CALCULATION OF THE EQUILIBRIUM PARAMETERS FOR THE COMPACT RING OF TTX

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Abstract

We study the effect of the Intra-Beam Scattering (IBS) on the compact low energy electron storage ring, proposed for Tsinghua Thomson Scattering X-ray source (TTX). For a single bunch with peak current at about 3.6 A and re-entrant type normal conducting RF cavity of 500 MHz with peak voltage at 5kV, the equilibrium horizontal and vertical emittances are 3.0 and 0.3 μ m, and the rms momentum spread and bunch length are about 0.14%, and 111 ps. The responding x-ray flux is about 5×10^{10}photons/s.

INTRODUCTION

We design a compact electron storage ring for Tsinghua Thomson scattering X-ray source. The proposed scheme stores a single bunch at 50 MeV in the ring. The electron beam will scatter an laser pulse stored in an optical cavity with the revolution frequency, which is 62.5 MHz. The ring is designed to be 4.8 m long, with 4 dipoles and 2 quadrupoles. Layout of the ring and the corresponding betatron amplitude functions are shown in Fig-1. We present the details of the design in this proceeding[1].

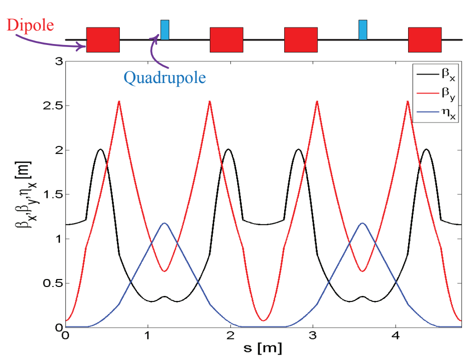

Figure 1: The Layout, Betatron Amplitude Function and Dispersion Function of the ring.

Since the energy of electrons are designed to be 50 MeV, it takes longer time for the electron beam to achieve equilibrium than in GeV level electron storage rings. In the process of achieving equilibrium, many effects, such as synchrotron radiation damping, quantum excitation, intra-beam scattering, residual gas scattering, and so on, can influence the equilibrium parameters. Hence, all of the effects mentioned above should be considered in the calculation. Based on some previous study, we believe that we need to pay more attention to IBS effect which plays a dominant role in the process of developing towards the equilibrium in this kind of low energy ring.

In this paper, we'll present a self-consistent algorithm to calculate the evolution of parameters and the equilibrium parameters calculated by this method. Using this method, we study the dependence of equilibrium parameters on the peak voltage of RF cavity and find the optimized voltage.

EMITTANCE EVOLUTION PROCESS

The equilibrium parameters of an electron beam are mainly determined by the characteristics of the storage ring. The major effects which influence the equilibrium emittance in three directions are synchrotron radiation damping, quantum excitation, and residual gas scattering (assuming no undulator or wiggler), etc. The mechanism of synchrotron radiation damping, quantum excitation, residual gas scattering are already well studied, one can have a good estimation of the equilibrium emittances analytically. But in a low energy storage ring like the ring described here, it is well accepted that intra-beams scattering plays a dominant role in the process of getting equilibrium. Take IBS into account, emittance evolution equations can be written as Eq(1), Eq(2), and Eq(3).

$$\frac{d\varepsilon_x}{dt} = -\frac{C_\gamma J_x}{\rho T_0}E^3\varepsilon_x + \frac{1}{C}\frac{3}{4} < H >_{dip} \frac{C_u C_\gamma \hbar c^2 E^5}{(mc^2)^3\rho_{dip}^2} + 3.0636\times 10^{-7}\times\frac{1}{E_k^2+2E_kE_0}P_g + \frac{2}{\tau_x}\varepsilon_x \quad (1)$$

$$\frac{d\varepsilon_z}{dt} = -\frac{C_\gamma J_z}{\rho T_0}E^3(\varepsilon_z - \kappa\varepsilon_x) + \frac{1}{C}\frac{3}{4}\frac{1}{\gamma^2} < \beta_z >_{dip} \frac{C_u C_\gamma \hbar c^2 E^5}{(mc^2)^3\rho_{dip}^2} + 3.0636\times 10^{-7}\times\frac{1}{E_k^2+2E_kE_0}P_g + \frac{2}{\tau_z}\varepsilon_z \quad (2)$$

$$\frac{d\delta}{dt} = -\frac{C_\gamma J_e}{2\rho T_0}E^3\delta + \frac{1}{C}\frac{3}{8}\frac{C_u C_\gamma \hbar c^2 E^5}{(mc^2)^3\rho_{dip}^2\delta} + 0 + \frac{1}{\tau_\delta}\delta \quad (3)$$

Since the growth times of IBS (τ_x in Eq(1), τ_z in Eq(2), τ_δ in Eq(3)) depend on the beam properties in 6-D phase space at that time, we need to calculate the evolution process in a self-consistent way. So, we rewrite these differential equations in the form of difference equations and divide the process into many time steps. Beam parameters and IBS

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growth time are updated at each step. 10% of coupling between horizontal and vertical directions are also included in the calculation.

The procedure of this calculation is shown as follows:

- 1, Set the initial beam parameters: ε_{x0}, ε_{z0}, δ_0, σ_{s0};
- 2, For a given energy and coupling coefficient, calculate IBS growth time;
- 3, Use the equations Eq(1) - (3), calculate the changing rate of emittances and momentum spread;
- 4, For a given time step, calculate the new beam parameters;
- 5, Go back to step-2 until equilibrium is achieved.

The key step in this procedure is to calculate the growth time of IBS effect which can be done by several methods. Among these methods, Piwinski's approach for weak focusing accelerators and Bjorken-Mtingwa's theory for strong focusing ones are well accepted and built in a lot of programs. By assuming a Gaussian beam in three directions, we can use Bjorken-Mtingwa's approach. In our calculation, the IBS module of MAD-8, which uses Bjorken-Mtingwa's approach, is selected.

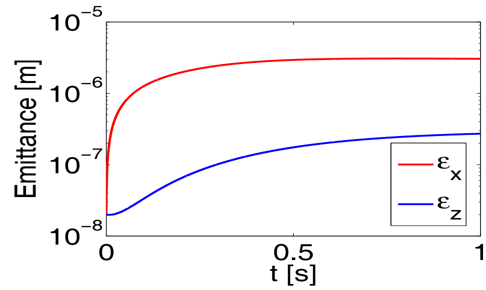
SIMULATION OF EVOLUTION PROCESS AND EQUILIBRIUM

The injected beam, comes from a 3m S-band linac and a photocathode RF gun, has 20 nm of both horizontal and vertical emittance. The peak RF voltage of 500 MHz cavity is selected as 5 kV in the calculation. We'll study the dependence of different parameters on the RF voltage in the next section. Assume the average pressure of vacuum is maintained to 1 nTorr as a constant. Initial bunch length and momentum spread are set matched to the RF bucket, which are 0.12343 m and 0.5%, respectively. The time step is chosen as 1000 turns. The change of transverse emittance, bunch length and momentum spread are shown in Fig-2.

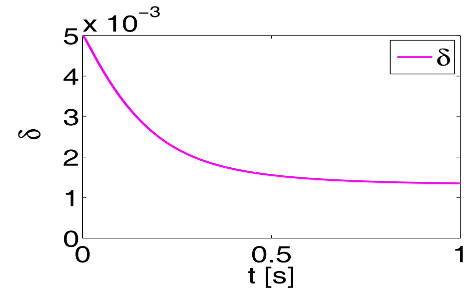
The results show a dramatic rise of the horizontal emittance, which changes from 20 nm to about 2.0 μm, in the first 0.2 s. The beam reaches equilibrium at about 0.3 s. During this period of time, the increase of vertical emittance (from 20 nm to about 65 nm) is less than the growth of horizontal emittance which is well matched with the prediction in Bjorken-Mtingwa's theory [2]. The equilibrium RMS momentum spread is about 0.135% and bunch length is 111.4 ps.

EQUILIBRIUM PARAMETERS' CHANGE VS RF VOLTAGE AND CORRESPONDING X-RAY FLUX

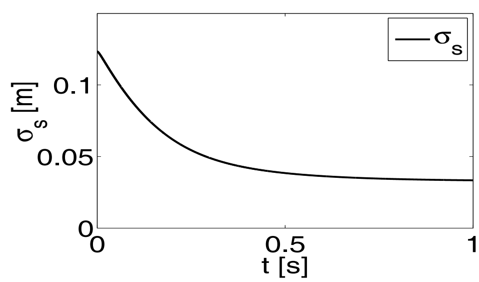
As mentioned above, the equilibrium parameters depend on the peak voltage of RF cavity.



(a) Transverse Emittance Evolution



(b) RMS Momentum Spread Evolution



(c) RMS Bunch Length Evolution

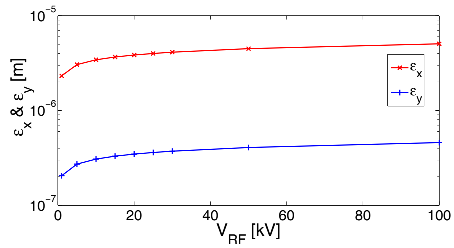
Figure 2: Evolution Process of the beam parameters: (a), Transverse emittance growth process. Horizontal emittance increases dramatically mainly due to IBS. Since 10% coupling is included, vertical emittance also increases. (b), Momentum spread reduces to equilibrium. (c), Bunch length reduces to equilibrium.

The Eq(1)-(3) show that RF voltage can influence the equilibrium parameters. When increasing the RF voltage, the equilibrium momentum spread is expected to increase because of the increasing bucket height. Meanwhile, the bunch length is inclined to reduce for preserving the longitudinal emittance. Therefore, it will cause higher growth rate of IBS due to the shorter bunch length.

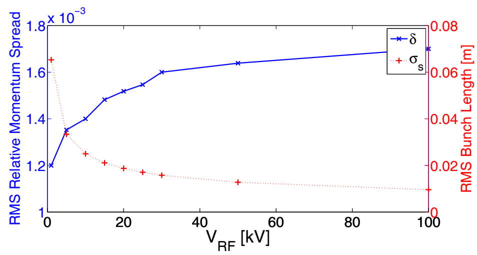
The simulation results are shown in Fig-3 and support the prediction.

In the previous section, we set the peak voltage of RF cavity as 5 kV to study the evolution process and equilibrium parameters. Actually, we need to choose the RF voltage carefully to optimize the performance. Touschek lifetime and generated X-ray photon flux are two major objectives in the optimization of performance.

As shown in the calculation of equilibrium parameters, it takes about 1 s to achieve equilibrium. In order to generate



(a) Transverse Emittance Evolution



(b) RMS Momentum Spread Evolution

Figure 3: Dependence of ε_x, ε_z, δ, and σ_s on RF voltage.

stable flux of X-ray, longer lifetime is always preferred. As shown in many studies, Touschek effect limits the lifetime in this kind of low energy storage ring[3][4]. The Touschek lifetime can be estimated as Eq-(4) [4].

$$\tau_{Touschek} = \frac{8\pi\sigma_z\sigma_s\sigma_{x\beta}\sqrt{1+\frac{\sigma_{xs}^2}{\sigma_{x\beta}^2}}}{Nr_0^2c}\frac{\gamma^2}{\lambda^3}\frac{1}{D_x(\xi)} \quad (4)$$

where $\lambda^{-1} = (\frac{\Delta E}{E})_{RF}$ stands for the energy acceptance. $D_x(\xi) = \sqrt{\xi} - \frac{3}{2}e^{-\xi} + \frac{\xi}{2}\int_\xi^\infty \frac{ln(u)e^{-u}}{u}du + \frac{1}{2}(3\xi - \xi ln(\xi) + 2)\int_\xi^\infty \frac{e^{-u}}{u}du$. In the expression of $D_x(\xi)$, $\xi = (\varepsilon_{RF}/\gamma\sigma_p)^2$.

As shown in Eq-(4), Touschek lifetime will increase when RF voltage increases.

Now, we're going to see how the dependence of photon flux on RF voltage. The number of photons per second generated by inverse Compton scattering process under the linear interaction can be described as Eq(5) [5]:

$$N = \frac{f\Sigma N_e N_l/(2\pi\sqrt{\sigma_{ey}^2+\sigma_{ly}^2})}{\sqrt{cos^2(\alpha/2)(\sigma_{ex}^2+\sigma_{lx}^2)+sin^2(\alpha/2)(\sigma_{ez}^2+\sigma_{lz}^2)}} \quad (5)$$

where N_e and $N_l = \lambda I/\hbar c$ are the total number of electrons in a bunch and total number of laser photons in one pulse, respectively. And f is the revolution frequency. α is the collision angle which is chosen as 177.25° in the calculation. Σ is the total cross section given by Klein-Nishina formula (in the Thomson limit, $F_{KN} = 1$) which is shown in Eq(6).

$$\Sigma = \int (\frac{d\sigma}{d\Omega})_{KN} d\Omega = \int \frac{r_e^2}{2}(1+cos^2\theta)F_{KN}d\Omega \quad (6)$$

The energy in one laser pulse is set as 1 mJ. The spot size of laser at the interaction point are $\sigma_{lx} = \sigma_{ly} = 85\mu$ m.

The dependence of Touschek lifetime and expected photon flux on peak RF voltage are shown in Fig-4.

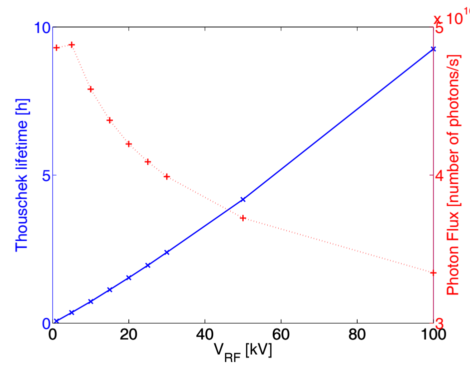


Figure 4: Touschek Lifetime and Photon Flux VS Peak RF Voltage.

As shown in Fig-4, if we choose the RF voltage as 5 kV, Touschek lifetime is about 0.4 h and the photon flux is about 5×10^{10}photons/s which is only a very small drop from the maximum value.

So we think that the optimized value of RF voltage is $V_{RF} = 5$kV. The corresponding Touschek lifetime and photon flux are expected to be about 0.4 h and 5×10^{10}photons/s.

DISCUSSION

In this paper, we present a self-consistent method to calculate the evolution process of transverse emittance, momentum spread and bunch length. We also calculate how the equilibrium parameters depend on RF voltage. At the last, Touschek lifetime and expected photon flux are calculated.

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