

# COHERENT THOMSON SCATTERING USING BEAM ECHO

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## Abstract

Longitudinal phase space of the beam is modulated by laser interacting in undulators. The beam can have higher frequency component using the beam echo than the laser as discussed by G. Stupakov et al. The modulated beam has a potential to emit coherent radiation with the wave length. We study a coherent short wave length ( $\sim$ nm) and/or short pulse (attosec) light source using the beam echo in a low energy accelerator  $\sim$ 100MeV.

## INTRODUCTION

Head-on collision of beam and laser pulse ( $\lambda_1$ ) emits radiation shorter wave length with the relation

$$\lambda_R = \frac{1 + a_1^2}{4\gamma^2} \lambda_1, \quad (1)$$

where  $a_1 = eA/mc$  is normalized vector potential of the laser field. The process is regarded as Thomson scattering when the recoil of the beam is negligible, Thomson scattering is really synchrotron radiation emitted from the perturbed beam due to the laser field.

When the beam has longitudinal density modulation equal to the wave length  $\lambda_R$ , the radiation from Thomson scattering becomes coherent. The longitudinal density modulation is induced by beam echo [1]. Echo with a short laser pulse gives localized density modulation with wave length  $< \lambda_R$  [2]. Extreme short pulse can be emitted by the coherent Thomson scattering.

In this paper, radiation field emitted by Thomson scattering is discussed for KEK-(c)ERL. The beam energy and its spread are 100 MeV and  $\sigma_\delta = 3 \times 10^{-4}$ , respectively. The bunch population and its length are  $N_e = 4.8 \times 10^7$  and  $\sigma_z = 0.6$ mm.

## ELECTRON MOTION AND ELECTRIC FIELD INDUCED BY THOMSON SCATTERING

Motion of electron interacting with laser and undulator fields is represented by Hamiltonian as follows

$$H = (1 + \delta) - \sqrt{(1 + \delta)^2 - \left(\mathbf{p} - \frac{\mathbf{a}}{\gamma}\right)^2 - \frac{1}{\gamma^2}} \quad (1)$$

where  $\mathbf{a}$  is vector potential for laser or undulator. For undulator,  $\mathbf{a}$  is expressed by

$$a_{u,x} = a_u \cos k_u s \quad (2)$$

where  $k_u = 2\pi/\lambda_u$  is wave number of undulator period. Laser field traveling to the beam direction is expressed by

$$a_{L,x} = a_L \cos(k_L s - \omega t + \phi) = a_L \cos(k_L z + \phi), \quad (3)$$

where  $z = s - ct$ . Laser field traveling against the beam direction is expressed by

$$\begin{aligned} a_{L,x} &= a_L \cos(-k_L s - \omega t + \phi) \\ &= a_L \cos[k_L(z - 2s) + \phi]. \end{aligned} \quad (4)$$

For pulse laser,  $a$  is a function of  $z$  as follows,

$$a_L = a_{L,0} \exp\left(-\frac{z^2}{2\sigma_z^2}\right) \quad \text{or} \quad a_L = a_{L,0} \exp\left(-\frac{(z - 2s)^2}{2\sigma_z^2}\right)$$

Equation of motion is expressed by

$$\begin{aligned} x' &= \frac{\partial H}{\partial p_x} = \frac{p_x - a_x/\gamma}{p_s} & p_x' &= \frac{\partial H}{\partial x} = 0 \\ z' &= \frac{\partial H}{\partial \delta} = 1 - \frac{1 + \delta}{p_s} & & \\ \delta' &= -\frac{\partial H}{\partial z} = \frac{1}{\gamma p_s} \left(p_x - \frac{a_x}{\gamma}\right) \frac{\partial a_x}{\partial z} \end{aligned} \quad (5)$$

where

$$p_s = \sqrt{(1 + \delta)^2 - \left(\mathbf{p} - \frac{\mathbf{a}}{\gamma}\right)^2 - \frac{1}{\gamma^2}}$$

Radiation field emitted by Thomson scattering is calculated from electron motion using the Heaviside-Feynman expression. Electric field emitted by  $i$ -th electron observed at  $(\mathbf{x}_o, t)$  is given by

$$\mathbf{E}_i(\mathbf{x}_o, t) = \frac{e}{4\pi\epsilon_0} \left[ \frac{\mathbf{R}_i}{R_i^3} + \frac{R_i}{c} \frac{d}{dt} \left( \frac{\mathbf{R}_i}{R_i^3} \right) + \frac{1}{c^2} \frac{d^2}{dt^2} \left( \frac{\mathbf{R}_i}{R_i} \right) \right] \quad (6)$$

where  $\mathbf{R}_i = \mathbf{x}_o - \mathbf{x}_i$  is a vector between observer ( $\mathbf{x}_o$ ) and electron ( $\mathbf{x}_i$ ), and observer time and electron local time is connected by

$$t = t_i + \frac{R_i}{c} \quad ct = s - z_i + R_i \quad (7)$$

Total electric field at observer position,  $\mathbf{x}_o$ , is given by summation for all electrons,

$$\mathbf{E}(\mathbf{x}_o, t) = \sum_{i=1}^{N_e} \mathbf{E}_i(\mathbf{x}_o, t) \quad (8)$$

## LONGITUDINAL ELECTRON DISTRIBUTION IN BEAM ECHO

Longitudinal density modulation is induced by laser-beam interaction in undulator. Electron motion is solved under the vector potential of  $\mathbf{a}_u + \mathbf{a}_L$  in Eq.(2) and (3). Simplified 1 dimensional model is written as follows,

$$H = -\frac{1}{2\gamma_z^2} \delta + \frac{1}{2\gamma_z^2} \delta^2 + \frac{\mathbf{a}_u \mathbf{a}_L}{\gamma^2} \quad (9)$$

$$\Delta\delta = \frac{a_u a_L}{\gamma^2} k_L L_u \sin k_L z \quad (10)$$

After passage of 1<sup>st</sup> undulator, energy modulation Eq.(10) is induced. Using undulator  $\lambda_{u1}=2$  cm,  $L_{u1}=30$  cm,  $a_u=1.46$ , the beam interacts with laser with  $\lambda_L=800$ nm. The energy spread is  $\sigma_\delta=3 \times 10^{-4}$ , the laser amplitude  $a_{L1}=2 \times 10^{-5}$  is required for  $\Delta\delta=3\sigma_\delta$  in the interaction of the undulator. Stripes of energy distribution ( $N_{\text{stripe}} \sim k_L \Delta\delta R_{56}^{(1)}=25$ ) are created in 1<sup>st</sup> slippage section with  $R_{56}^{(1)}=3$ mm. The energy modulation is applied again for the striped beam in 2<sup>nd</sup> (short) undulator interacting with the laser ( $L_{u2}=4$  cm,  $a_{L2}=1.3 \times 10^{-3}$ ). Then after 2<sup>nd</sup> slippage section ( $R_{56}^{(2)} \sim 0.1$ mm), the longitudinal phase space distribution as shown in Figure 1 is realized [1,2]. The global structure in the phase space depends on the 2<sup>nd</sup> interaction of 800nm laser, while the fine structure induced by 1<sup>st</sup> interaction and slippage is found as shown in lower picture. The fine structure contains high frequency and short pulse component. Coherent Thomson radiation is produced by collision with a long wave length laser pulse ( $\lambda \sim 2$ mm) corresponding the fine structure satisfying the wave length relation Eq.(1).

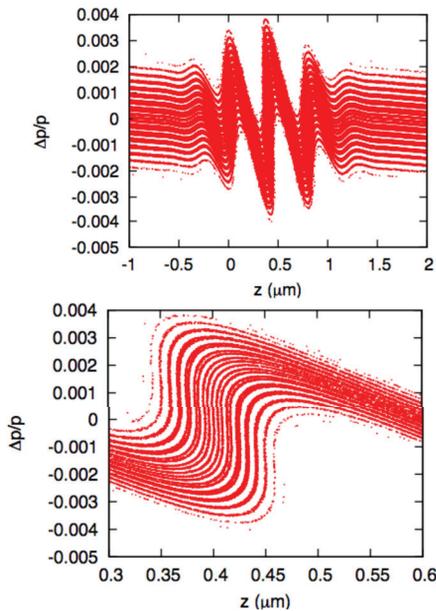


Figure 1: Electron distribution in the longitudinal phase space using beam echo. Bottom picture is expanded in z.

### ELECTRIC FIELD EMITTED BY THOMSON SCATTERING OF ECHO BEAM

The echo beam collides with a laser pulse as shown in Figure 2. The wave length of collision laser is  $\lambda_{L,c}=2.16$  mm. The wave length of radiation is chosen to be the same as the density modulation of the beam,  $\sim 20$  nm in Figure 1.

Electric field emitted by electrons are calculated by Eq. (6) and (8). Figure 3 shows electric field emitted by Thomson scattering of 10 electrons and the laser pulse as

a test. 10 pulses of the emitted electric field are seen in the top plot. Field emitted by an electron is focused in the bottom plot. The wave length of the field is 20 nm. Electric field emitted by the whole beam is given by superposition of the field. The strength of the electric field is  $O(N^{1/2})$  for random particle distribution, while  $O(N)$  for coherent density distribution in z projection. Figure 4 shows particle distribution before and after the final slippage, which correspond to random and coherent distribution, respectively. Figure 5 shows electric field for the distribution given in Figure 4. The field strength for coherent distribution is larger than that for random distribution. Macro-electron statistics of the simulation is chosen the same as real electron statistics. S/N for coherent signal is not high, because the bunch population is low for KEK-cERL,  $N_e=4.8 \times 10^7$ .

For short pulse generation, phase space modulation is local as shown in Figure 1. Figure 6 shows electric field emitted by Thomson scattering. Very short pulse is seen under background. The detailed oscillation of the field is seen in bottom picture. The pulse length is 50-100 nm, 150-300 attosec.

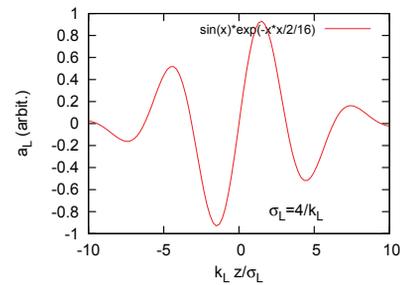


Figure 2: Laser pulse collides with echo beam.

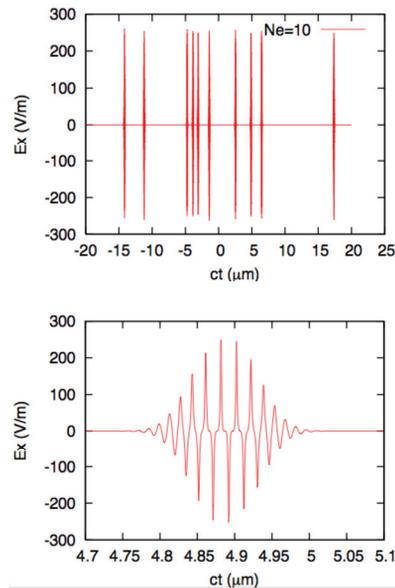


Figure 3: Electric field emitted by Thomson scattering of 10 electrons and laser pulse. Field emitted by an electron is focused in the bottom plot.

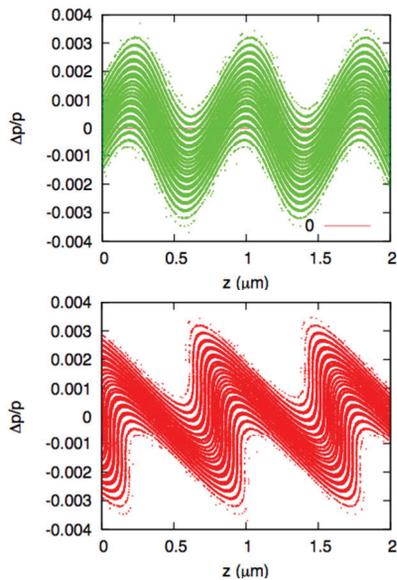


Figure 4: Longitudinal phase space distribution of echo beam. Top and bottom plots depict before and after the final slippage.

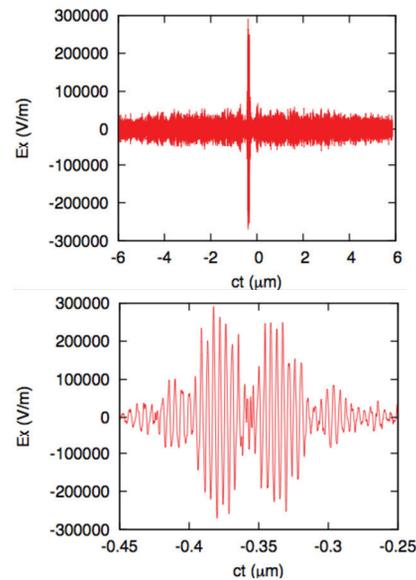


Figure 6: Electric field emitted by Thomson scattering of echo beam in Figure 1 and laser pulse.

## SUMMARY AND CONCLUSIONS

We have studied coherent Thomson scattering of echo-beam and a long wave length laser ( $\lambda \sim 2\text{mm}$ ) in KEK-cERL. For 100MeV electron beam, Using two undulator  $\lambda_{u1} = 2\text{ cm}$ ,  $L_u = 30\text{ cm} + 4\text{ cm}$  and laser  $\lambda_L = 800\text{ nm}$ , required bunch density modulation  $\lambda_p = 10\text{ nm}$  is induced. Collision with a short pulse with the wave length ( $\lambda \sim 2\text{mm}$ ) is expected to emit 200 attosec pulse.

## ACKNOWLEDGMENT

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## REFERENCES

- [1] G. Stupakov, Phys. Rev. Lett. 102, 074801 (2009).
- [2] D. Xiang et al., Phys. Rev. ST-AB 12, 060701 (2009).

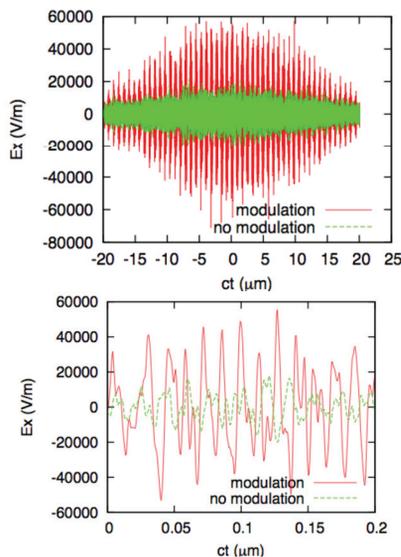


Figure 5: Electric field emitted by Thomson scattering of echo beam in Figure 3 and laser pulse. Green and red lines draw field given by beam before and after the final slippage.