ENVELOPE PERTURBATIONS IN A SPACE-CHARGE-DOMINATED ELECTRON BEAM*

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Abstract

Linear perturbation analysis of the RMS envelope equations predicts a frequency splitting of the transverse envelope resonances with the onset of space charge. These resonances are a potential source of beam degradation for space-charge-dominated particle accelerators and storage rings. We use WARP for both envelope code integration and particle-in-cell (PIC) simulations to predict the behavior of these resonances for an existing alternating gradient lattice storage ring. The focus of these simulations is tailored toward examining physics that is scalable to future high-intensity accelerators. This paper provides detailed simulation results for the University of Maryland Electron Ring (UMER), a high intensity 10 keV electron storage ring.

INTRODUCTION

When designing a high intensity accelerator, it is important to account for the envelope resonances that are present due to space charge. To understand these resonances, we must first look at the envelope equations first derived by Kapchinsky and Vladimirsky [1] and generalized to the RMS case by Lapostolle and Sacherer [2,3]. Largely known as the K-V equations, they can be written in terms of the transverse envelopes X(s) and Y(s) as follows:

$$X'' + \kappa_x^2(s)X - \frac{2K}{(X+Y)} - \frac{\varepsilon_x^2}{X^3} = 0 \qquad (1a)$$

$$Y'' + \kappa_y^2(s)Y - \frac{2K}{(X+Y)} - \frac{\varepsilon_y^2}{Y^3} = 0 \qquad (1b)$$

Where $\kappa_{x,y}^2$ is the periodic transverse focusing force, K is the generalized perveance, and $\varepsilon_{x,y}$ is the RMS emittance. Linear perturbation analysis of the K-V equations yields information about the envelope modes. Two of these modes can have exponential growth due to space charge forces, the even (breathing) mode and the odd (quadrupole) mode. In the smooth approximation, these modes can be described by their relative spatial frequencies:

$$\varphi_{odd} = \sqrt{3\sigma^2 + \sigma_0} \cdot S \qquad (2)$$

$$\varphi_{even} = \sqrt{2(\sigma^2 + \sigma_0)} \cdot S \qquad (3)$$

Where σ_0 is the zero current phase advance, σ is the space charge depressed phase advance, and S is the lattice period. The transverse cross section of the envelope modes is shown in Fig. 1.

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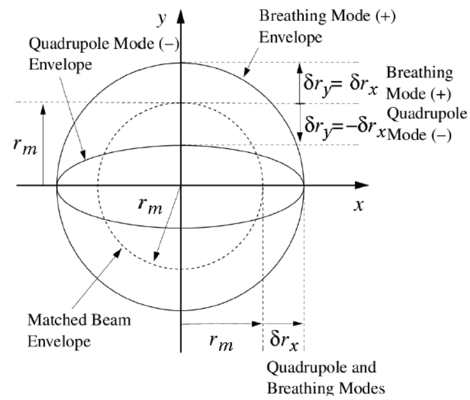

Figure 1: Transverse picture of the envelope modes [4].

However, the envelope modes do not become unstable within the smooth approximation, because this analysis breaks down for σ_0 greater than 90° [5]. Since we are interested in the unstable case of these modes, it is necessary to examine the exact solutions. By performing a generalized linear perturbation, it is possible to extract the envelope mode eigenfunctions. The eigenvalues corresponding to the unstable modes lie either on the real axis or off the unit circle [5]. Simulations for UMER incorporate an alternate gradient (AG) FODO lattice, and so the smooth approximation is not an adequate model. In Fig. 2, we show how the relative frequencies from simulations of UMER compare to the values obtained by the smooth approximation.

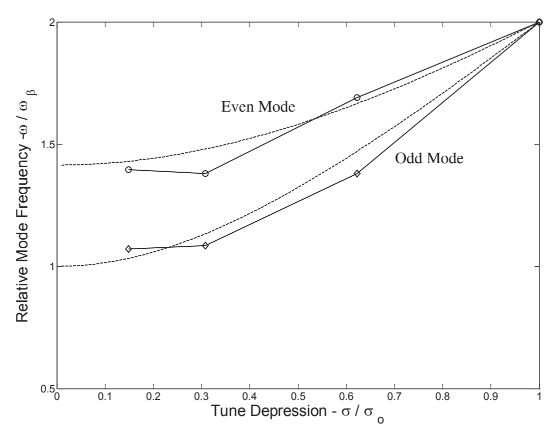

Figure 2: Relative envelope mode frequencies as a function of the tune depression for UMER operating parameters, plotted next to a calculation for the smooth approximation (dashed lines) shown in Eq. 2,3.

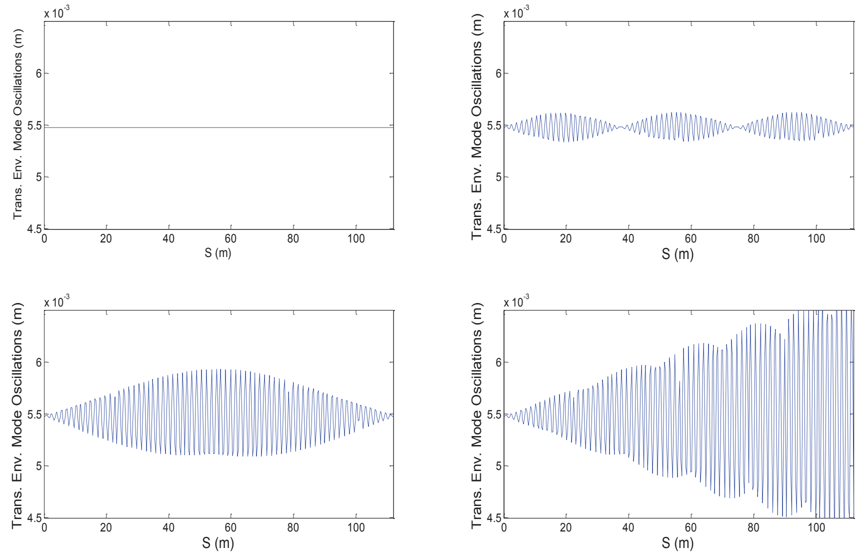

Figure 3: WARP envelope code simulations of the transverse envelope displacement near the quadrupole mode resonance sampled once per betatron frequency. Beam is kicked once per lattice period off resonance (top left), close to resonance (top right, bottom left), and on resonance (bottom right) for 10 turns.

ENVELOPE SIMULATIONS

Simulations are run through a particle-in-cell (PIC) code called WARP, developed by J-L Vay et al [6]. WARP has a built-in envelope solver that numerically solves by integrating equation 1. Starting with a matched beam, we apply a 10% mismatch to the initial amplitude conditions. Using the envelope code, we sample the resulting envelope amplitude once per betatron period and plot it as a function of time. Taking the Fourier transform, we can extract the resulting mismatch frequency. This is the desired frequency for the applied kick. For a lossless beam with an emittance of 25 mm-mrad, the mode frequencies are listed in table 1.

Table 1: Kick Frequencies from WARP Envelope Code

Beam Current	Odd Mode Freq	Even Mode Freq
6 mA	48.1 Mhz	53.6 Mhz
23 mA	36.9 Mhz	48.0 Mhz
40 mA	33.7 Mhz	46.0 Mhz

The resonant frequency peak in the Fourier transform has a small FWHM, so it can be very difficult to identify resonant behaviour. Off resonance the envelope oscillations form beats, and the behavior is fairly uniform across a large range of frequencies, also adding to the difficulty of determining the resonance frequency. In the simulation, the perturbative kicks are applied either once per turn or once per period to the transverse envelope velocities. This results in the behavior shown in Fig. 3. To find the bandwidth of the resonant frequency, the beam is kicked for three turns and then left to coast. The Fourier transform of the coasting transverse displacement is taken. The peak value at the kick frequency in the Fourier plot is recorded for several frequency values. The magnitude of this value is a good measure of the strength of the resonant growth. Plotting these values as a function of frequency provides a good quantitative estimate for the resonant bandwidth. This is shown in Fig. 4.

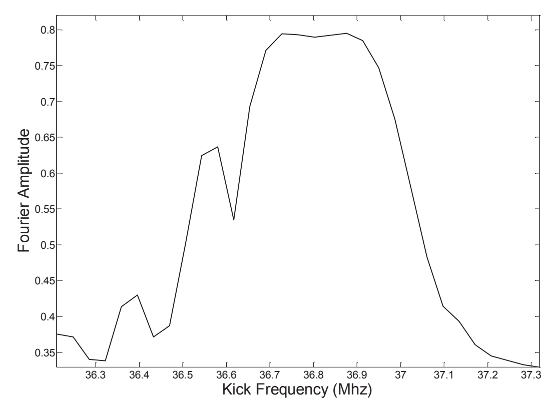

Figure 4: Plot showing the resonant bandwidth for the quadrupole envelope mode in a 23 mA beam.

With a FWHM of approximately 0.5 Mhz, it can be difficult to identify the resonances without the proper analysis.

Nonlinearities

We have determined that the resonant frequency is phase dependent. If a phase shift is applied to the sinusoidal kick at a resonant frequency, the result will often no longer be resonant. The mechanism behind this is not obvious, especially because the simulation starts with a matched beam. Also, the horizontal and vertical

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transverse displacements are slightly asymmetric. When applying the kick, it is important that the amplitudes be normalized in such a way to excite the correct mode. It is easy to mistakenly excite both envelope modes in a simulation, and the results are then harder to interpret.

PARTICLE-IN-CELL (PIC) SIMULATIONS

As a more accurate way to simulate a beam in UMER, particle simulations are run through WARP. We start with a semi-Gaussian distribution of 20,000 particles in the middle of a quadrupole magnet. Using the results from the envelope code to match the beam, we iterate the beam slice through the UMER geometry, removing the complicating corrections of the dipole magnets and the earth field (we assume a linear beam trajectory). To kick the beam, we apply a kick to the particle velocities in much the same way as in the envelope simulations, however this time the kick must be proportional to each particle's transverse displacement. The resulting resonant frequencies are very similar to the envelope code, however there are a few nuances uncovered through this method.

Emittance growth is present from the resonant beam kicks. Concurrently, the trace space area grows and shows a halo formation. Further proof of a halo formation is shown by the rotation in phase space. This behavior is shown in Fig. 5.

Nonlinearities

When running the PIC simulation, we noticed that the mode frequencies obtained from the Fourier transform of the initially mismatched beams were dependent on the initial mismatch. This implies the mode frequencies have intrinsic amplitude dependence. We plan to further study this phenomena and what it implies for future simulations and experiments.

CONCLUSIONS

Envelope perturbations arising from resonant modes are present in space-charge-dominated particle accelerators and storage rings. It is important to understand these resonances when building and operating a high intensity accelerator to avoid emittance growth and beam loss. These modes were simulated and analyzed for a particular alternating-gradient lattice electron storage ring. Envelope and PIC WARP simulations are compared. In the future, we plan to model an electric quadrupole kicker in the PIC code, and develop an experiment to show these results empirically.

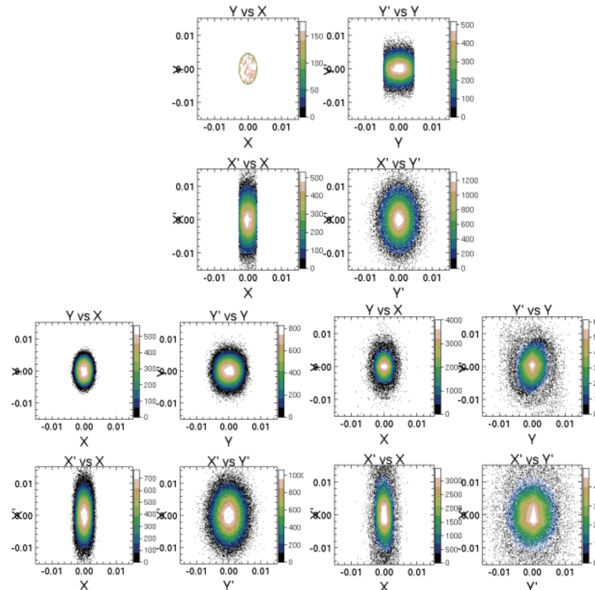

Figure 5: Halo growth in the 6th Turn. Initial conditions (top), no kick (bottom left), and resonant kick (bottom right)

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