DESIGN ISSUES OF LOW ENERGY BEAM TRANPORT

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Abstract

Low energy beam transport (LEBT) is an important element of accelerator facilities used to provide beam matching between particle source and accelerator structure, perform required beam diagnostics measurements, dispose extra particle components, and create necessary time structure of the beam. Most existing ion LEBT are based on solenoid focusing. On this paper we discuss the matched design criteria for ion LEBT with magnetostatic focusing. Specifically, we show how the dynamics in LEBT can be optimized in terms of maximizing acceptance of the channel and transported beam current.

LATTICE OF PERIODIC SOLENOID CHANNEL

Consider a focusing lattice consisting of a periodic sequence of focusing solenoids of length *D*, field *B*, distance between lenses *l*, and period *L*= *l*+*D* (see Fig. 1). A matched beam reaches it's maximum size in the center of solenoids, and minimum size in the middle of drift space (see Fig. 2). The transformation matrix in rotating frame through a period of the structure between centers of solenoids is given by

$$\begin{pmatrix} \cos\frac{\theta}{2} & \frac{D}{\theta}\sin\frac{\theta}{2} \\ -\frac{\theta}{D}\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{pmatrix} \begin{pmatrix} 1 & l \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\frac{\theta}{2} & \frac{D}{\theta}\sin\frac{\theta}{2} \\ -\frac{\theta}{D}\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{pmatrix}$$

$$= \begin{pmatrix} \cos\theta-\frac{l}{2D}\theta\sin\theta & \frac{D}{\theta}\sin\theta+l\cos^2\frac{\theta}{2} \\ -\frac{\theta}{D}\sin\theta+l(\frac{\theta}{D})^2\sin^2\frac{\theta}{2} & \cos\theta-\frac{l}{2D}\theta\sin\theta \end{pmatrix}, \quad (1)$$

where $\theta = qBD/(2mc\beta\gamma)$ is the rotational angle of particle trajectory in a solenoid. The matrix of transformation through the period of the structure between centers of drift space is:

$$\begin{pmatrix} \cos\theta-\frac{l}{2D}\theta\sin\theta & l\cos\theta-\frac{l^2\theta}{4D}\sin\theta+D\frac{\sin\theta}{\theta} \\ -\frac{\theta}{D}\sin\theta & \cos\theta-\frac{l}{2D}\theta\sin\theta \end{pmatrix}. \quad (2)$$

From the matrices (1) and (2), the value of betatron tune shift per period, μ_o, is determined by

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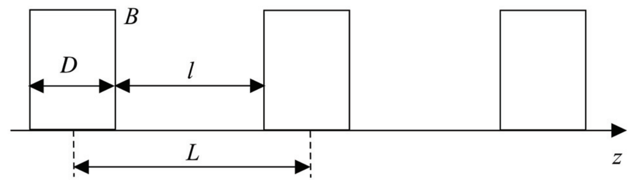

Figure 1: Periodic structure of focusing solenoids.

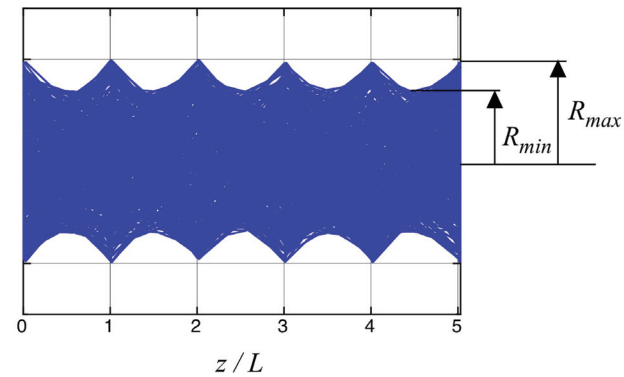

Figure 2: Matched beam in periodic focusing structure.

$\cos\mu_o = \cos\theta - \theta\sin\theta(L-D)/(2D)$. Adopting the expansions $\cos\xi = 1-\xi^2/2+\xi^4/24$ and $\sin\xi = \xi-\xi^3/6$, the value of betatron tune shift per period reads:

$$\mu_o = \theta\sqrt{\frac{L}{D}}\sqrt{1-\frac{\theta^2}{6}[1-\frac{1}{2}(\frac{D}{L}+\frac{L}{D})]}. \quad (3)$$

Thus, the maximum and minimum values of beta-function $\beta_{max/min} = m_{12}/\sin\mu_o$ in the channel are given by:

$$\beta_{max} = \frac{L\cos^2\frac{\theta}{2}[1-\frac{D}{L}(1-\frac{\tan\theta/2}{(\theta/2)})]}{\sin\mu_o}, \quad (4)$$

$$\beta_{min} = \frac{(L-D)\cos\theta-\frac{(L-D)^2\theta}{4D}\sin\theta+D\frac{\sin\theta}{\theta}}{\sin\mu_o}. \quad (5)$$

Eqs. (4), (5) determine the maximum $R_{max} = \sqrt{\beta_{max}\ni}$ and minimum $R_{min} = \sqrt{\beta_{min}\ni}$ matched envelope of the beam with unnormalized emittance of $\ni$ and negligible beam current, *I* = 0. Acceptance of the channel with aperture radius of *a* is given by $A = a^2/\beta_{max}$.

If the length of the lens is significantly smaller than the period of the structure, $D/L << 1$, focusing properties of the solenoid can be represented by a thin lens with a

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focal length of $f = D/\theta^2$. Phase advance of the structure is simplified by the expressions $\cos\mu_o \approx 1-\theta^2 L/(2D)$, from which the phase advance is determined as $\mu_o \approx \theta\sqrt{L/D}$. From the condition $|\cos\mu_o| \le 1$, the stability criteria for particle oscillations is expressed as $0 \le L \le 4f$. Acceptance of the channel is simplified as $A \approx (a^2/L)\sin\mu_o$ and has a maximum at the value of $\mu_o \approx \pi/2$, or $f = L/2$ [1]. In this case, the ratio of matched beam sizes is $R_{\max}/R_{\min} = \sqrt{2}$.

AVERAGED ENVELOPE EQUATION FOR SPACE-CHARGE-DOMINATED BEAM

Analysis presented above gives us the ability to match a beam with negligible current. For analysis with non-zero current, $I \ne 0$, let us use the KV envelope equation for a round beam envelope $R(z)$ in an axially-symmetric channel:

$$\frac{d^2R}{dz^2} - \frac{\ni^2}{R^3} + k(z)R - \frac{P^2}{R} = 0\,, \qquad (6)$$

where $P^2 = 2I/(I_c\beta^3\gamma^3)$ is the generalized beam perveance, $I_c = 4\pi\varepsilon_o mc^3/q =$ $3.13\times10^7 A/Z$ [Amp] is the characteristic beam current, and $k(z) = [qB(z)/(2mc\beta\gamma)]^2$ is the focusing term. The square of magnetic field along the structure can be expanded as Fourier series:

$$B^2(z) = B^2[\frac{D}{L} + \frac{2}{\pi}\sum_{k=1}^{\infty}\frac{\sin(\pi kD/L)}{k}\cos(2\pi kz/L)]\,. \qquad (7)$$

Taking advantage of this expansion, the solution of Eq. (6) in smooth approximation i.e. $R(z) = R_{aver}(z)[1+\vartheta(z)]$, can be represented as a combination of a slow variable $R_{aver}(z)$ and a quickly oscillating component $\vartheta(z)$. The slow component is described by the envelope equation in a continuous focusing field with constant focusing frequency μ_o:

$$\frac{d^2R_{aver}}{dz^2} - \frac{\ni^2}{R_{aver}^3} + \mu_o^2 R_{aver} - \frac{P^2}{R_{aver}} = 0\,, \qquad (8)$$

while the rapidly varying component is defined by the oscillating focusing term of the envelope equation:

$$\vartheta(z) \approx \frac{\mu_o^2}{2\pi^2}\frac{\sin(\pi D/L)}{(\pi D/L)}\sin(2\pi z/L)\,. \qquad (9)$$

The maximum value of the fast oscillating function, $\vartheta_{\max} = (\mu_o^2/2\pi^2)\sin(\pi D/L)/(\pi D/L)$, determines the minimum and maximum matched beam envelope in presence of space charge:

$$R_{\max/\min} = R_{aver}(1 \pm \frac{\mu_o^2}{2\pi^2}\frac{\sin(\pi D/L)}{(\pi D/L)})\,, \qquad (10)$$

where $R_{aver} = R_{aver}(0)\sqrt{b_o + \sqrt{1+b_o^2}}$ is the average matched beam envelope, $R_{aver}(0) = \sqrt{\ni L/\mu_o}$ is the matched average beam size with negligible space charge, and $b_o = (\beta\gamma)^{-3}(I/I_c)(R_{aver}(0)/\ni)^2$ is the space charge parameter. The envelope equation gives an approximation to the acceptance of the channel:

$$A_{env} = \frac{a^2\mu_o}{L(1+\vartheta_{\max})^2}\,, \qquad (11)$$

and to the maximum beam current:

$$I_{\max} = \frac{I_c}{2}\frac{\mu_o}{L}A_{env}(\beta\gamma)^3[1 - (\frac{\ni}{A_{env}})^2]\,. \qquad (12)$$

The validity of the above formulae is largely determined by the validity of the smooth approximation in the envelope equation, which holds for the values of $\mu_o \le 60^o$.

SPACE-CHARGE LIMITED BEAM CURRENT

Eq. (12) gives an approximate value of the maximum beam current in a periodic structure. To determine a more exact value for space - charge limited beam current, consider beam transport in drift space between lenses described by envelope equation (6) without a focusing term:

$$\frac{d^2R}{dz^2} - \frac{\ni^2}{R^3} - \frac{P^2}{R} = 0\,. \qquad (13)$$

Equation (13) can be integrated [2]:

$$(\frac{dR}{dz})^2 = (\frac{dR}{dz})_o^2 + (\frac{\ni}{R_o})^2(1-\frac{R_o^2}{R^2}) + P^2\ln(\frac{R}{R_o})^2\,. \qquad (14)$$

Now consider the space charge dominated regime, where beam emittance can be neglected, $\ni \approx 0$. At the middle point between lenses, $z = z_o$, the beam has a waist size $R_o = R_{min}$ and zero divergence, $R_o' = 0$. Thus, equation (14) can be rewritten in this case as $(d\bar{R}/dZ)^2 = \ln\bar{R}$, where $\bar{R} = R/R_{\min}$, and $Z = \sqrt{2}\,zP/R_{\min}$. Consequently, expansion of beam radius in drift space from $\bar{R} = 1$ to $\bar{R}_{\max} = R_{\max}/R_{\min}$ is determined by the integral:

$$\frac{1}{\bar{R}_{\max}}\int_1^{\bar{R}_{\max}}\frac{d\bar{R}}{\sqrt{\ln\bar{R}}} = \sqrt{2}P\frac{(z-z_o)}{R_{\max}}\,. \qquad (15)$$

The left hand side of Eq. (15) has a maximum value of 1.082 for $\bar{R}_{\max}$ = 2.35 [3]. As already alluded to above, the maximum radius is achieved in the channel at $z - z_o = L/2$, which in turn yields $P_{\max} L/(\sqrt{2}R_{\max}) = 1.082$. From this expression, the maximum transported current in the channel is

$$I_{\lim} = 1.17 I_c (\beta\gamma)^3 (\frac{R_{\max}}{L})^2 . \quad (16)$$

The divergence of the beam at the lens can be estimated from Eq. (14):

$$\frac{dR_{\max}}{dz} = \sqrt{\frac{4 I_{\lim}}{I_c (\beta\gamma)^3} \ln(\frac{R_{\max}}{R_{\min}})} \approx 2\frac{R_{\max}}{L} . \quad (17)$$

Total change in slope of the beam envelope at the lens has to be equal to twice the value of $dR_{\max}/dz$ determined by Eq. (17). Therefore, the required focal length of the lenses is $f \approx L/4$, and the maximum space charge limited beam current is achieved in a structure where $\mu_o \approx 180^o$. Such transports are usually unstable, and can be used only with a limited number of focusing elements.

APPLICATION TO INJECTOR LEBT

Most existing ion LEBTs utilize 2 or 3 solenoids with intermediate equipment (deflectors, bending magnets, Wien filters, emittance stations) to match the beam from the exit of ion source column to the subsequent RF structure. Consider a LEBT comprised of 2 solenoids, separated by a distance L (see Fig. 3). The beam is characterized by a certain emittance $\backepsilon$ and effective current $I = I_o(1-\eta)$, where I_o is the total beam current and η is the space-charge neutralization factor. Initial envelope parameters R_s, R'_s are determined by extraction conditions from the ion source column. Final beam parameters R_f, R'_f, are determined by the matching conditions at the front end of the RF accelerator. The purpose of the design is then to find appropriate solenoid parameters, and distances d_1, d_2.

Analysis of the previous section allows us to select periodic matched beam envelopes, corresponding to minimal values of the beam size at the center of solenoid. Minimization of the beam size R_{max} allows us to minimize solenoid power consumption, beam losses, and space-charge induced beam emittance growth. Value of R_{max} can be approximately obtained from Eq. (10). More precise value of R_{max} is determined by variation of the value of R_{min} at the middle point between solenoids, $z = z_o$, and searching for the smallest value of the beam size at the center of solenoids via an exact solution of the envelope equation in drift space between solenoids (see Fig. 3). Then, the distances d_1, d_2 are defined by integration of equation (14) to establish points where the beam radius evolves from initial value of R_o to R_{max} [2]:

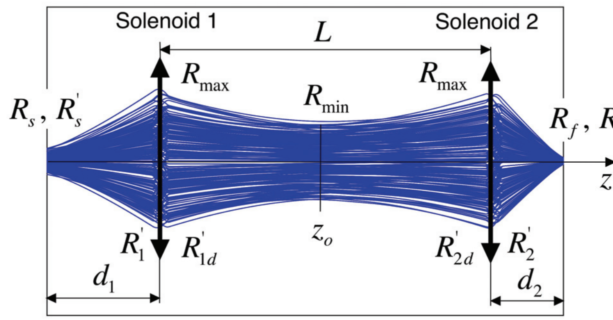

Figure 3: LEBT with two focusing solenoids.

$$z = \frac{R_o^2}{2\backepsilon}\int_1^{(\frac{R_{\max}}{R_o})^2} \frac{ds}{\sqrt{[1+(\frac{R_o R'_o}{\backepsilon})^2]s + (\frac{P R_o}{\backepsilon})^2 s \ln s - 1}} . \quad (18)$$

In Eq. (18), the values of R_o, R'_o correspond to either R_s, R'_s or R_f, R'_f. Slopes of beam envelopes at solenoids R'_1, R'_2 can be found from Eq. (14):

$$R' = \sqrt{(R'_o)^2 + (\frac{\backepsilon}{R_o})^2[1-(\frac{R_o}{R})^2] + \frac{2I}{I_c(\beta\gamma)^3}\ln(\frac{R}{R_o})^2} . \quad (19)$$

The values of R'_{1d}, R'_{2d} are determined by Eq. (19) assuming $R_o = R_{\min}$, $R'_o = 0$. Then, focal lengths of solenoids f_1, f_2, are determined by the total change in the slope of the beam at each solenoid:

$$f_1 = \frac{R_{\max}}{|R'_{1d}| + |R'_1|} , \qquad f_2 = \frac{R_{\max}}{|R'_{2d}| + |R'_2|} . \quad (20)$$

After that, the magnetic field within each solenoid is determined by $B = 2mc\beta\gamma/(q\sqrt{f D})$.

SUMMARY

In this work, we have determined matched beam transport conditions for a periodic structure of focusing solenoids in both, emittance-dominated and space-charge-dominated regimes. A closed-form expression for the maximal limited beam current in the considered structure is obtained. The developed analysis is subsequently applied to the problem of beam matching in a typical LEBT with two solenoids.

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