CSR-DRIVEN LONGITUDINAL SINGLE BUNCH INSTABILITY THRESHOLDS*

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INTRODUCTION

The interaction of a short bunch of electrons with its own coherent synchrotron radiation, CSR, leads to low longitudinal instability thresholds. According to Bane, et al. [1] these thresholds follow a quite simple scaling law. More detailed theoretical calculations for the underlying shielded CSR-impedance have been performed for BESSY II and the Metrology Light Source, MLS. It is found that there are additional parameter regions where the instability is weak and thus the thresholds depend on the longitudinal damping time, T_{long}, and the synchrotron tune, $1/T_{syn}$. Most of the observed longitudinal instability features [2, 3] are in surprisingly good agreement with the theoretical findings.

CALCULATION OF INSTABILITY THRESHOLD CURRENTS

The theoretical problem is tackled by numerically solving the Vlasov-Fokker-Planck-equation, VFP, which describes the temporal evolution of the distribution of particles in the longitudinal phase space. Calculations were performed for the shielded CSR- and some of the other fundamental interactions and for various operating conditions of the MLS and the BESSY II storage ring. Details of the numerical approach, the parameters of the two rings, and the way the shielded CSR-wake is treated can be found in [4]. Electrons are assumed to move on a circle between two perfectly conducting infinite plates. The impact of the shielded CSR-wake on the longitudinal beam dynamics can be understood better on the basis of the result for some of the simpler interaction mechanism.

Resistive Impedance

One of those is the interaction of the bunch of electrons with the resistive impedance of the vacuum chamber. The VFP-equation was solved with the corresponding δ-function wake. This also served as a test of the developed code. The results are presented in the left of Figure 1 and are in very good agreement with the semi-analytical calculations of Oide [5] shown as a thick solid line. Please note for later discussions, the instability is weak and depends on the longitudinal damping time: threshold currents are proportional to the square root of T_{syn}/T_{long}. These calculations were performed with a constant momentum compaction factor, α, and the standard operating parameters for the Diamond Light Source [6] and the BESSY II storage rings. The synchrotron tune and thus the zero current bunch length, σ_0, were varied by changing the RF cavity voltage. In all cases the quadrupole mode is the first unstable mode.

Broad-Band-Resonator

Equally fundamental is the interaction of the bunch with a resonator-like impedance. Further tests of the code were therefore performed with the wake corresponding to the impedance of a broad-band-resonator, BBR. A comparison with previous theoretical results [7] is shown in the centre of Figure 1 where the dimensionless strength parameter, S_{BBR}, is displayed as a function of the normalized resonance frequency, the product of resonance frequency and zero current bunch length, $2\pi F_{res}\cdot\sigma_o$. In the calculations the quality factor, $Q=1$, and the bunch length was kept fixed. The frequency of the resonator, F_{res}, was varied. S_{BBR} is given by:

$$S_{BBR} = \frac{2Nr_e}{\gamma c Z_0 \nu_s \sigma_\varepsilon} \cdot \frac{2\pi F_{res} R_s}{Q}$$

With the shunt impedance, R_s, the number of particles in the bunch, N, the classical radius of the electron, r_e, the Lorentz factor, γ, the speed of light, c, the vacuum impedance, $Z_0=377\Omega$, the synchrotron tune, ν_s, and the natural relative energy spread, σ_ε. The agreement with earlier calculations is quite good [7], however, the best analytical approach by Oide and Yokoya fails to predict the finer details. Especially around the step in the (azimuthal) mode number (F_{inst}/F_{syn}) the agreement is poor. Note that the jumps are a consequence of the resonance-like features of the wake and the first unstable azimuthal mode relative to the synchrotron frequency increases proportional to $2\pi F_{res}\cdot\sigma_o$.

Shielded CSR-Impedance

Details of the calculations and some results for this wake obtained for a constant RF-cavity voltage have been published already [4]. Recent calculations were performed for a fixed momentum compaction factor and the bunch length variation achieved by changing the RF-voltage. The theoretical results for BESSY II are shown in the right of Figure 1 together with the simple scaling law extracted from similar calculations performed by Bane, et al. [1] which is represented by the thick solid black line: $S_{csr} \sim 0.5 + 0.12X$. S_{csr} and X are given by:

$$S_{csr} = \frac{Nr_e}{2\pi\nu_s\gamma\sigma_\varepsilon} \cdot \frac{\rho^{1/3}}{(c\sigma_0)^{4/3}} \text{ and } X = \frac{\rho^{1/2}c\sigma_0}{h^{3/2}}.$$

With the bending radius, ρ, and $2h$, the separation of the shielding plates. These authors have already observed the much lower thresholds for shorter bunches. Their explanation was a loss of synchrotron tune spread in this region [8]. Indeed, there the solution of the Haissinski equation shows a transition from bunch lengthening to bunch shortening, similar to the impact of a BBR-wake. Bane, et al., have also pointed out that in this region the
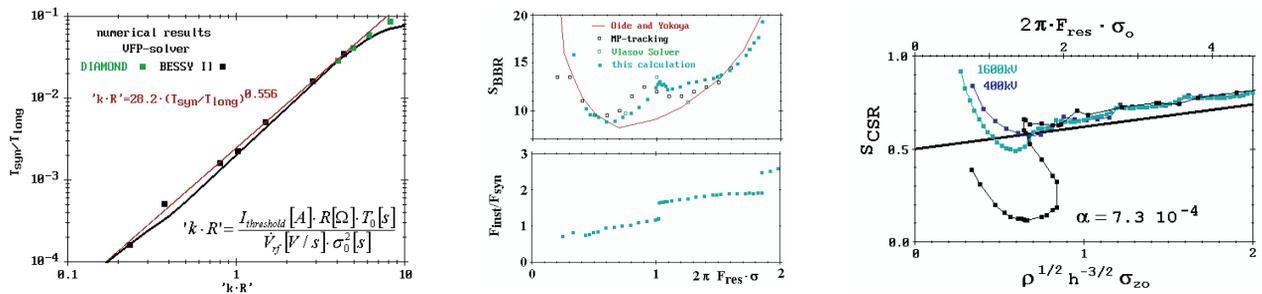
Figure 1: Thresholds predicted by solving the VFP-equation for BESSY II: left – with a purely resistive δ-function wake, centre – with a broad-band-resonator wake, and right – with the wake of the shielded CSR-impedance.

instability is weak and there can be more than one threshold. The bunch can become stable again at higher currents. This half island of stability is clearly visible in the data calculated with a fixed momentum compaction factor, α. The shortest bunch corresponds to V_{rf}=100 MV. A similar behaviour exists for the BBR-wake at around $2\pi F_{res} \cdot \sigma_o \approx 1.8$. The step-like increase of the thresholds is related to jumps in the azimuthal mode of the instability which can be observed with the BBR- and the CSR-wake for longer bunches. BBR- and CSR-impedance are similar because of their resonance-like features. The resonance frequency of the CSR-impedance is given by: $F_{res}/c = (\pi\rho/24h^3)^{1/2}$ [9]. The width of this resonance is much broader and therefore Q<1. For very short bunches the resistive part of the CSR-impedance dominates the beam dynamics. As a consequence the thresholds will be higher than expected by the linear scaling law. This is a new observation compared to the previous investigations.

COMPARISON WITH EXPERIMENTAL RESULTS

The results are presented in Figure 2 and the agreement between experimental threshold currents for the MLS [3] and calculations with bunches longer than ~1mm is very good. Very long bunches behave as described by the linear scaling. Bunches shorter than ~2.5 mm suffer from the weak instability and have lower thresholds currents. This is the region of the hump in Figure 2. Because the longitudinal damping time, T_{long}, is constant the height of the hump depends on T_{syn}: The

higher the synchrotron frequency, $1/T_{syn}$, the lower is the threshold. Below a bunch length of 1 mm experiment and theory deviate from each other. The experimentally determined thresholds are much lower than predicted. According to the theory the threshold current should no longer be proportional to V_{rf} but rather should approach proportionality with $V_{rf}^{1/2}$. The agreement with the predictions for very short bunches is much better in case of BESSY II [4]. Therefore tracking studies were performed for the MLS in order to shed some light on the observed discrepancies at this storage ring.

Figure 3: Waterfall display of the Fourier transformed CSR-signal as measured at BESSY II.

Importance of Multi Particle Tracking

In the experiments the power of coherent synchrotron radiation up to a few THz [10] is detected with a fast InSb-bolometer as a function of time and bunch currents. At the longitudinal instability threshold the particle distribution will become a function of time. A very sensitive indication of these non-stationary particle distributions is therefore the occurrence of spectral features in the Fourier transform of the detected ore calculated time dependent CSR-signal. Usually, these spectral features are lines at frequencies close to multiples of the zero current synchrotron frequency. An example of a measured spectral waterfall display is shown in Figure 3 for a ~1.5 ps long bunch in BESSY II. The actual onset of the instability is around 12 μA with a line of roughly 3 times the zero current synchrotron frequency of 1 kHz. Lines corresponding to the dipole and quadrupole modes appear already at much lower currents due to the high sensitivity of the THz-detector in use. The frequency of the lines shifts as a function of current and their height
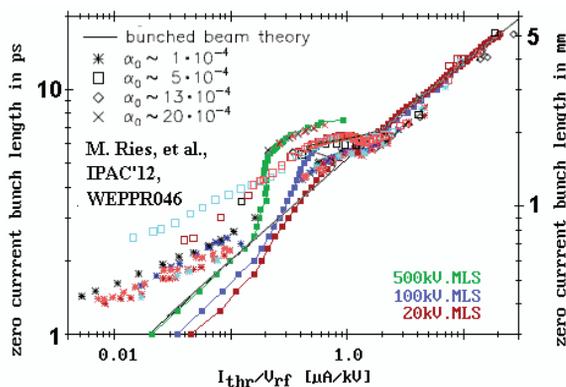
Figure 2: Comparison of theoretical and experimental results for the MLS.
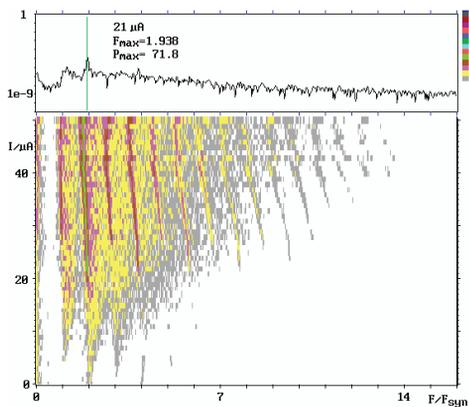
Figure 4: Fourier transformed CSR-signal as calculated by tracking of 10^6 particles for the MLS operating with V_{rf}=330 kV and α=1.3·10^{-4}

increases quite linearly with current. Only the line at around 3 kHz shows the instability driven exponential increase, at least over a limited range of currents.

Initially it was thought that the noise-free description of the evolution of the ensemble of particle with a distribution function governed by the VFP-equation would be superior to results obtained by multi particle tracking [11]. Today, and because threshold currents are comparatively low for short bunches, realistic multi particle tracking can be performed with the actual number of particles. Spectra of the calculated CSR-signal at the MLS are shown in Figure 4. Similar to the measurements presented in Figure 3, and Figure 1 of [3], lines show up at very low currents and they have to be attributed to the equilibrium fluctuations of the ensemble due to the finite number of particles. This Schottky noise effect scales inversely with the number of particles and can be seen more clearly in Figure 5 where the calculated bunch length, energy spread, and the spectral peaks are shown as a function of current. Tracking simulations were performed with a fixed number of 10^6 particles or the actual number of particles which approaches $3 \cdot 10^7$. Compared to these simulations the solution of the VFP-equation is effectively noise-free. Above the threshold the
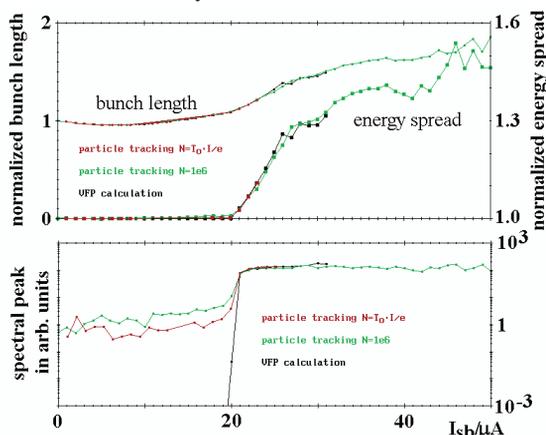
Figure 5: Results of multi particle tracking calculation in comparison with the numerical solution of the VFP-equation with the parameters used for Figure 4.

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agreement between all simulations is perfect. The signal of the instability threshold is the thin green spectral line in Figure 4 which occurs at around 20 µA where also the relative energy spread starts to increase. Thresholds have to be extracted carefully in case the first unstable mode is the quadrupole or dipole mode. Threshold currents should be determined from the steepness of the spectral peaks as the current is increased. Especially when a direct measurement of the energy spread is missing.

CONCLUSION

In conclusion, the very broad resonance created by the shielding plates is very important for the CSR-driven longitudinal beam dynamics. Theory predicts a strong instability mechanism for bunches much longer than $1/(2\pi F_{res})$, with $F_{res}/c=(\pi\rho/24h^3)^{1/2}$, and thresholds are described well by Bane's scaling law. For shorter bunches the instability is weak and thresholds depend in addition on longitudinal damping time and synchrotron tune. Threshold currents can be lower or higher than predicted by the scaling law. For very short bunches the wake of the shielded CSR looks like a resistive δ-function wake. For these short bunches the threshold currents are higher and their experimental determination based on the spectra of the time varying CSR-power requires special care because the Schottky noise produces lines similar to the CSR-driven instability itself. Nevertheless, the observation of these lines below the actual threshold can serve as an additional longitudinal beam diagnostics. In addition, the dominant resonant frequency of the impedance driving the longitudinal instability can be extracted from the measured frequencies of the first unstable mode as a function of the bunch length.

ACKNOWLEDGMENT

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