SUPPRESION OF HALO FORMATION IN FODO CHANNEL WITH NONLINEAR FOCUSING

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Abstract

The focusing properties of a quadrupole FODO channel with inserted multipole lenses are analyzed via an application of the averaging method. A general expression for the averaged focusing potential is obtained as a function of the position of multipole lenses with respect to FODO quadrupole lenses. Obtained results are subsequently applied to the problem of intense beam transport in a combined FODO structure. Accordingly, numerical and analytical treatments of high-brightness beam dynamics with suppressed space-charge induced halo formation are presented.

INTRODUCTION

Formation of a beam halo is a key issue for many existing and proposed accelerator projects. Small beam losses in the linac produce radio-activation, which degrade accelerator components, compromising their reliability as well as hinder or prevent hands-on maintenance. Traditional accelerator designs utilize linear focusing elements (quadrupoles, solenoids) to provide stable particle motion. High intensity non-uniform beams are intrinsically mismatched with such structures, which result in beam emittance growth and halo formation. In Ref. [1] it was proposed to use a higher-order multipole (duodecapole) component in a quadrupole focusing-defocusing (FD) channel to prevent halo formation and emittance growth of a space-charge-dominated beam. The performed analysis can be extended to a FODO quadrupole structure with arbitrary multipole lenses.

FODO STRUCTURE WITH HIGHER-ORDER MULTIPOLES

Consider a FODO quadrupole focusing structure with inserted multipole lenses (see Fig. 1). The beamline can be treated as a superposition of two focusing structures with the same period L, consisting of quadrupole lenses with a gradient $G_2 = B_{pole} / R_{pole}$, and a higher-order multipoles with a field gradient $G_m = B_{pole} / R_{pole}^{m-1}$, shifted along the longitudinal coordinate, z, by the distance Δ. The index m is related to number of poles, $2m$, required to excite the corresponding multipole i.e. m = 3 for sextupole, m = 4 for octupole, m = 5 for decapole, m = 6 for duodecapole, etc. The Lorentz force acting on a charged particle, arising from the magnetic field of the combined structure can be represented as

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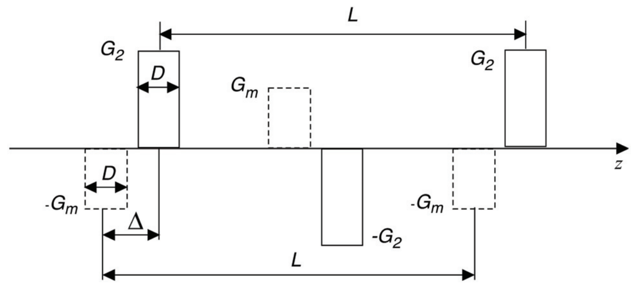

Figure 1. Combined FODO stricture with quadrupoles $_2G$ and multipoles G_m lenses.

$$\vec{F} = v_z[-\vec{i}_x B_y(2) + \vec{i}_y B_x(2)]G(z) + v_z[-\vec{i}_x B_y(m) + \vec{i}_y B_x(m)]G(z-\Delta), \quad (1)$$

where $v_z = \beta c$ is the beam velocity, $B_x(2)$, $B_y(2)$ are field components of the quadrupole lenses, $B_x(m)$, $B_y(m)$ are field component of the multipole lenses, and $G(z)$ is the longitudinal field dependence expanded in Fourier series:

$$G(\xi) = \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{2n-1}\sin(2n-1)\pi\frac{D}{L}\ \sin 2\pi(2n-1)\frac{\xi}{L} . \quad (2)$$

According to the averaging method [2], particle motion in an oscillating field

$$\ddot{\vec{r}} = \frac{q}{m\gamma}\vec{F}(\vec{r},t) , \quad (3)$$

$$\vec{F}(\vec{r},t) = \sum_{n=1}^{\infty}[\vec{F}_n^s(\vec{r})\sin(\omega_n t) + \vec{F}_n^c(\vec{r})\cos(\omega_n t)] , \quad (4)$$

can be approximated by the following Hamiltonian:

$$H = \frac{\dot{\vec{R}}^2}{2} + \frac{q^2}{4(m\gamma)^2}\sum_{n=1}^{\infty}\frac{(\vec{F}_n^s)^2 + (\vec{F}_n^c)^2}{\omega_n^2} . \quad (5)$$

Calculation of the potential part of the Hamiltonian, Eq. (5), gives:

$$U_{eff} = (\frac{\mu_o \beta c}{L})^2[\frac{r^2}{2} + f\varsigma r^m\cos(m-2)\theta + \varsigma^2\frac{r^{2(m-1)}}{2}], \quad (6)$$

where μ_o is the phase advance of transverse oscillations attained within a single period of FODO quadrupole channel:

$$\mu_o = \frac{L}{2D}\sqrt{1-\frac{4}{3}\frac{D}{L}\frac{qG_2D^2}{mc\beta\gamma}}\,, \qquad (7)$$

ς is the ratio of field components:

$$\varsigma = \frac{G_m}{G_2}\,, \qquad (8)$$

and the function f depends on the mutual positions of multipole lenses with respect to the quadrupoles:

$$f = \frac{1-\frac{4}{3}\frac{D}{L}-4\frac{\Delta^2}{LD}(1-\frac{1}{3}\frac{\Delta}{D})}{1-\frac{4}{3}\frac{D}{L}}\,, \quad \Delta < D\,, \qquad (9)$$

$$f = \frac{1-4\frac{\Delta}{L}}{1-\frac{4}{3}\frac{D}{L}}\,, \quad \Delta > D\,. \qquad (10)$$

Note that $f = 0$ for $\Delta = L/4$, i.e. when multipole lenses are placed in the centers of drift spaces of FODO structure between quadrupole lenses. In this case, effective potential does not depend on the azimuthal angle:

$$U_{eff}(r) = (\frac{\mu_o\beta c}{L})^2[\frac{r^2}{2}+\varsigma^2\frac{r^{2(m-1)}}{2}]\,. \qquad (11)$$

Similar problem for $m = 3$ was treated in Ref. [3].

In Ref [4] it was shown, that self-consistent potential of the stationary high-brightness, space-charge dominated beam, U_b, is opposite to external focusing potential:

$$U_b = -\frac{\gamma^2}{1+\delta}U_{eff}\,, \qquad (12)$$

where $\delta \approx b^{-1}$ is a small parameter inversely proportional to the dimensionless beam brightness $b = (2/\beta\gamma)(I_o/I_c)(R/\varepsilon)^2$, I_o is the beam current, $I_c = 4\pi\varepsilon_o mc^3/q =$ 3.13 x 10^7 (A/Z) [Amp] is the characteristic beam current, R is the beam size and ε is the normalized beam emittance. Space charge density distribution of a stationary beam is determined from Poisson's equation:

$$\rho(r,\theta) = -\varepsilon_o[\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial U_b}{\partial r})+\frac{1}{r^2}\frac{\partial^2 U_b}{\partial\theta^2}]\,. \qquad (13)$$

Taking into account Eqs. (6), (12), (13), the space charge distribution of the matched stationary beam in the considered structure is:

$$\rho(r,\theta) = \rho_o[1+\varsigma^2(m-1)^2 r^{2(m-2)}$$

$$+2(m-1)f\varsigma r^{(m-2)}\cos(m-2)\theta]\,. \qquad (14)$$

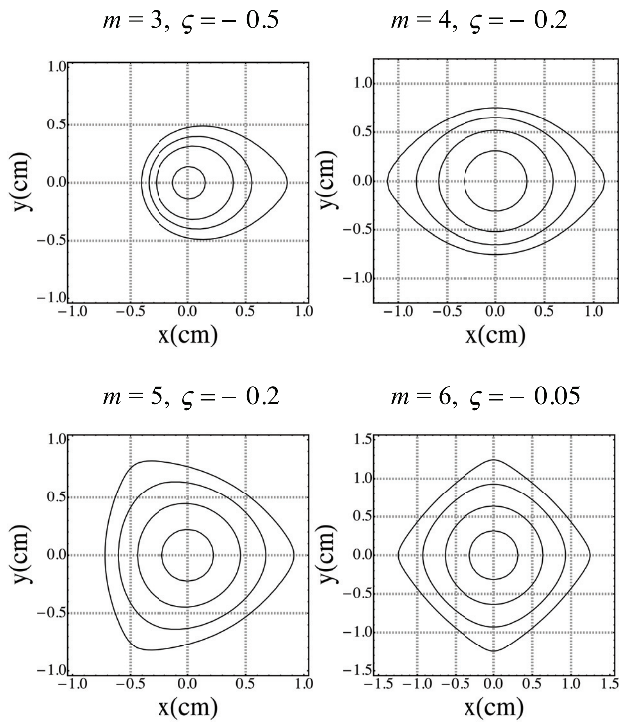

Figure 2. Equipotential lines of the effective potential, Eq. (6), in a quadrupole-multipole focusing channel.

Fig. 2 illustrates a family of equipotential lines of effective potential, Eq. (6), in the considered beamline for different multipole lenses incorporated into quadrupole lenses, $\Delta = 0$. Equipotentials are functions of radius and azimuthal angle. Analysis shows, that among all presented cases, the quadrupole-duodecapole channel ($m = 6$) provides the best matching for the beam being cut along equipotential lines.

Fig. 3 illustrates the results of BEAMPATH simulation of a 35 keV, 11.7 mA, 0.045 π cm mrad proton beam in a FODO quadrupole channel. A space-charge-dominated beam with an initially parabolic distribution function

$$f = f_o\{1-\frac{1}{2}[\frac{x^2}{R_x^2}+\frac{y^2}{R_y^2}+\frac{p_x^2}{(\varepsilon/R_x)^2}+\frac{p_y^2}{(\varepsilon/R_y)^2}]\}\,, \qquad (15)$$

and with the ratio of depressed - to - undepressed betatron tune shift of $\mu/\mu_o = 0.4$ is a subject of strong emittance growth and halo formation. Fig. 4 illustrates the dynamics of the beam with the same parameters in a FODO structure with combined quadruple-duodecapole lenses, correspondig to $\Delta = 0$. The quadrupole gradient is kept constant along the structure while the duodecapole component gradually decreases from its nominal value to zero at a distance of 7 FODO periods. The injected beam with the same distribution, Eq. (15), was truncated along equipotential lines of effective potential, Eq. (6).

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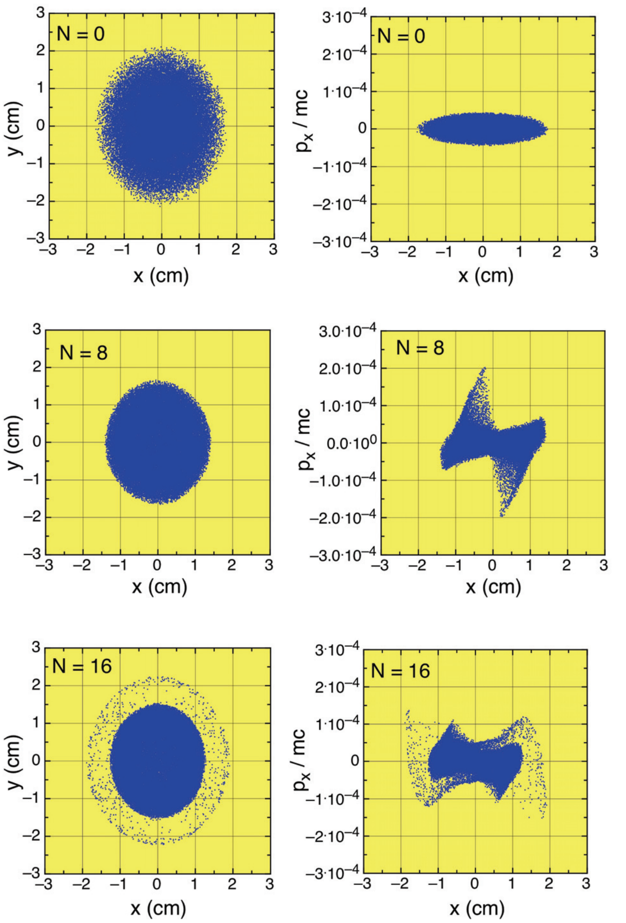

Figure 3. Emittance growth and halo formation of the 35 keV, 11.7 mA, 0.045 π cm mrad proton beam in a FODO quadruple channel with the period of L = 15 cm, lens length of D = 5 cm, and quadrupole field gradient of G_2 = 0.03579 T/cm. Numbers indicate FODO periods.

Adiabatic decline of the duodecapole component results in transformation of truncated, non-uniform beam, into a beam, matched with the structure. Such matching provides significant suppression of halo formation. Fig. 4 further illustrates the fraction of particles outside the elliptical area of the beam core $2.5\sqrt{<x^2>}\ x\ 2.5\sqrt{<y^2>}$ for both cases. The fraction of halo particles oscillates along the structure, with significantly reduced number of halo particles in the quadruple-duodecapole structure, than in a pure quadruple channel.

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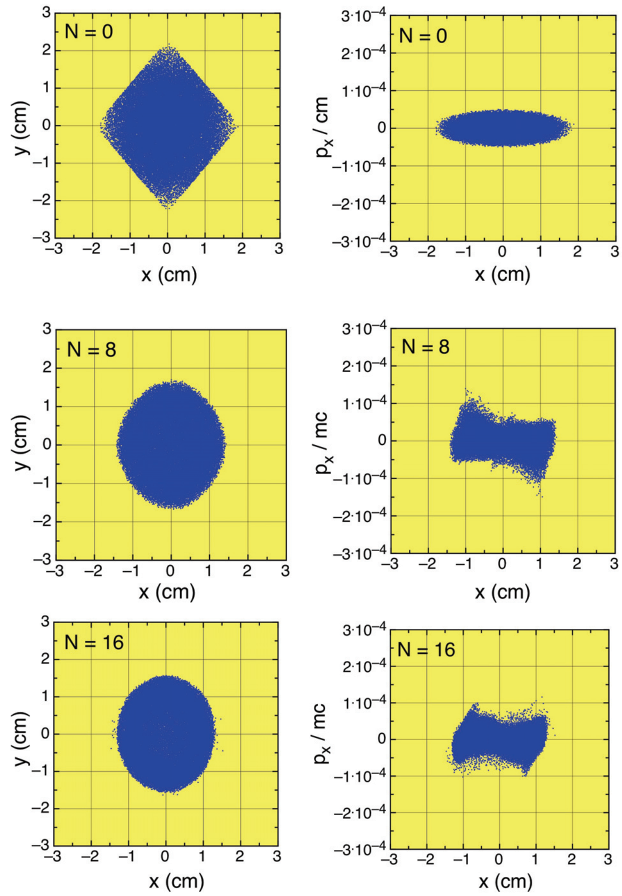

Figure 4. Adiabatic matching utilized to avoid halo formation of a 35 keV, 11.7 mA, 0.045 π cm mrad proton beam in a FODO quadruple-dodecapole channel. The channel is characterized by the period of L = 15 cm, lens length of D = 5 cm, quadrupole field gradient of G_2 = 0.03579 T/cm and adiabatic decline of duodecaple component from $G_6 = -1.756 \cdot 10^{-4}$ T/cm^5 to zero at the distance of 7 periods. Numbers indicate FODO periods.

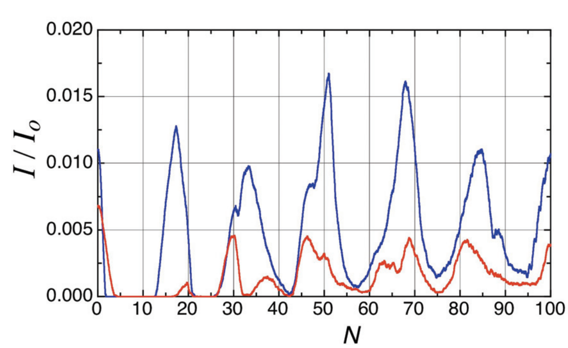

Figure 5. Fraction of particles outside the beam core $2.5\sqrt{<x^2>}\ x\ 2.5\sqrt{<y^2>}$ as a function of FODO periods: (blue) quadrupole channel, (red) quadrupole-duodecapole channel.