BEAM EMITTANCE GROWTH EFFECTS IN HIGH-INTENSITY RFQ

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Abstract

Beam dynamics in an RFQ are strongly affected by coupling between transverse and longitudinal particle oscillations. The adiabatic process of high-intensity bunched beam formation results in equipartitioning in the RFQ, which determines the longitudinal beam emittance. Avoiding parametric resonances is an important design criterion to prevent significant emittance growth of the beam. Manufacturing errors can result in beam emittance growth and reduction of beam transmission. This paper will present the results of a study where analytical and numerical evaluations were performed to determine the effect of the aforementioned factors on beam quality in a high-current RFQ.

BEAM EQUIPARTITIONING

The Hamiltonian of the averaged particle motion in an RF linac is given by [1]:

$$H = \frac{p_x^2 + p_y^2 + p_z^2}{2m} + qU_{ext} + qU_b \qquad (1)$$

$$U_{ext} = \frac{U_L T}{\pi}[I_o(k_z r)\sin(\varphi_s - k_z\zeta) + k_z\zeta\cos\varphi_s] + \frac{m\Omega_r^2 r^2}{2q} \qquad (2)$$

where p_x and p_y are transverse momenta, $p_z = P_z - P_s$ is the deviation of longitudinal momentum from the momentum of the synchronous particle, $\zeta = z - z_s$ is the longitudinal deviation from the synchronous particle, U_L is the intervane voltage, $T = (\pi/4)A$ where A is the efficiency of acceleration, $k_z = 2\pi/\beta\lambda$ is the wave number, φ_s is the synchronous phase, and Ω_r is the transverse oscillation frequency without vane modulation:

$$\Omega_r = \frac{\omega}{\sqrt{2\pi^2}}\chi\frac{qU_L}{mc^2}(\frac{\lambda}{2a})^2 \qquad (3)$$

where $\omega = 2\pi c/\lambda$ is the circular frequency, χ is the focusing efficiency, a is the radius of aperture, and U_b is the space charge potential.

The self-consistent solution for a stationary matched beam distribution can be expressed as a function of the Hamiltonian. A convenient way to do so is to use an exponential function of the form $f = f_o(-H/H_o)$:

$$f = f_o\exp(-\frac{p_x^2 + p_y^2 + p_z^2}{2mH_o} - \frac{qU_{RFQ}}{H_o} - \frac{qU_b}{H_o}). \qquad (4)$$

Eq. (4) indicates that the rms momentum spread in a stationary beam distribution is the same in all 3 directions $mH_o = \langle p_x^2\rangle = \langle p_y^2\rangle = \langle p_z^2\rangle$, from which the equipartitioning condition [2] can be derived:

$$\eta = \frac{\varepsilon_z R}{\varepsilon_t R_z} = 1 \qquad (5)$$

where $\varepsilon_x = \varepsilon_y = \varepsilon_t$, and ε_z are normalized transverse and longitudinal beam emittances, and $R_x = R_y = R$, R_z are averaged bunch sizes. In an RFQ linac, continuous beam is initially matched in the radial direction and a well-bunched beam is formed during adiabatic increase of the vane modulation. In the presence of sufficient collective interaction through space charge, the bunch evolves into a equilibrium state where the equipartitioning condition, Eq. (5), is fulfilled.

Figs. 1 - 3 illustrate BEAMPATH beam dynamics simulation results for an RFQ linac designed using PARMTEQM for acceleration of protons from 35 keV to 750 keV. Fig. 3 shows the variation in the equipartitioning parameter, Eq. (5), along the RFQ structure for different values of beam current. At negligible beam current, the beam occupies all available area in longitudinal phase space (see Fig. 2). With high values of beam current, the space charge field of the beam provides strong coupling between degrees of freedoms and results in an adjustment of the longitudinal beam emittance such that the equipartitioning condition is $\varepsilon_z = \varepsilon_t(R_z/R)$ is fulfilled. This results in the longitudinal beam emittance for space-charge dominated beams being smaller than that of an emittance-dominated beam (see Fig. 2). For the considered case, equipartitioning appears at values of beam current $I > 20$ mA. Within the range of $0 < I < 20$ mA, equipartitioning is not observed because of insufficient interaction between particles (see Fig. 3).

TRANSVERSE PARAMETRIC RESONANCE

The transverse particle motion in an RF linac is affected by coupling with longitudinal motion because of the dependence of the RF defocusing force on the RF phase. Neglecting space-charge forces, the transverse particle oscillation in an RF field in the smooth approximation is determined by equation [1]

$$\frac{d^2x}{dt^2} + x[\Omega_{rs}^2 - \frac{\Omega^2}{2}\text{ctg}\varphi_s\Phi\sin(\Omega t + \psi_o)] = 0 \qquad (6)$$

where Φ is the amplitude and Ω is the frequency of longitudinal oscillations:

$$\Omega^2 = \omega^2\frac{qU_L T}{\pi}\frac{|\sin\varphi_s|}{m(\beta c)^2}. \qquad (7)$$

The transverse oscillation frequency of a synchronous particle in the presence of an RF field is given by:

$$\Omega_{rs}^2 = \Omega_r^2 - \frac{\Omega^2}{2}. \qquad (8)$$

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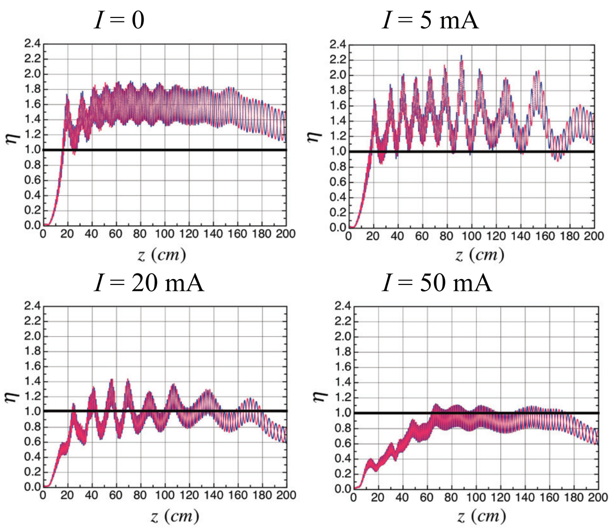

Figure 1. Equipartitioning parameter, Eq. (5), along RFQ structure for different values of beam current.

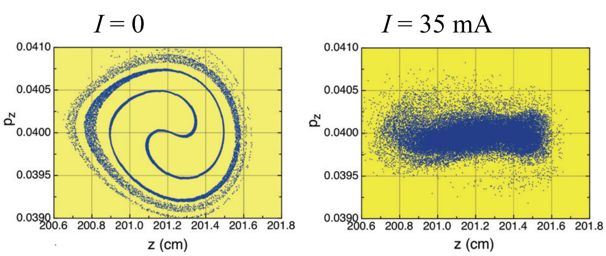

Figure 2. Output longitudinal phase space.

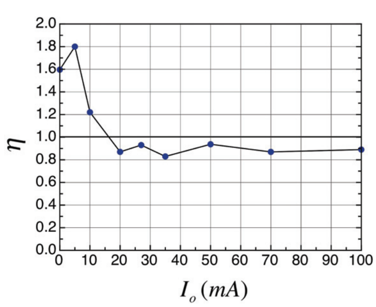

Figure 3. Equipartitioning parameter at the end of RFQ buncher section as a function of beam current.

Because of the periodic variation of the transverse oscillation frequency with longitudinal frequency, parametric resonance occurs when $\Omega_{rs}=(n/2)\,\Omega$, n = 1, 2, 3… The regions of parametric instability are [3]

$$\frac{\sqrt{b_n}}{2}<\frac{\Omega_{rs}}{\Omega}<\frac{\sqrt{a_n}}{2} \qquad (9)$$

where for the first two regions of instability, n = 1, 2, the parameters a_n, b_n are:

$$a_1=1+q-\frac{q^2}{8}-\frac{q^3}{64}\ , \qquad b_1=1-q-\frac{q^2}{8}+\frac{q^3}{64} \qquad (10)$$

$$a_2=4+\frac{5q^2}{12}-\frac{763q^4}{13824}, \qquad b_2=4-\frac{q^2}{12}+\frac{5q^4}{13824} \qquad (11)$$

and the parameter $q=\Phi/|tg\varphi_s|\approx\varphi_s/tg\varphi_s$.

In an RFQ linac, the transverse oscillation frequency is typically larger than the longitudinal oscillation frequency, and the first $n=1$ parametric resonance instability region is avoided. The potentially dangerous region in this case is the second parametric resonance bandwidth where n = 2. Instabilities of higher-order resonance regions are typically unimportant [1].

LONGITUDINAL PARAMETRIC RESONANCE

Injection of low-velocity particles into an RFQ results in dependence of the longitudinal oscillation frequency on transverse particle position. Neglecting space-charge forces, the equation of small-amplitude longitudinal oscillations for off-axis particles is given by [1]:

$$\frac{d^2\zeta}{dt^2}+\Omega^2 I_o(k_z r)\zeta=\frac{\Omega^2}{k_z|tg\varphi_s|}[I_o(k_z r)-1]\ . \qquad (12)$$

Averaged transverse oscillations can be approximated by $r=R\cos\Omega_{rs}t$. Periodic function $I_o(k_z R\cos\Omega_{rs}t)$ can be expanded in Fourier series [1]:

$$I_o(k_z R\cos\Omega_{rs}t)=I_o^2(\frac{k_z R}{2})+2\sum_{m=1}^{\infty}I_m^2(\frac{k_z R}{2})\cdot\cos 2m\Omega_{rs}t \qquad (13)$$

Because the amplitudes of the terms of the Bessel function drop off quickly, only the first two terms are important, resulting in the following equation of motion:

$$\frac{d^2\zeta}{dt^2}+\Omega^2\zeta[I_o^2(\frac{k_z R}{2})+2I_1^2(\frac{k_z R}{2})\cdot\cos 2\Omega_{rs}t]$$
$$=\frac{\Omega^2}{k_z\,tg\varphi_s}[I_o^2(\frac{k_z R}{2})-1+2I_1^2(\frac{k_z R}{2})\cos 2\Omega_{rs}t] \qquad (14)$$

Analysis of longitudinal parametric instabilities (see Refs. [1, 3]) includes (i) consideration of a Mathieu-type equation parametric resonance instability neglecting the right-side part of Eq. (14), and (ii) external resonances, taking into account the right-hand external driving force of Eq. (14). Longitudinal parametric resonances occur when the following condition is fulfilled:

$$\frac{\Omega_{rs}}{\Omega}=\frac{I_o(\frac{k_z R}{2})}{n} \qquad n=1,2,3,\ldots \qquad (15)$$

with the region of parametric instability defined as:

$$\frac{I_o^2(\frac{k_z R}{2})}{a_n}<(\frac{\Omega_{rs}}{\Omega})^2<\frac{I_o^2(\frac{k_z R}{2})}{b_n} \qquad (16)$$

where a_n, b_n are given by Eqs. (10), (11), and the parameter

$$q=(\frac{\Omega}{\Omega_{rs}})^2 I_1^2(\frac{k_z R}{2})\,. \qquad (17)$$

The first significant parametric resonance area is when n = 1. This leads to the following resonance bandwidth defined by Eq. (16):

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$$I_o^2(\frac{k_z R}{2}) - I_1^2(\frac{k_z R}{2}) < (\frac{\Omega_{rs}}{\Omega})^2 < I_o^2(\frac{k_z R}{2}) + I_1^2(\frac{k_z R}{2}). \quad (18)$$

An external resonance occurs when the transverse oscillation frequency is $\Omega_{rs} = (\Omega/2) I_o(k_z R/2)$. Both external and parametric resonances can be avoided simultaneously when $\Omega_{rs}/\Omega > I_o(k_z a/2)$ [1].

Fig.4 illustrates two different cases of RFQ beam dynamics. The RFQ presented on the left side, was designed to avoid parametric-resonance bandwidths, Eqs. (9), (18). The other RFQ shown on the right side of Fig. 4 was designed using a more conventional approach, resulting in the parametric resonance conditions being met at a certain distance along the RFQ (see Fig. 4b). This results in noticeable transverse emittance growth with respect to the first case, where the transverse emittance remains nearly constant (see Fig. 4c). In both cases, the beams eventually become equipartitioned (see Fig. 4d), with the longitudinal emittance also satisfying the equipartitioning condition, Eq. (5).

EFFECT OF MANUFACTURING ERRORS ON BEAM EMITTANCE

Manufacturing errors are a source of additional beam emittance growth and beam losses. The increase of the amplitude of longitudinal oscillations at each cell due to random errors can be described as an increase in the rms relative momentum spread within the bunch [4]:

$$\frac{<\delta p_z>^2}{P_z^2} = \frac{\pi^2}{2}(\frac{\Omega}{\omega})^4 ctg^2\varphi_s(<\frac{\delta U_L}{U_L}>^2 + <\frac{\delta T}{T}>^2)$$
$$+ \frac{\pi^2}{2}(\frac{\Omega}{\omega})^2 [1+\pi^2(\frac{\Omega}{\omega})^2] < \frac{\delta L}{L}>^2. \quad (19)$$

The increase of the transverse amplitude of particle oscillations at each cell is estimated as [4]

$$<\frac{\delta R}{R}>^2 = 2(<\frac{\delta r_o}{R}>^2 + <\frac{\delta U_L}{U_L}>^2 + 4<\frac{\delta R_o}{R_o}>^2) \quad (20)$$

where δr_o is the axis displacement, and δR_o is the error in average radius of the vane structure $R_o = a/\sqrt{\chi}$.

Simulations of beam dynamics in the RFQ with $U_L = 72.6\ kV$ (see Fig. 4a) were performed with the following parameters randomly varied and uniformly distributed within an interval of $[-\Delta, \Delta]$ at each RFQ cell: cell size L, aperture a, maximum deviation of electrodes from the axis ma, and deviation of electrodes from optical axis δx_o, δy_o. Values of the beam emittances at the end of the RFQ structure were compared with emittance values at the exit of the ideal structure. Fig. 5 illustrates the effect of transverse and longitudinal beam emittance growth as functions of the maximum manufacturing random error Δ. Results of the simulations indicate that an error $\Delta < 20$ µm results in acceptable emittance growth of 20% , while larger errors result in significant emittance growth.

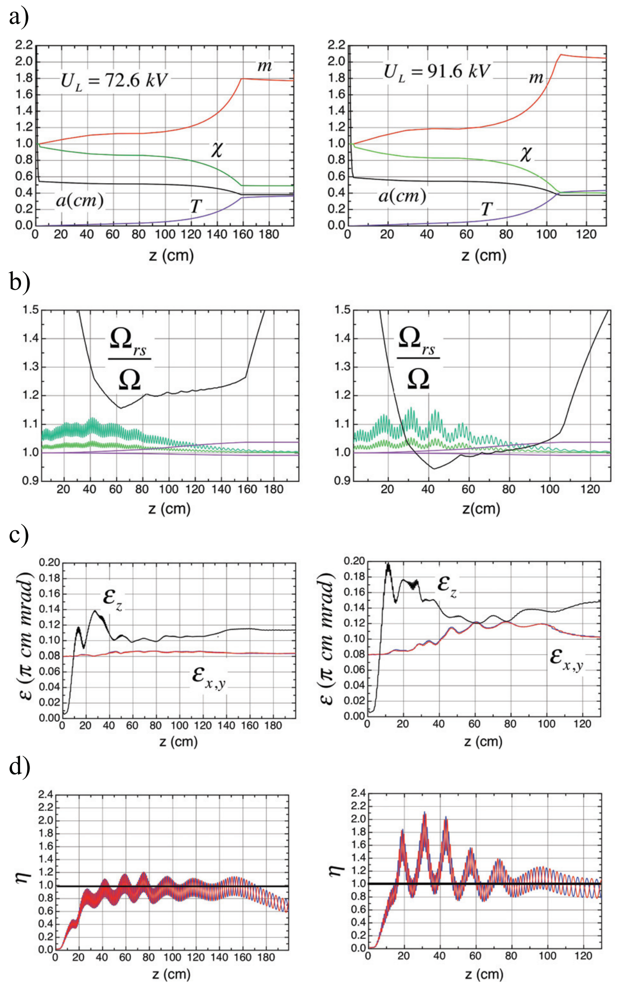

Figure 4. Beam dynamics in RFQ with beam current I=35 mA: (left) avoiding parametric resonances, (right) including parametric resonances: (a) RFQ parameters, (b) parametric resonance bandwidth: (green) Eq. (18), (red) Eq. (9), (c) beam emittances, (d) equipartitioning parameter, Eq. (5).

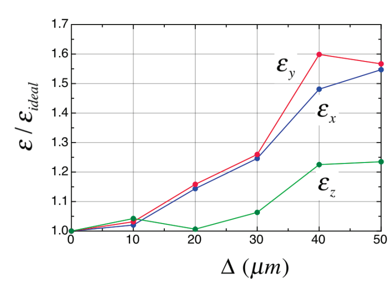

Figure 5. Effect of manufacturing errors on beam emittance.

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