ON THE CHOICE OF LINAC PARAMETERS FOR MINIMAL BEAM LOSSES

M. Eshraqi, ESS, Lund, Sweden
J-M. Lagniel, GANIL, Caen, France

Abstract

In high intensity linear accelerators, the tune spreads induced by the space-charge forces in the radial and longitudinal planes are key parameters for halo formation and beam losses. For matched beams they are the parameters governing the number of resonances (including coupling resonances) which affect the beam and determine the respective sizes of the stable and halo areas in phase space. The number and strength of the resonances excited in mismatched beams leading to even higher amplitude halos are also directly linked to the tune spreads. In this paper, the equations making the link between the basic linac parameters (rf frequency, zero-current phase advances, beam intensity and emittances) and the tune spreads are given. A first analysis of the way these linac parameters can be chosen to minimize the tune spreads is presented. The ESS linac parameters are used for this study.

INTRODUCTION

In high intensity linear accelerators where the beam power is in the order of hundred kilowatts up to few megawatts, activation due to loss of halo particles is a crucial parameter affecting the design as well as the cost of the accelerator. There has been several studies demonstrating that the tune spreads induced by the space charge forces in the two radial and longitudinal planes are key parameters for halo formation and beam losses [1–3].

With σ_{0t} and σ_t (σ_{0l} and σ_l) the transverse (longitudinal) phase advances without and with space charge, the transverse and longitudinal relative tune spreads are given by

$$\zeta_t = \frac{\sigma_{0t} - \sigma_t}{\sigma_{0t}} \qquad \zeta_l = \frac{\sigma_{0l} - \sigma_l}{\sigma_{0l}} \qquad (\zeta = 1 - \eta), \qquad (1)$$

where $\eta = \sigma/\sigma_0$ is the tune depression. These relative tune spreads are parameters which give a "measure" of the space-charge nonlinear effects on the beam dynamics as a function of the beam current I with

$$\lim_{I\to 0} \zeta = 0 \qquad \lim_{I\to +\infty} \zeta = 1. \qquad (2)$$

Relative tune spreads are good indicators of the strength of the space charge force, e.g., in a linac with a relative tune spread of 0.6, the space charge force is 84% of the external focusing force.

In a beam with space charge, the particle phase advances range from the phase advance with space charge for particles with low amplitudes to the zero current phase advance for large amplitude ones (see Fig. 1). The relative tune spreads are then the key parameters governing the number of resonances between the particle oscillations and the space-charge force oscillations, the space-charge "excitation force" oscillations being mainly due to the non-continuous character of the radial and longitudinal focusing systems (matched beam, Fig. 1) and/or to beam mismatches (see Fig. 2).

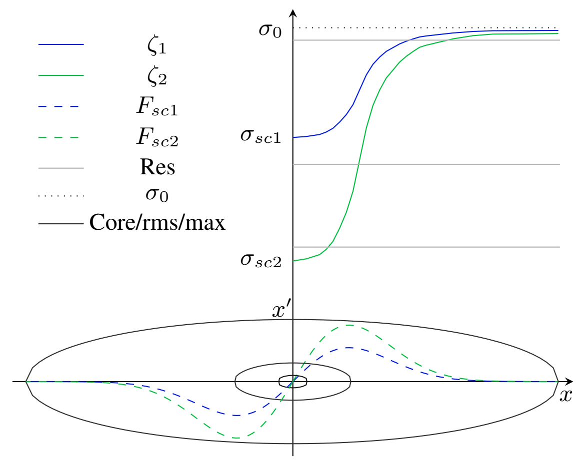

Figure 1: Schematic representation of the particle phase advances as a function of their amplitude for two relative tune spreads with two beam currents I_1 and $I_2 > I_1$.

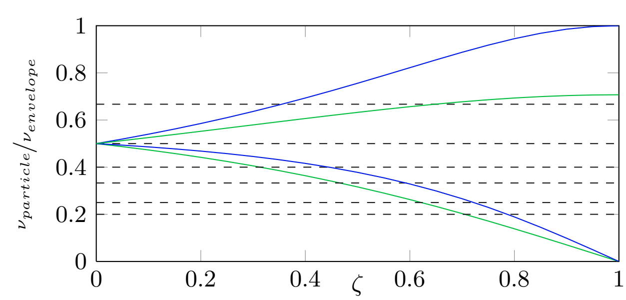

Figure 2: Ranges of resonances excited by a mismatch as a function of ζ, the relative tune spread. For a given ζ value, each pair of curves defines the lower and upper limits of the resonances ($\nu = {}^2/_3, {}^1/_2, {}^1/_3$...) which are excited by the odd (blue lines) and even (green lines) mismatch modes [2].

Higher ζ means higher number of resonances and higher excitations of those resonances for both matched and mismatched beams, then higher halo densities at larger amplitudes induced by the resonance overlap mechanism which explains the particle diffusion at such large amplitudes [1, 2].

ISBN 978-3-95450-122-9

Copyright © 2013 by JACoW — cc Creative Commons Attribution 3.0 (CC-BY-3.0)

The logical question raised by this analysis is "What is the best choice of linac parameters to minimize the relative tune spreads?" or, in other words, "How can we make high-power linac less sensitive to space charge?". The study presented here has been motivated by these questions.

THEORY

The linac beam physics in smooth approximation can be studied using 10 parameters. The beam current (I), the particle charge (q), rest mass (m_0c^2) and energy (W), the rf frequency (f) and the bunch spacing (N) are used to calculate the space charge factor (K, see Eq. 4 and [4]). The four other parameters are the transverse and longitudinal rms emittances (ϵ_t, ϵ_l) and zero current phase advances per meter (σ_{0t}, σ_{0l}).

Once these parameters defined, the system of 4 equations (Eqs. 3) governing the phase advances per unit of length in transverse and longitudinal planes can be solved. Then the relative tune spreads in transverse and longitudinal planes can be calculated using Eq. 1.

$$\sigma_t a^2 = 5\,\epsilon_{rms\;t} \tag{3a}$$

$$\sigma_l b^2 = 5\,\epsilon_{rms\;l} \tag{3b}$$

$$\sigma_t^2 = \sigma_{0t}^2 - K\frac{1-ff_z(a,b)}{2a^2b} \tag{3c}$$

$$\sigma_l^2 = \sigma_{0l}^2 - K\frac{ff_z(a,b)}{2a^2b}, \tag{3d}$$

a and b are the equivalent uniformly charged ellipsoid semi-axes dimensions in the radial and longitudinal directions respectively, $ff_z(a,b)$ is the longitudinal bunch form factor [4] and K is the space charge factor,

$$K = \frac{3eN\lambda I}{4\pi\epsilon_0 c m_0 c^2 \beta_s^2\gamma_s^3}, \tag{4}$$

where λ is the rf wave length, ϵ_0 is the vacuum permittivity and β_s and γ_s are the relativistic factors.

The system of equations 3 can be numerically solved using σ_t and σ_l as search parameter, the ellipsoid dimensions a and b being given by eqs. 3a and 3b respectively.

STUDY

Current Dependence of ζ

The evolution of the relative tune spreads as a function of the beam current can be more easily grasped in the case of a spherical bunch, $a = b$ then $ff_z = 1/3$. In this case, the set of equations 3 leads to:

$$\epsilon_{rms\;t}\,\sigma_l = \epsilon_{rms\;l}\,\sigma_t$$

$$\sigma_t^2 + \alpha\frac{I}{\epsilon_t^{3/2}}\sigma_t^{3/2} - \sigma_{0t}^2 = 0, \tag{5}$$

where the current independent parameter $\alpha = K/(3\times\sqrt{5^3}I)$ has been introduced to emphasize the dependence on current (I). One can easily write a similar equation for the longitudinal beam parameters. Equation 5 and its longitudinal counterpart show that:

$$\lim_{I\to+\infty}\sigma = 0 \quad\to\quad \lim_{I\to+\infty}\zeta = 1.$$

Equation 5 shows that the phase advances with space charge become null only when the current becomes infinite, consequently there is no theoretical current limit in linacs. Assuming constant rms emittances, the decrease of the tunes with space charge as the intensity increases induces larger and larger bunch dimensions but the motion never becomes unstable. However, the practical limit comes from the radial and longitudinal available apertures. Equation 5 also shows, as expected, that the key factor which determines the phase advances with space charge, then the relative tune spreads, is the bunch charge density $I/\epsilon^{3/2}$, not the beam current I.

The behavior for the ellipsoidal bunches is plotted in Fig. 3, the dependence on $\sqrt{I}$ is visible from the plot.

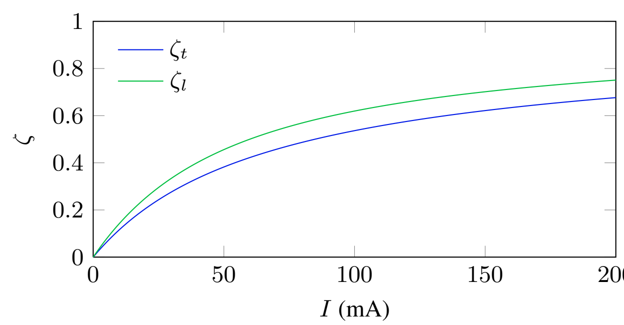

Figure 3: Evolution of ζ_t and ζ_l as a function of current at the entrance of the ESS DTL [5].

Emittance Dependence of ζ

The emittance dependence of the relative tune spread or phase advance with current in spherical bunches can be derived by differentiating Eq. 5 with respect to emittance. The derivative has a positive value, which means higher emittances result in higher values of phase advance with current, and therefore to lower value of ζ. This is true even in the case of ellipsoidal bunches as shown in Fig. 4.

Figure 4 shows that higher emittances result in lower relative tune spread which will lead to lower halo production and therefore lower losses and activation. The trend is much faster in the plane on which the emittance is varied, nonetheless the other plane also acquires a lower relative tune spread due to lower charge density in the bunch. On the other hand, higher emittances results in bigger beam sizes ($\sim\sqrt{\epsilon}$). This behavior means that there is an optimized couple of emittances which minimizes the beam losses for a particular linac design. Such a minimum will be achieved as a compromise between bigger emittances that lower the halo production and lower emittances associated with smaller beam core sizes but not necessarily to denser "tails". This study is not finished yet.

Copyright © 2013 by JACoW — cc Creative Commons Attribution 3.0 (CC-BY-3.0)

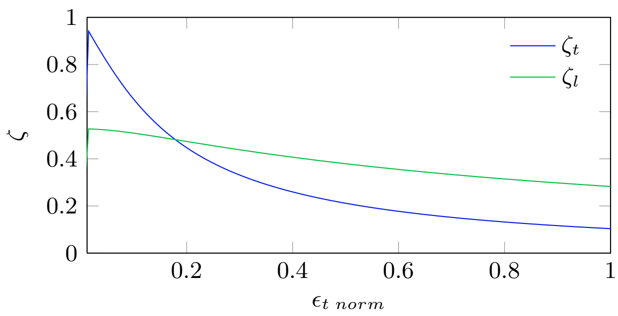

Figure 4: Evolution of ζ_t and ζ_l as a function of transverse normalized emittance at entrance to ESS DTL.

Zero Current Phase Advance Dependence of ζ

While it seems counter intuitive at first, the space charge effects decrease when both zero current phase advances increase, *i. e.* when both transverse and longitudinal bunch dimensions decrease (See Fig. 5). The explanation of this effect is that the space charge forces increase less than the external forces when increasing both zero current phase advances. Figure 6 shows the behavior of the relative tune spreads when only one zero current phase advance is varied.

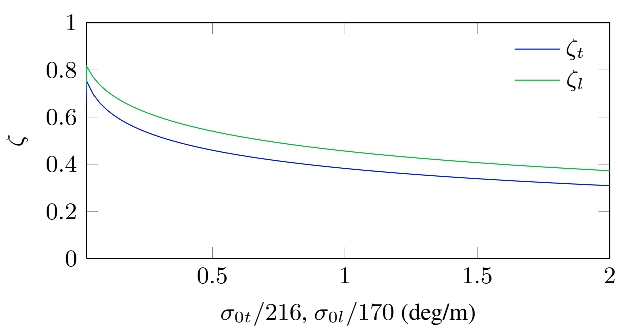

Figure 5: Evolution of ζ_t and ζ_l when both σ_{0t} and σ_{0l} are multiplied by the same factor (input of the ESS DTL).

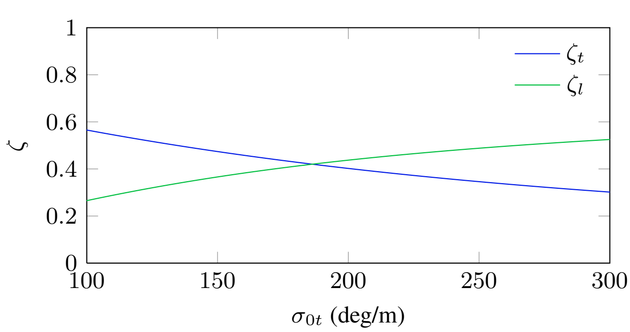

Figure 6: Evolution of ζ_t and ζ_l as a funtion of zero current transverse phase advance (input of the ESS DTL).

Condition of Equal Relative Tune Spread, $\zeta_t = \zeta_l$

For every set of beam and linac parameters there is a unique working point $(\sigma_{0t}, \sigma_{0l})$ on which the transverse and longitudinal relative tune spreads are equal. At any other working point, one of the two relative tune spreads is bigger than the other, making the equal ζ point the least sensitive working point to space charge forces.

The numerical search for the $\zeta_t = \zeta_l$ condition can be done substituting the expressions of σ_t and σ_l from Eq. 3c and 3d in Eq. 1. As shown in Fig. 7, this condition is satisfied for a given zero current phase advance ratio when the normalized emittance ratio is fixed, whatever the values of the other parameters. The $\zeta_t = \zeta_l$ condition leads therefore to a "universal" relation between the normalized emittances and the zero current phase advances to apply to minimize halo formation.

The condition of $\zeta_t = \zeta_l$ is a better rule to follow with respect to equipartition rule to minimize halo formation.

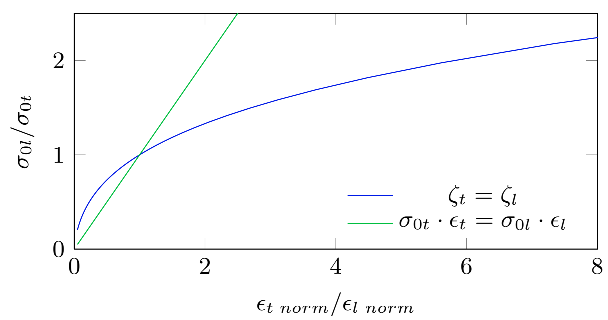

Figure 7: Zero current phase advance ratio as a function of the normalized emittance ratio to satisfy $\zeta_t = \zeta_l$.

CONCLUSION

It has been shown that for every linac, the parameters and in particular the external focusing forces can be chosen to minimize the effects of space charge. This is specially important in high power linacs where the space charge forces are in the same order of the external forces. Numerical tools have been built to solve the equations concerning the choice of linac parameters to minimize the relative tune spreads.

REFERENCES

[1] A. M. Sessler. "Collective phenomena in accelerators". *in proc.* LINAC'72.

[2] J. M. Lagniel. "Halos and chaos in space-charge dominated beams". *in proc.* EPAC'96.

[3] M. Pabst, K. Bongardt, A. Letchford, "Progress on intense proton beam dynamics and halo formation". *in proc.* EPAC'98.

[4] P. M. Lapostolle, and A. L. Septier. *Linear Accelerators*. Amsterdam, North Holland Publishing Company 1970.

[5] M. Eshraqi. "End to end beam dynamics of the ESS linac". *in proc.* IPAC'12.

Copyright © 2013 by JACoW — cc Creative Commons Attribution 3.0 (CC-BY-3.0)

ISBN 978-3-95450-122-9