.

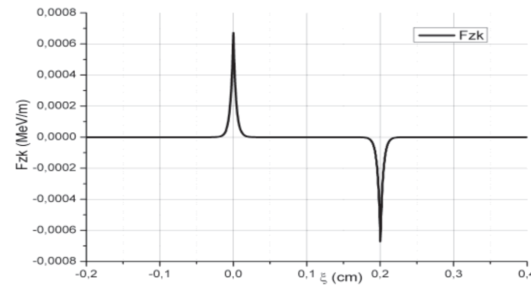

Figure 3: The axial profile of the coulomb part of axial force.

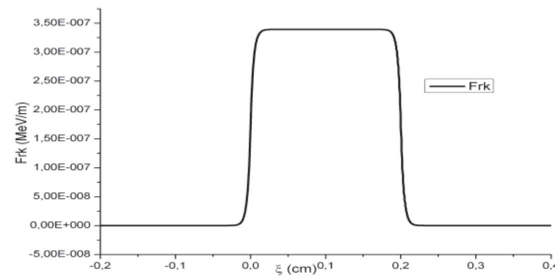

Figure 4: The axial profile of the coulomb part of transverse force.

The figures show that the Coulomb force component are much smaller than the total force. When increasing energy the influence of the Coulomb field becomes more imperceptible. That means that we can do not consider the influence of the Coulomb field in our calculations.

CONCLUSIONS

Our calculations have shown that the Coulomb field does not have a noticeable effect on the dynamics of bunches in the plasma-dielectric structure with energies ranking at about 15 MeV. Increasing energies makes the influence of the Coulomb field more imperceptible. Thus there is no need to consider quasistatic field Influence on bunches focusing by wakefields in the plasma-dielectric waveguide.

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