THE INFLUENCE OF THE MAGNETIC FIELD ERRORS IN THE CYCIAE-100 CYCLOTRON*

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Abstract

The main magnet size of CYCIAE-100 is 2.31 m in height and 6.16 m in diameter and the outer radius of the sector is 2.0 m, and the total iron weight is about 415t. The magnetic field cannot be absolutely ideal because of imperfections during manufacturing and installation of this big magnet. Therefore the influence of the magnetic field errors on the beam behaviour should be studied to provide the reference for magnet mapping and shimming. The tolerances for the magnetic field errors are given in this paper based on analytic calculations and numerical simulations. The resonances v_r=1, $2v_r$=2 driven by the 1st, 2nd harmonic magnetic field error are considered, which will result in the radial emittance growth .Besides that, the resonances cause the vertical emittance growth are considered. The maximum allowable field errors for CYCIAE-100 are presented in this paper.

INTRODUCTION

The main magnet of the CYCIAE-100 cyclotron is under construction[1]. When the magnet is designed, most of dangerous resonances are avoided. But the magnetic field cannot be absolutely ideal during manufacturing and installation such as misalignment or magnetic non-uniformity of the magnet sectors, so the influence of the magnetic field errors on the beam behavior need be considered. Magnetic field errors in a cyclotron will excite coherent oscillations through displacing the center of orbit or distorting the transverse phase space. And the resonances driven by the field errors cause growth in the circulating emittance due to the precessional mixing. This effect is especially important in the CYCIAE-100 cyclotron because there are a number of different turns in the extracted beam. In this paper we will discuss the effect of first and second harmonic magnetic field error on the radial motion based upon analytic calculations and numerical simulations.

FIRST HARMONIC FIELD ERROR

The Emittance Growth

W. Kleeven gave a Hamiltonian function, describing the v_r=1 resonance driven by a first harmonic field error. (Eq. 1, ref. 2). With this Hamiltonian the motion equation of the orbit centre can be determined as following:

$$\left(x_c+\frac{rA_1}{2(\upsilon_r-1)}\right)^2+\left(y_c+\frac{rB_1}{2(\upsilon_r-1)}\right)^2=\left(\frac{rb_1}{2(\upsilon_r-1)}\right)^2 \quad (1)$$

Where $A_1=b_1\cos\phi$; $B_1=b_1\sin\phi$, the first harmonic field expanded in a Fourier series. b_1 is the radio of the amplitude of first harmonic field to the average magnetic field. r is the radius of the particle orbit, v_r is the relative radial oscillation frequency.

From this expression it follows that the particle precesses under the influence of a first harmonic field in a circular path about the equilibrium orbit center displaced by a distance d1, given by

$$d_1=\frac{b_1 r}{2(\upsilon_r-1)} \quad (2)$$

The displacement of the orbit center caused by the first harmonic field error will excite coherent oscillations and increase the beam emittance. If the field error area is wide enough, when the beam will be affected by this error for large turn numbers, the phase advance can reach 2π and there will be a complete processional mixing. Under this condition the emittance is called circulating emittance. The formula can be derived

$$\varepsilon_c=\pi\left(x_0+d_1\right)\cdot\frac{\upsilon_r\left(x_0+d_1\right)}{r}=\varepsilon\cdot\left(1+\frac{d_1}{x_0}\right)^2 \quad (3)$$

Where

$\varepsilon=\pi x_0\cdot\frac{\upsilon_r x_0}{r}$, is the initial emittance;

$x_0=\sqrt{\frac{\varepsilon r}{\pi\upsilon_r}}=\sqrt{\frac{\varepsilon_n r}{\beta\gamma\cdot\pi\upsilon_r}}=\sqrt{\frac{\varepsilon_n R_\infty}{\pi\upsilon_r}}$, is beam size.

The emittance growth is defined as the radio of the circulating emittance to the initial emittance, $f_1=\varepsilon_c/\varepsilon$. When the limitation of f_{1m} is given, the upper limit for the first harmonic field amplitude is determined.

$$d_1\le\left(\sqrt{f_{1m}}-1\right)\left(\frac{R_\infty\varepsilon_n}{\pi\upsilon_r}\right) \quad (4)$$

For CYCIAE-100 cyclotron the emittance growth is calculated using formula (3),R_∞=430cm, for an emittance growth of 100% (f_{1m}=2) and for an initial normalized emittance 4π mm-mrad, the maximum allowable first harmonic error is 2.2G.

The CYCLONE [3] program was used to simulate the emittance growth caused by the first harmonic field error through multi-particle tracking. 8 particles were selected from the eigen-ellipse of a certain energy with a normalized emittance 4π mm-mrad; the first harmonic field error was included in magnetic field to simulate the motion of the 8 particles. Record the coordinates of particles in phase space at the same azimuth, one point per turn. Because of the wide range of the first harmonic field and the fact that the turn numbers is large enough, the phase space is filled up, then produce the circulating emittance which is the maximum ellipse area in Fig. 1. The emittance growth f_1 at different energy calculated by program simulation and by analytic formula is given in Table 1. We can see a good agreement between the numerical results and analytical ones.

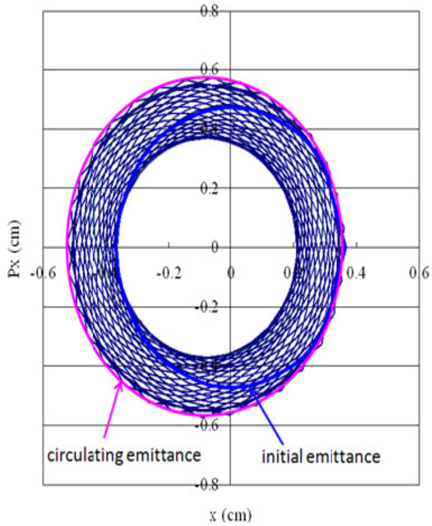

Figure 1: At 1MeV, the circulating emittance produced by phase space filling up due to the first harmonic field error simulated by CYCLONE.

Table 1: The Emittance Growth

E_k(MeV)	f_1 by formula	f_1 by simulation
1	1.471	1.474
5	1.767	1.746
10	1.865	1.832
20	2.004	1.966
50	1.881	1.861
100	1.647	1.625

The Influence of the Azimuthal and Radial Distribution

The effect of the first harmonic field error on beam behaviour depends not only on the amplitude but also on the radius range. In order to take this effect into account the simulation were done for the acceleration process. We first determined the "Accelerated Equilibrium Orbit (AEO)" for ions in (x, Px) phase space by running backwards from 100MeV to low energy. Then we superimposed the first harmonic field imperfections on the ideal field map from 3D FEL program and observed the coherent oscillation.

We used the first harmonic field with 10G amplitude, Gaussian shaped bump with 2σ=6cm width, to calculate the radial amplitudes of coherent oscillation at various radii, the results are shown in Fig. 2 and Fig. 3. The horizontal ordinate in Fig. 3 is the first harmonic bump central radius. Figure 3 shows the results when the bump was centered at 47cm, the field bump of 10G builds up an oscillation of 7mm. Similarly, the 5G bump produces an oscillation of 3.48mm. It was found to be almost linearly proportional to the first harmonic amplitude and almost independent of its phase, see Fig. 4.

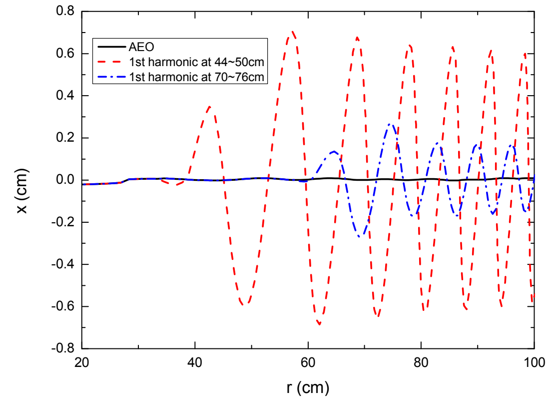

Figure 2: The radial coherent oscillations due to the first harmonic field error at different radius range.

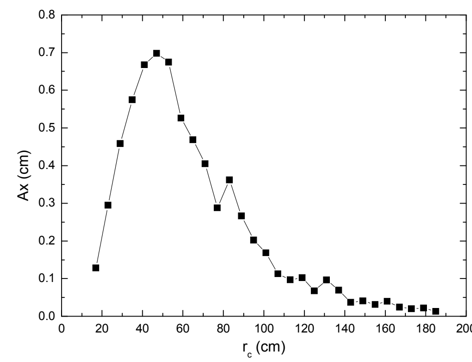

Figure 3: The coherent oscillation amplitudes excited by the first harmonic field bumps, which are Gaussian shaped, at different radius ranges.

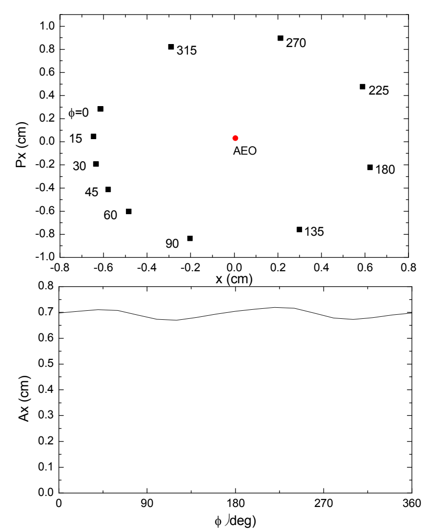

Figure 4: The coherent oscillation changes with the azimuthal of the first harmonic field error, the amplitude is 10G, and the angle Φ denotes field bump azimuth. The upper figure is the ions in (x, Px) space at 50MeV accelerated from low energy. The bottom figure is the oscillation amplitude changes with the bump azimuth, and the bump position is at 44~50cm.

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The results show that in CYCIAE-100 cyclotron the most sensitive region for the first harmonic bump is 44～50cm, where the Gaussian bump of 10 G produces an oscillation amplitude of about 7 mm. Based on the beam dynamics request the radial beam size (half size) is limited within 5mm, then the upper limit of the first harmonic field error amplitude at the most sensitive region should be less than 7G.

SECOND HARMONIC FIELD ERROR

The $2\upsilon_r = 2$ resonance is driven by a second harmonic field error and its gradient. The second harmonic distorts the shape of the radial phase space. The frequency of the perturbed oscillation is given by[2]:

$$\upsilon_r^* = \sqrt{C_0^2 - C_2^2}$$

Where $C_0 = \upsilon_r - 1,\ C_2 = 0.5A_2 + 0.25A_2'$, $A_2 = b_2/\bar{B},\ A_2' = r\frac{dA_2}{dr}$.

Thus, the $2\upsilon_r = 2$ resonance will be crossed and the motion becomes unstable if $C_2^2 > C_0^2$, that means if

$$\left|0.5A_2 + 0.25A_2'\right| > \left|\upsilon_r - 1\right|.$$

For the CYCIAE-100, C_0=0.01 (at central region), B=7250G, we find b2<72.5G and $\left|db_2/dr\right| < 14.5\,G/cm$.

The radial phase space is distorted by a second harmonic field error, the emittance will be increased due to the precessional mixing. Then the emittance growth is,

$$f_2 = \frac{\varepsilon_c}{\varepsilon} = \sqrt{\frac{(C_0 + C_2)}{(C_0 - C_2)}}$$

When f_2=1.5 for 50% emittance growth, then $b_2 < 28G,\ db_2/dr < 5.6\,G/cm$. At 1MeV, C_0=0.03, when f_2=1.5, then $b_2 < 85G$. The simulation result is calculated using a second harmonic field with the amplitude of 85G, the radio of the circulating emittance to the initial emittance is about 1.5, which shows good agreement with analytic result. The influence of the second harmonic field error is very small at the large radius.

WALKINSHAW RESONANCE

This is a resonance coupling horizontal space and vertical space driven by radial derivatives of main field. Walkinshaw resonance does not cause instability but it will become very dangerous if the horizontal beam size is large. In this case the vertical beam size will be too large to lose due to the exchanges energy between the horizontal and vertical oscillations.

Ref[2] gives the maximum vertical amplitude growth per turn from the resonance, that is $\left(dz/dn\right)_{\max} = 4\pi g'' r J_0$.

The tune diagram of CYCIAE-100 cyclotron is given in Ref[1]. We can see it cross the $\upsilon_r = 2\upsilon_z$ resonance at r=35cm, E=3.2MeV, υ_z=0.52. If x_0=z_0=5mm, the constant J_0=2.6×10^{-4}, g''≈0.07, then $\left(dz/dn\right)_{\max} \approx 0.1mm/turn$. Figure 5 shows the vertical beam profiles for centering and off-centering beam. We can see that the effect from Walkinshaw resonance at low energy is not too much, and this resonance crossing is much rapid.

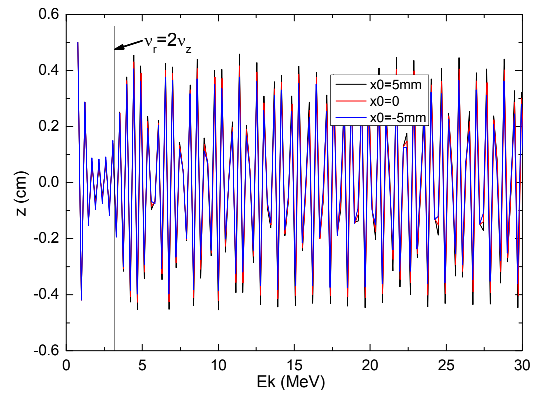

Figure 5: The oscillations in vertical envelope due to the Walkinshaw resonance for centering and off-centering beam.

CONCLUSIONS

In this paper the analytical descriptions of the orbit center displacement and the emittance growth driven by a first harmonic and a second harmonic field error are given, which provide a quick understanding and a quantitative estimate for the influence on the beam. Some of the calculations have been done by numerical orbit tracing and show good agreement with analytical results. According to the results in this paper, for CYCIAE-100 cyclotron, at the radius range of 44～50 cm the amplitude of the first harmonic field error should be limited within 7G. Furthermore it should be avoided that the first harmonic field error existed in a range wide enough. The magnetic field tolerances for the CYCIAE-100 cyclotron are given in this paper which can be the reference to the shimming for the first harmonic and second harmonic field errors.

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