



has only a small number of corrector quadrupole elements with independent power supplies. Nonetheless, each of the triplet quadrupoles is treated as independent in this analysis. The optics distortions are expected to be small, so the primary goal of the LOCO analysis for the PSB is to improve the model of the lattice by determining the distribution of linear errors throughout the machine. The set of variable parameters used in the analysis was the strength and tilt of each of the 48 quadrupoles, and the calibrations and tilts of the 26 dipoles and 32 BPMs in each ring. Each ring was measured and analyzed independently.

## DATA ANALYSIS

The orbit response fitting was done using a MADX model of the PSB with the design lattice. The strengths of the triplet quads and all corrector magnet elements were set according to the operational magnet currents, but no field errors were included in the initial model. The tune was matched to the measured tune at the beginning of the iterative fitting process.

Analysis of the orbit response matrix in the PS Booster is complicated by the fact that the betatron phase advance between periods is close to  $\pi/2$  in both planes. Closed orbit distortion propagates with the same frequency as the betatron tune, and beta beating propagates with twice that frequency. In a case where quadrupoles are located every  $\pi/2$  betatron oscillations, there are combinations of quadrupoles that are in phase with each other with respect to beta beating, but anti-phase with respect to orbit distortion. Therefore one must be careful to avoid arriving at a solution which converges to match the measured orbit response but also has unconstrained growth of beta beating.

### Unconstrained Fitting

Without any constraints, the fit to the measured orbit response in the PSB quickly converges to a solution with a small chi squared value, but the relative error among the triplet quadrupoles was as large as 15%, which is too large to be realistic (see Fig. 3).

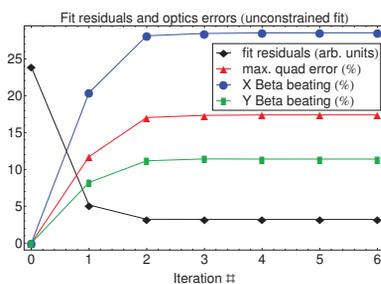


Figure 3: Results of fit with no constraints on fit parameters. Values of  $\chi^2$ , the maximum triplet quadrupole error, and the magnitude of beta beating are shown over six iterations.

### Constrained Fitting

The quadrupole strengths, or any other parameters, can be constrained by adding an additional weighting term to the penalty function

$$\chi^2 = \sum_{i,j} \frac{1}{\sigma_{ij}^2} \left( \left( \frac{\partial x_i}{\partial \theta_j} \right)_{meas} - \left( \frac{\partial x_i}{\partial \theta_j} \right)_{model} \right)^2 + \sum_q (w_q \Delta K_q)^2 \quad (2)$$

where  $\Delta K_q$  is the change to the  $q^{th}$  parameter and  $w_q$  is a weighting factor [2]. This essentially limits the step size of the parameter at each iteration by imposing a cost for each change. The most effective weighting factors for parameters in a given lattice are typically determined through trial and error.

A heavy weighting factor was chosen for the quadrupole strength and tilt parameters, and the constrained fit residuals and optics deviation over fifteen iterations are shown in Fig. 4. The fit residuals still quickly converge to the same minimum as was reached in the unconstrained fit, but the quadrupole strength and tilt error parameters are much smaller. After only about two iterations the fit residuals become as small as the uncertainty of the orbit response measurement. Further iterations result in increasing beta distortion but no further change to the difference between model and measured orbit response.

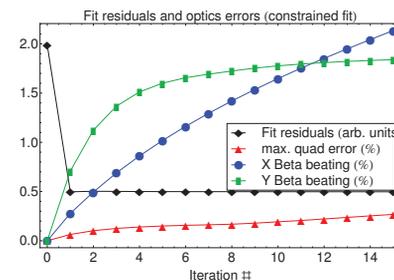


Figure 4: Results of a constrained fit. Values of  $\chi^2$ , the maximum triplet quadrupole error, and the magnitude of beta beating are shown over fifteen iterations.

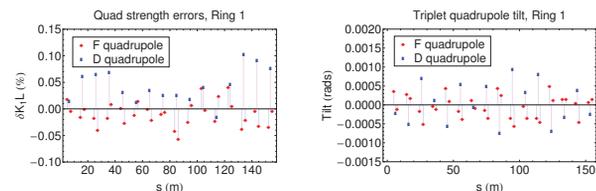


Figure 5: Triplet quadrupole strength and tilt errors from LOCO fit. Each of the 48 magnets (16 triplets) has individual strength and tilt parameters.

The beta beating results shown here are obtained by stopping iterations when the RMS difference between the mea-

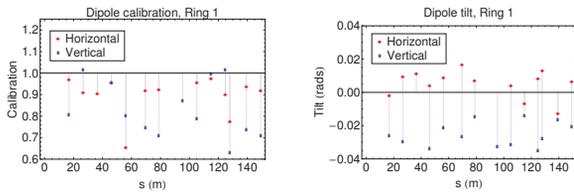


Figure 6: Dipole corrector calibration and tilt errors from LOCO fit. Each horizontal and each vertical corrector had an individual calibration and tilt parameter.

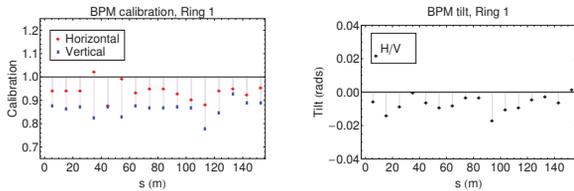


Figure 7: BPM calibration and tilt errors from LOCO fit. Calibrations are fit separately in each plane, but H and V pickups in the same package are given a single tilt error.

sured and model response becomes smaller than the measurement error, which happened after only two iterations. The resulting triplet quadrupole strength and roll errors, dipole calibrations and tilts, and BPM calibrations and tilts are shown in Figs. 5, 6, and 7. Beta beating and dispersion are shown in Figs. 8 and 9. With this fitting method, the resulting calibration parameters are excessively sensitive to the choice of constraints. Further study is needed to determine a method to reliably limit excursions of quadrupole parameters, and to determine whether the magnitude of the resulting dipole and BPM calibrations can be explained relative their expected calibration and alignment tolerances.

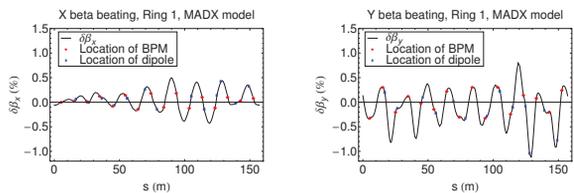


Figure 8: Horizontal and vertical beta beating from MADX model with LOCO fit parameters.

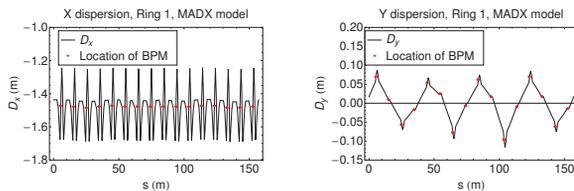


Figure 9: Horizontal and vertical dispersion from MADX model with LOCO fit parameters.

### Transverse Coupling

Some degree of transverse coupling is clearly seen in the orbit response and dispersion measurements. The coupling predicted by the LOCO optimization parameters can be useful for qualitatively gauging the goodness of the LOCO fit. Figure 10 shows a measurement of minimum tune separation and the MADX prediction. The coupling is weaker in the LOCO-calibrated model than was measured, which may indicate that the constraints on quadrupole tilt parameters were too severe. Further study is needed to determine whether the results of LOCO fitting are consistent with measured coupling.

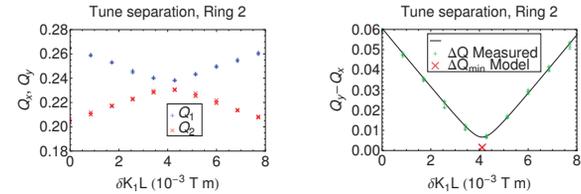


Figure 10: Measured minimum tune separation, and predicted value from calibrated MADX model.

### CONCLUSIONS

The preliminary analysis of orbit response data in the PS Booster suggests beta beating of less than one percent, but these results are not yet definitive. Due to the  $\pi/2$  phase advance per period it is necessary to put constraints on the parameters to avoid unreasonably large results, and the distribution of focusing errors found by this fitting method is very sensitive to the choice of constraints. The calibrated model shows some transverse coupling, but slightly less than is measured directly. Studies continue in order to fully understand the effects of constraint methods on the solution, and to find a solution that does not depend strongly on the constraints.

### REFERENCES

- [1] J. Safranek, "Linear Optics from Closed Orbits (LOCO)-An Introduction," Beam Dynamics Newsletter No. 44, p.43 (2007).
- [2] X. Huang et al., "LOCO with Constraints and Improved Fitting Technique," Beam Dynamics Newsletter No. 44, p.60 (2007).