DECOUPLING CAPABILITIES STUDY OF THE TRANSVERSE EMITTANCE TRANSFER SECTION*

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Abstract

Flat beams can be created successfully in electron machines by applying effective stand-alone solenoid fringe fields in the electron gun. Extension of this method to ion beams was proposed conceptually. The present paper is on the decoupling capabilities of an ion beam emittance transfer line. The proposed beam line provides a single-knob-tool to partition the horizontal and vertical emittances, while keeping the product of the two emittances constant as well as the transverse rms Twiss parameters $\alpha_{x,y}$ and $\beta_{x,y}$ in both planes. It is shown that this single knob is the solenoid field strength.

INTRODUCTION

From first principles beams are created round without any coupling among planes. Their rms emittances as well as their eigen-emittances are equal in the two transverse planes. Thus, any transverse round-to-flat transformation requires a change of the beam eigen-emittances by a non-symplectic transformation [1]. Such a transformation can be performed by placing a charge state stripper inside an axial magnetic field region as proposed in [2]. Inside such a solenoid stripper, transverse inter-plane correlations are created non-symplectically. Afterwards they are removed symplectically by a decoupling section including a regular quadrupole triplet and a skew quadrupole triplet.

The planned EMTEX (emittance transfer experiment) beam line for the demonstration of transverse rms emittance transfer is shown in Fig. 1. A quadrupole triplet and a skew quadrupole triplet separated by a drift space are employed to remove these correlation symplectically. The section from the solenoid exit to the skew triplet exit will be called decoupling section in the following. A final quadrupole triplet is used for matching to the existing beam line followed by a beam current transformer and an emittance measurement unit.

STRIPPING INSIDE A SOLENOID

Let C_0 denote the second moment matrix at the entrance of the solenoid. If the beam has equal horizontal and vertical rms emittances and no inter-plane correlations, the beam matrix can be simplified to

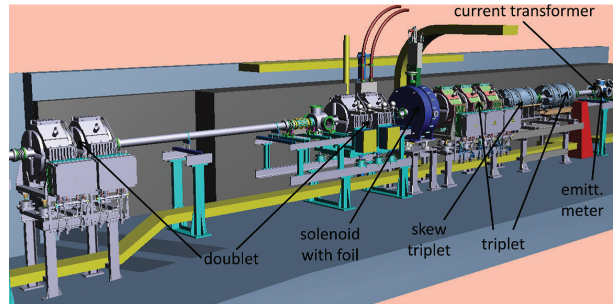

Figure 1: Layout of the EMTEX section.

$$C_0 = \begin{pmatrix} \varepsilon\beta & 0 & 0 & 0 \\ 0 & \frac{\varepsilon}{\beta} & 0 & 0 \\ 0 & 0 & \varepsilon\beta & 0 \\ 0 & 0 & 0 & \frac{\varepsilon}{\beta} \end{pmatrix}. \quad (1)$$

Assuming a very short solenoid, its transport matrix can be divided into two parts

$$R_{in,out} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & \pm k_{in,out} & 0 \\ 0 & 0 & 1 & 0 \\ \mp k_{in,out} & 0 & 0 & 1 \end{pmatrix} \quad (2)$$

If the beam has the same rigidity at the solenoid entrance and exit, k_{in} is equal to k_{out}. The first part describes the entrance fringe field and the second part is the exit fringe field. In here the focusing strength of the solenoid is

$$k = \frac{B}{2(B\rho)}. \quad (3)$$

B is the on-axis magnetic field strength and $B\rho$ is the beam rigidity. In order to change the eigen-emittances a non-symplectic transformation has to be integrated into the round-to-flat transformation section. The transformation through the solenoid is non-symplectic if the beam rigidity is abruptly changed in between the entrance and exit fringe fields, thus the beam properties are reset inside the solenoid. The non-symplectic transformation is accomplished for instance by changing the beam rigidity $B\rho$ in between the fringe fields from $(B\rho)_{in}$ to $(B\rho)_{out}$ through charge state stripping. Defining

$$\delta q := \frac{(B\rho)_{in}}{(B\rho)_{out}} \quad (4)$$

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the exit fringe field transfer matrix changes to

$$R'_{out} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -\delta qk & 0 \\ 0 & 0 & 1 & 0 \\ +\delta qk & 0 & 0 & 1 \end{pmatrix}. \quad (5)$$

The focusing strength of the solenoid k is calculated from the unstripped charge state. After the stripper the beam passes through the exit fringe field with reduced beam rigidity and the beam matrix C_1 after the exit fringe field becomes

$$C_2 = \begin{pmatrix} \varepsilon_n R_n & ak\varepsilon_n\beta_n J_n \\ -ak\varepsilon_n\beta_n J_n & \varepsilon_n R_n \end{pmatrix}, \quad (6)$$

where $a := \delta q - 1$ and

$$\varepsilon_n = \sqrt{\varepsilon\beta(\frac{\varepsilon}{\beta} + a^2k^2\varepsilon\beta + \Delta\varphi^2)}, \quad \beta_n = \frac{\beta\varepsilon}{\varepsilon_n}, \quad (7)$$

introducing the 2×2 sub-matrices R_n and J_n

$$R_n = \begin{pmatrix} \beta_n & 0 \\ 0 & \frac{1}{\beta_n} \end{pmatrix}, J_n = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \quad (8)$$

Inter-plane correlations are created and the rms emittances and eigen-emittances after the solenoid with stripper foil read

$$\varepsilon_{x,y} = \varepsilon_n, \quad \varepsilon_{1,2} = \varepsilon_n(1 \pm ak\beta_n). \quad (9)$$

and the four-dimensional rms emittance is

$$\varepsilon_{4d} = \varepsilon_1\varepsilon_2 = \varepsilon^2 + \varepsilon\beta\Delta\varphi^2. \quad (10)$$

The four-dimensional rms emittance increase is proportional to the beam sizes on the stripper foil. It is purely from scattering in the foil; it is not caused by the shift of beam rigidity inside the longitudinal magnetic field. The parameter t is introduced to quantify the inter-plane coupling

$$t = \frac{\varepsilon_x\varepsilon_y}{\varepsilon_1\varepsilon_2} - 1 \geq 0 \quad (11)$$

DECOUPLING SECTION

The simplest skew decoupling section contains three skew quadrupoles with appropriate betatron phase advances in each plane. Let R_q be the 4×4 matrix corresponding to a certain arrangement of quadrupoles and drift spaces and assume that this channel is represented by an identity matrix in the x-direction and has an additional 90° phase advance in y-direction as in [3]

$$R_q = \begin{pmatrix} I_n & O_n \\ O_n & T_n \end{pmatrix}. \quad (12)$$

Here the 2×2 sub-matrices O_n, T_n and I_n are defined as

$$O_n = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, T_n = \begin{pmatrix} 0 & u \\ -\frac{1}{u} & 0 \end{pmatrix}, I_n = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad (13)$$

If the quadrupoles are tilted by 45° the 4×4 transfer matrix can be written as

$$\overline{R} = R_r R_q R_r^T = \frac{1}{2}\begin{pmatrix} T_{n+} & T_{n-} \\ T_{n-} & T_{n+} \end{pmatrix}. \quad (14)$$

where

$$R_r = \frac{\sqrt{2}}{2}\begin{pmatrix} I_n & I_n \\ -I_n & I_n \end{pmatrix}, \quad T_{n\pm} = T_n \pm I_n. \quad (15)$$

The beam matrix C_3 after the decoupling section is

$$C_3 = \overline{R}C_2\overline{R}^T = \begin{pmatrix} \eta_+\Gamma_{n+} & \zeta\Gamma_{n-} \\ \zeta\Gamma_{n-} & \eta_-\Gamma_{n+} \end{pmatrix}, \quad (16)$$

and the 2×2 sub-matrices $\Gamma_{n\pm}$ are defined through

$$\Gamma_{n\pm} = \begin{pmatrix} u & 0 \\ 0 & \pm\frac{1}{u} \end{pmatrix}, \quad (17)$$

with

$$\eta_\pm = \frac{\varepsilon_n}{2}(\frac{\beta_n}{u} + \frac{u}{\beta_n} \pm 2ak\beta_n), \quad (18)$$

and

$$\zeta = \frac{\varepsilon_n}{2}(-\frac{\beta_n}{u} + \frac{u}{\beta_n}). \quad (19)$$

Assuming that this beam matrix is diagonal, its x-y component vanishes

$$\zeta\Gamma_{n-} = O_n \quad (20)$$

solved by

$$u = \beta_n. \quad (21)$$

This result was found earlier in [3] for instance. However, the major steps have been repeated here since they will be referred to later.

Suppose that the decoupling transfer matrix $\overline{R}$ is able to decouple the two transverse planes of C_2. We still do not know how this transfer beam line looks in detail, but anyway we calculate the final rms emittances obtaining

$$\varepsilon_{x,y} = \frac{\varepsilon_n}{2}(\frac{\beta_n}{u} + \frac{u}{\beta_n} \pm 2ak\beta_n). \quad (22)$$

For a given solenoid strength k_0, referring to the unstripped beam, the corresponding quadrupole gradients of the decoupling section are determined using a numerical routine, such that finally the rms emittances are equal to the eigen-emittances. If these optimized gradients are applied to remove inter-plane correlations produced by a different solenoid strength k_1, the resulting rms emittances and eigen-emittances at the exit of the decoupling section are calculated to be

$$\varepsilon_{x,y} = \frac{\varepsilon_n(k_1)}{2}\left[\frac{\beta_n(k_1)}{\beta_n(k_0)} + \frac{\beta_n(k_0)}{\beta_n(k_1)} \pm 2ak_1\beta_n(k_1)\right] \quad (23)$$

and

$$\varepsilon_{1,2} = \varepsilon_n(k_1)\left[1 \pm ak_1\beta_n(k_1)\right] \quad (24)$$

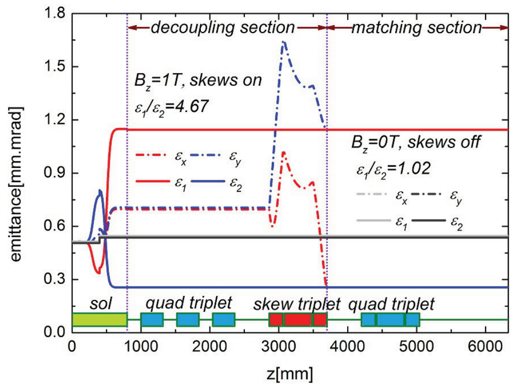

Figure 2: Evolution of the rms emittances and eigen-emittances along the EMTEX beam lines for two cases.

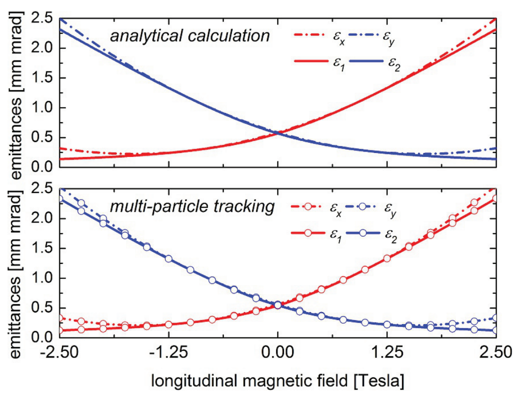

Figure 3: Eigen-emittances and rms emittances calculated by analytical method and by multi-particle tracking through the EMTEX beam line.

with the parameter t

$$t = \frac{a^4\,\varepsilon^2\beta^2}{(\frac{\varepsilon}{\beta}+\Delta\varphi^2)(\frac{\varepsilon}{\beta}+a^2k_0^2\varepsilon\beta+\Delta\varphi^2)}\frac{(k_1^2-k_0^2)^2}{4}\,. \quad (25)$$

In the same way the rms Twiss parameters of a beam coupled by k_1 but decoupled by $\overline{R}(k_0)$ are found from Eq. (16) as

$$\tilde{\alpha}_x = \tilde{\alpha}_y = 0, \quad \tilde{\beta}_x = \tilde{\beta}_y = \beta_n(k_0)\,, \quad (26)$$

showing that the rms Twiss parameters after decoupling do not depend on the coupling solenoid strength k_1 if the decoupling section was set assuming a coupling strength k_0. EMTEX beam line uses more elements than a single skew triplet because of finite apertures and gradients of a real experiment. Fig. 2 illustrates the multi-particle beam dynamics simulations of the transverse emittance transfer beam line. The final eigen-emittances and rms emittances at the exit of the skew quadrupole triplet calculated using Eq. (24) and those obtained from tracking through EMTEX are compared in Fig. 3. The remarkable result is that both decoupling matrices work effectively for a wide range of longitudinal magnetic field values, i.e. the beam is well decoupled for a wide range of longitudinal magnetic fields

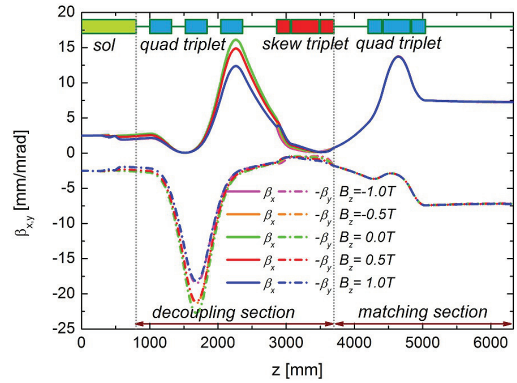

Figure 4: Horizontal and vertical beta-functions of the beam along the EMTEX beam line for different solenoid field strengths.

around the gradients the quadrupoles have been optimized for [4]. To exclude that this is casual for this one beam line, the beam line has been modified by prolonging or shortening drifts and quadrupole field lengths. For all modifications (all using a regular quadrupole triplet followed by a skew quadrupole triplet) the same behavior of decoupling performance was observed.

Another convenient feature, which can be explained for the generic case of decoupling according to Eq. (24), seems to manifest as a general rule in numerical matrix as well as in tracking calculations: the shape of the transverse beta-functions after the decoupling section does not practically depend on the solenoid field strength. In other words, the two transverse rms ellipses after decoupling are just changed in size through the solenoid field; their orientation and shape remains uneffected by the solenoid strength. This matching capability is illustrated in Fig. 4.

CONCLUSION AND OUTLOOK

The beam line decoupling performance was found to be very stable w.r.t. the magnetic field strength of the solenoid, i.e. the same decoupling gradients can be applied for a wide range of solenoid fields without relevant reduction of the decoupling performance. After the beam is decoupled its rms Twiss parameters do not practically depend on the solenoid field strength that created the coupling. Although the results were illustrated using specific beam parameters, they apply for any other set of beam parameters transported through the proposed kind of beam line. For the time being we can explain the result for a generic case but not the generality of which it has been observed.

REFERENCES

[1] A.J. Dragt, Phys. Rev. A **45**, 4 (1992).

[2] L. Groening, Phys. Rev. ST Accel. Beams **14**, 064201 (2011).

[3] K.-J. Kim, Phys. Rev. ST Accel. Beams **6**, 104002 (2003).

[4] C. Xiao, O.K. Kester, L. Groening, H. Leibrock, M. Maier, P. Rottländer, Phys. Rev. ST Accel. Beam **16**, 044201 (2013).

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