THE BEAM GAS COULOMB SCATTERING IN ELECTRON STORAGE RING*

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Abstract

In this paper, the collision formulas applying to any elastic collision was derived. Using the formulas, we have investigated the beam lifetime and the particle distribution's change caused by the coulomb scattering with the residual gas in HLSII. It is shown that the estimated beam lifetime shows a good agreement with the theoretical value. And the particle distribution in a bunch has diffusion in transverse direction due to the beam gas coulomb scattering.

INTRODUCTION

In e-storage ring, there is always some residual gas floating near the beam orbit, no matter how well vacuum condition. The electrons will inevitably collide with them. As the direct consequence, the motion state of the particle has changed. If the motion exceeds the limited conditions, the corresponding particle will be lost. The impact between electron and molecules is quite complex, here we only consider the coulomb scattering on the nucleus. Compared to the previous study on beam gas elastic scattering, we compute the momentum change of the particles in three directions, not just in transverse direction as before. On the other hand, in order to get the post-momentum of particle more accurately, we do as much as possible to restore the collision process. But one point which needs attention is following.

For convenience to treat the tracking of beam particles, the 6-D particle coordinates $(x, y, \sigma, x', y', p_\sigma)$ are chosen taking s as independent variable, in which $\sigma = s - v_0 t$, $p_\sigma = \Delta E / (E_0 \beta^2)$,while the motion of residual gas is in the laboratory coordinate system(S). To compute the collision between the two, first of all, you must convert the two into the same coordinate system.

With the collision formulas, we will investigate the beam lifetime and the particle distribution's change caused by the coulomb scattering with the residual gas.

ELASTIC COLLISION BETWEEN TWO RELATIVISTIC PARTICLES

The conservation of energy and momentum is the foundation of dealing collision problem. In order to give the simple form of conservation law, the center of mass frame (CM) was chosen to calculate the post-momentum. Accordingly, in order to obtain the parameters of particles in CM, we have to perform a Lorentz transformation from S to CM. Then the new momentums are transformed back to the S. The following derivation is based on the theory of [3] [4] [5] [6].

Set two collided particles with the rest mass m_{10} and m_{20} respectively. For convenience we suppose both of the S and CM belong to rectangular coordinate system, and the axes parallel to each other. The parameters of two particles before collision in S and CM are given below.

$$\text{S:}\quad \vec{v}_{1,2}, \beta_{1,2} = v_{1,2}/c, \gamma_{1,2} = 1/\sqrt{1-\beta_{1,2}^2}, m_{1,2}, \vec{p}_{1,2}, E_{1,2}$$

$$\text{CM:}\quad \vec{\tilde{v}}_{1,2}, \tilde{\beta}_{1,2} = \tilde{v}_{1,2}/c, \tilde{\gamma}_{1,2} = 1/\sqrt{1-\tilde{\beta}_{1,2}^2}, \tilde{m}_{1,2}, \vec{\tilde{p}}_{1,2}, \tilde{E}_{1,2}$$

The above parameters with symbol ' are corresponding parameters after collision.

The velocity of CM relative to S is given by [3]

$$\beta_i = \frac{\sum p_{ilab} c}{\sum E_{lab}} = \frac{p_{1i} + p_{2i}}{(m_1 + m_2)c}, i = a, b, c \tag{1}$$

According to the Lorentz transformation, the component of $\vec{\tilde{p}}$ can be obtained by [3].

$$\tilde{p}_i = p_i + (\frac{\gamma - 1}{\beta^2})(\vec{\beta} \cdot \vec{p}) \cdot \vec{\beta} - \gamma \frac{E}{c} \vec{\beta} \tag{2}$$

In CM system, the total momentum is zero. Besides the energy of each particle don't change in elastic collision, neither the magnitude of momentum with a deflection of angle χ .Therefore, the following relationships exist in the CM [4].

$$\vec{\tilde{p}}_1 = -\vec{\tilde{p}}_2, \vec{\tilde{p}}'_1 = -\vec{\tilde{p}}'_2, \left|\vec{\tilde{p}}_1\right| = \left|\vec{\tilde{p}}'_1\right|, \left|\vec{\tilde{p}}_2\right| = \left|\vec{\tilde{p}}'_2\right| \tag{3}$$

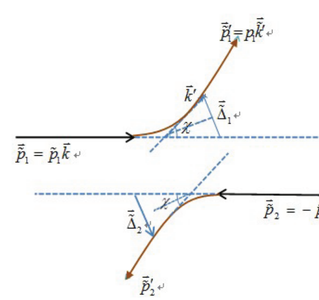

Figure 1: The schematic diagram of the collision in CM.

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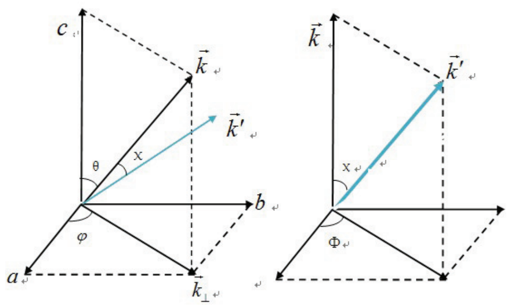

Figure 2: the vector diagram of $\vec{k}$ and $\vec{k}'$.

In Fig.1, $\vec{k}$ is the unit vector of $\vec{\tilde{p}}_1$, while $\vec{k}'$ is the unit vector of $\vec{\tilde{p}}_1'$.The fig.2 shows the $\vec{k}$ and $\vec{k}'$.

If the component of $\vec{k}'$ is known, $\vec{\tilde{p}}_1'$ will be easy to get. In the left of Fig.2, $\vec{k}$ and θ, φ

$$\begin{pmatrix} \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\phi \\ -\sin\phi & \cos\phi & 0 \\ \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \end{pmatrix}\begin{pmatrix} k_a \\ k_b \\ k_c \end{pmatrix}=\begin{pmatrix} 0 \\ 0 \\ k \end{pmatrix}\Leftrightarrow\begin{pmatrix} k_a \\ k_b \\ k_c \end{pmatrix}=\begin{pmatrix} k\sin\theta\cos\varphi \\ k\sin\theta\sin\varphi \\ k\cos\theta \end{pmatrix} \quad (4)$$

Similarly, on the right of Fig.2 there is a relationship between $\vec{k}'$ and $\vec{k}$

$$\begin{pmatrix} \cos\chi\cos\Phi & \cos\chi\sin\Phi & -\sin\Phi \\ -\sin\Phi & \cos\Phi & 0 \\ \sin\chi\cos\Phi & \sin\chi\sin\Phi & \cos\chi \end{pmatrix}\begin{pmatrix} k_a' \\ k_b' \\ k_c' \end{pmatrix}=\begin{pmatrix} 0 \\ 0 \\ k' \end{pmatrix}\Leftrightarrow\begin{pmatrix} k_a' \\ k_b' \\ k_c' \end{pmatrix}=\begin{pmatrix} k'\sin\chi\cos\Phi \\ k'\sin\chi\sin\Phi \\ k'\cos\chi \end{pmatrix} \quad (5)$$

So $\vec{k}'$ in (a, b, c) has following relationship.

$$\begin{pmatrix} \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\phi \\ -\sin\phi & \cos\phi & 0 \\ \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \end{pmatrix}\begin{pmatrix} k_a' \\ k_b' \\ k_c' \end{pmatrix}=\begin{pmatrix} k'\sin\chi\cos\Phi \\ k'\sin\chi\sin\Phi \\ k'\cos\chi \end{pmatrix} \quad (6)$$

It can be driven

$$\begin{pmatrix} k_a' \\ k_b' \\ k_c' \end{pmatrix}=\begin{pmatrix} k'\cos\chi\cos\phi\sin\theta+k'\sin\chi\cos\theta\cos\phi\cos\Phi-k'\sin\chi\sin\phi\sin\Phi \\ k'\cos\chi\sin\phi\sin\theta+k'\sin\chi\cos\theta\sin\phi\cos\Phi+k'\sin\chi\cos\phi\sin\Phi \\ k'\cos\theta\cos\chi-k'\sin\chi\cos\Phi\sin\theta \end{pmatrix} \quad (7)$$

Taking k'=k=1 and (5) into (8).

$$\begin{pmatrix} k_a' \\ k_b' \\ k_c' \end{pmatrix}=\begin{pmatrix} k_a\cos(\chi)+k_ak_c/k_\perp\sin(\chi)\cos\Phi-k_b/k_\perp\sin(\chi)\sin\Phi \\ k_b\cos(\chi)+k_bk_c/k_\perp\sin(\chi)\cos\Phi+k_a/k_\perp\sin(\chi)\sin\Phi \\ k_c\cos(\chi)-k_\perp\sin(\chi)\cos\Phi \end{pmatrix} \quad (8)$$

Then the post-collision momentum in CM for an elastic collision is given by

$$\vec{\tilde{p}}_1' = \tilde{p}_1\vec{k}' = \vec{\tilde{p}}_1\cos\chi + \vec{h}\sin\chi \quad (9)$$

Where $\vec{h}=(h_a,h_b,h_c)$ with

$$\begin{aligned} h_a &= (\tilde{p}_{1a}\tilde{p}_{\alpha c}\cos\Phi - \tilde{p}_{1b}\tilde{p}_1\sin\Phi)/\tilde{p}_{1\perp} \\ h_b &= (\tilde{p}_{1b}\tilde{p}_{\alpha c}\cos\Phi + \tilde{p}_{1a}\tilde{p}_1\sin\Phi)/\tilde{p}_{1\perp} \\ h_c &= -\tilde{p}_{1\perp}\cos\Phi \end{aligned} \quad (10)$$

The $\vec{\tilde{p}}_1$ can be driven by transforming $\vec{\tilde{p}}_1'$ back to the S system using the Lorentz transformation.

The above formulas apply to any elastic collision between two particles. Noting for that χ will be different for different types of elastic collision process.

SIMULATION METHOD

The description of the simulation method [1][2]

The simulation is based on the macro-particle method. The macro-particles transfer through the ring and encounter coulomb scattering with residue gas molecule simultaneously.

The initial 6-D coordinates of n macro-particles are given randomly with specified variances. Each macro-particle (i) has a particle number (N_i). ΣN_i is the total number of particles in a bunch. Let p be the probability that an electron undergoes a random process in one turn. The probability that each macro-particle undergoes a random process is N_ip. For each macro-particle, one random number $R\in\infty[0,1]$ is chosen, if $R< N_ip$, we separate one electron from the i-th macro-particle as a new macro-particle. The new macro-particle with the same coordinates to the parental collides with a molecule of the residual gas. During the collision, the (x', y', p_σ) of the particle was changed, while the (x, y, σ) was considered the same as before. In the presence of the dynamic aperture，this process will lead to the loss of particles. Then the beam lifetime of the beam will be estimated in the simulation by counting the number of the particles extending beyond the dynamic aperture.

In the method, the collision probability p can be got by the following expression [7]:

$$p(t)=1-\exp(-\sigma\rho v\Delta t) \quad (11)$$

Where σ is the cross section of coulomb scattering with residual gas, ρ is the volume density of gas, v is the beam electron's speed. And the σ is determined by [8]

$$\frac{d\sigma}{d\Omega}=\left(\frac{2Zr_e}{\gamma}\right)^2\frac{1}{(\chi^2+\chi_{\min}^2)^2} \quad (12)$$

where Ω is the solid angle, χ the scattering angle, Z the atomic number, re the classical electron radius, γ the Lorentz factor and $\chi_{\min}=Z^{1/3}\alpha/\gamma$ the screening of the atomic electrons with α is fine-structure constant.

In the paper, we track 10^5 initial macro-particles for $2*10^3$turns. And supposing that the residual gas is similar to the air, each molecule has two atoms with atomic number 7.25.

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About the beam optics, the linear matrix between two adjacent elements is adopted.

Table 1: Nominal HLS Parameters

Parameter	Description	Value
E	Beam Energy	0.8GeV
C	Circumference	66.1308m
I	Electric Current	300mA
h	Harmonic number	45
β	Av.beta function	8.50/5.25m
a_{Acc}	Transverse acceptance	46/28mm.mrad
σ_s	RMS Bunch Length	1.47519cm
Au	physical aperture	38/10mm
P	Vacuum pressure	1ntorr

The Lifetime of Beam

Once a particle's amplitude exceeds an aperture, this particle will be lost. Usually, the beam coulomb lifetime related to the vacuum can be obtained by [9]

$$\tau_{coul} \approx 10 \frac{E^2 a_{Acc}}{P < \beta >} \quad (15)$$

In the calculation, using the units in table 1, the unit of τ_{coul} will be hour

In the simulation, lost particles are counted per turn, the simulation lifetime of beam can be calculated by

$$\tau = \frac{N}{-dN/dt} \quad (16)$$

Where N is the total number of particles in a bunch.Fig.3 shows the number of lost particles growing with time.

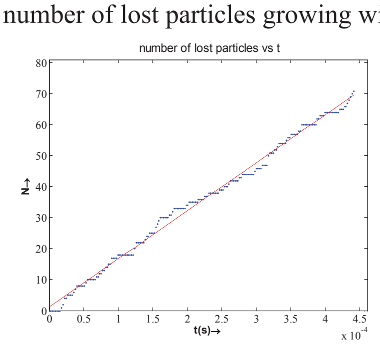

Figure 3: the number of lost particles vs time.

In the conditions of table 1, the two kinds of lifetime are 34.3834h and 16.4655h respectively.

There is a deviation between the two, but acceptable. The reason may be the choice of scattering angle. In this paper, the scattering angle is chosen by [9]

$$\chi = \sqrt[4]{a_{Accx} a_{Accy} / (\beta_x \beta_y)} \times \sqrt{U} \quad (17)$$

U is a uniform random number in [0 1].

The Distributions of Particles

In fig.4, the red dots represent the initial position of particles, while the blue is the position after 2000 turns. Visually, the particles have a diffusion in both x and y due to the beam gas coulomb scattering. And one thing is for sure, the peripheral particles in a bunch after 2000 turns are mostly new macro-particles.

Furthermore, the energy of particles wouldn't change in the case of elastic collision. The result analysis of numerical simulation also shows that the beam gas coulomb scattering doesn't have effect on p_σ.

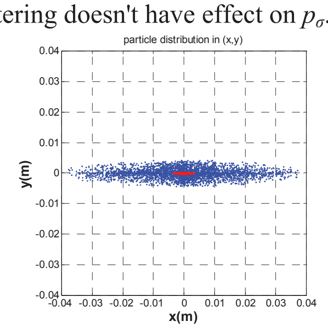

Figure 4: the transverse distribution of particles.

SUMMARY AND OUTLOOK

In the simulation, the scattering angle χ has a close relationship with the lost speed of the new created macro-particles. So the choice of χ which accord with the real collision is a difficult problem. The χ in this paper may not be quite accurate.

From the simulation, it can be seen the main effect of gas scattering is on the transverse under the linear transport. In the future work, more random processes including inelastic scattering with residual gas and IBS and synchrotron radiation will be added to the simulation.

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