CHROMATIC SEXTUPOLE PAIR OPTIMIZATION METHODS FOR ENLARGING DYNAMIC APERTURE

Zhenghe Bai#, Lin Wang$, Qika Jia, Qingkun Zhang, Weimin Li, NSRL/USTC, Hefei, P. R. China

Abstract

Based on the step-by-step chromaticity compensation method [1] and artificial intelligence algorithms, we propose new numerical methods, called chromatic sextupole pair optimization methods, for enlarging the dynamic aperture of electron storage rings. In the new methods, the decision variables related to chromatic sextupole pairs are optimized using artificial intelligence algorithms to enlarge the dynamic aperture. At the end, we demonstrate that the new methods are equivalent to the recently used numerical method, in which the decision variables, sextupole strengths, are optimized using artificial intelligence algorithms.

INTRODUCTION

In an electron storage ring, a large enough dynamic aperture (DA) is desired for high beam injection efficiency and long beam lifetime. In the analytical approach for DA optimization, such as widely-used single resonance method, the DA is indirectly optimized by minimizing (or maximizing) a penalty function, which is a weighted sum of some linear and nonlinear quantities. The optimization results depend on the weight settings and the lattice designers' experience, and usually are not optimal solutions. In the numerical approach, including the step-by-step chromaticity compensation method, the scanning method and the artificial intelligence (AI) algorithms, the DA is directly optimized based on particle tracking. Especially in recent years, the AI algorithms, like genetic algorithms, have been successfully applied to search for optimal solutions for the optimization of DA.

In this paper, we point out that the deficiencies of the step-by-step chromaticity compensation method can be cured by introducing AI algorithms, and then propose new numerical methods, called chromatic sextupole pair optimization methods, for the optimization of DA.

STEP-BY-STEP CHROMATICITY COMPENSATION METHOD

The schematic diagram of the step-by-step chromaticity compensation method [1] is shown in Fig. 1, where (ξ_{x0}, ξ_{y0}) are natural chromaticities, (ξ_{x1}, ξ_{y1}) are compensated chromaticities, and the line from A to B is the chromaticity compensation path. The chromaticities are compensated by many steps, and at each step the chromaticities are only compensated by small values. At each step there are $N_{SF} \times N_{SD}$ pairs of focusing and defocusing chromatic sextupoles to be examined (N_{SF} and N_{SD} are the numbers of the families of focusing and defocusing chromatic sextupoles, respectively), and the pair demonstrating the best DA is chosen and fixed. Such a procedure is repeated until the chromaticity compensation is finished.

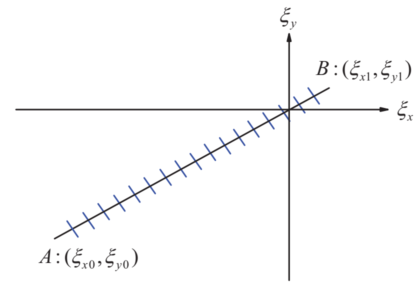

Figure 1: The schematic diagram of the step-by-step chromaticity compensation method.

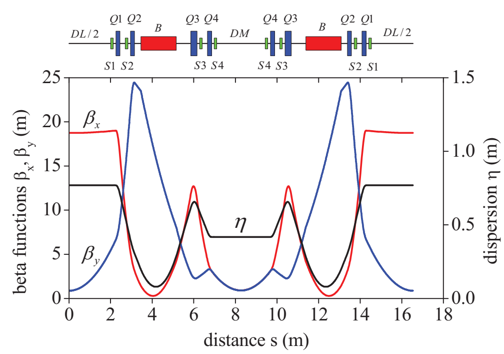

Figure 2: The magnet layout and linear optics functions of one period of the studied lattice.

As an example of application, we take one lattice of the HLS-II storage ring with natural emittance of about 15 nm-rad shown in Fig. 2. There are two families of focusing sextupoles: S1 and S3, and two defocusing families: S2 and S4. The quantitative criterion of DA used here is based on the clipped area of DA, which is provided in the code ELEGANT, and the symmetry of DA is considered in the quantitative criterion in the form of weighted sum. We tried different values of the number of steps N: 10, 20, 30, 40 and 50, and the quantitative values of DAs obtained at these values are shown in Fig. 3. We can see that at $N = 20$ steps the quantitative value of DA is largest, i.e., the DA is best. When N is too large, due to too small nonlinear perturbation caused by each chromatic sextupole pair, the comparison of DAs obtained at different chromatic sextupole pairs is meaningless, especially in the later part of the optimization process. So the problem of the settings of N is a deficiency of this method. In addition, due to that the tunes of off-momentum particles change as the chromaticities are step-by-step compensated, the off-momentum DAs cannot be directly included in the optimization. The DA obtained at $N = 20$ steps is shown in Fig. 6 (blue line) in the next section, and the number of times that each chromatic sextupole pair is chosen as the best pair is shown in Fig. 4.

#baizheng@mail.ustc.edu.cn, $wanglin@ustc.edu.cn

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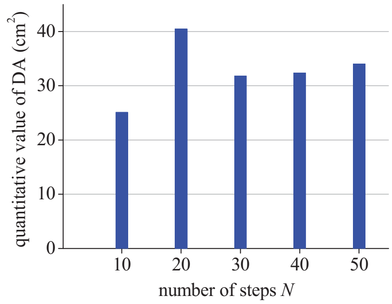

Figure 3: The quantitative values of DAs obtained at different numbers of steps N.

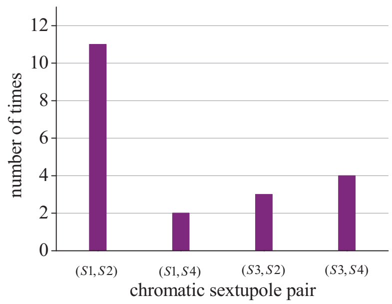

Figure 4: The number of times that each chromatic sextupole pair is chosen as the best pair.

CHROMATIC SEXTUPOLE PAIR OPTIMIZATION METHODS

Method 1: Chromatic Sextupole Pair Optimization with Chromaticity Compensation Path

After the optimization of DA using the step-by-step chromaticity compensation method, we can know the number of times that each chromatic sextupole pair is chosen as the best pair, for example as shown in Fig. 4. We can imagine that if we use another optimization process to obtain the same number of times as shown in Fig. 4, the optimized DA must be the same as that in Fig. 6 (blue line). So we can convert such an optimization problem into a usual optimization problem, in which the decision variables are the number of times that each chromatic sextupole pair is chosen as the best pair and the optimization objective is DA. Such a usual optimization problem can be easily solved by AI algorithms.

In the converted optimization problem, the number of decision variables $M = N_{SF} \times N_{SD}$, and the decision variables can be written as $(N_1, N_2, \ldots, N_m, \ldots, N_M)$, where N_m represents the number of times that m^{th} chromatic sextupole pair is chosen as the best pair and N_m is a nonnegative integer. The sum of all N_m is equal to the number of steps N, i.e.,

$$N_1 + N_2 + \cdots + N_m + \cdots + N_M = N \,. \tag{1}$$

For a given value of N, m^{th} chromatic sextupole pair contributes the chromaticities by values of $\left((\xi_{x1} - \xi_{x0}) \times N_m / N, (\xi_{y1} - \xi_{y0}) \times N_m / N\right)$. The decision variables are optimized to enlarge DA using AI algorithms.

We call this new method the chromatic sextupole pair optimization (CSPO) method. In the CSPO method in this subsection, for any solution, if the natural chromaticities are compensated by each chromatic sextupole pair successively, the increase of chromaticities will be along the path from A to B in Fig. 1. So we call this CSPO method the CSPO with chromaticity compensation path (Method 1). In Method 1, N can be set to be a very large number, and the off-momentum DAs can also be directly included in the optimization.

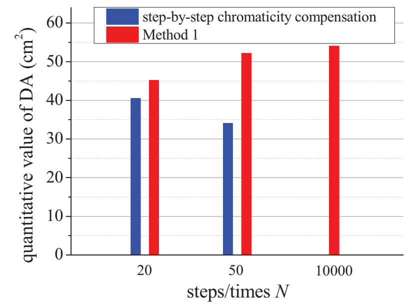

Figure 5: The quantitative values of DAs obtained by the step-by-step chromaticity compensation method (blue column) and Method 1 (red column) at different N.

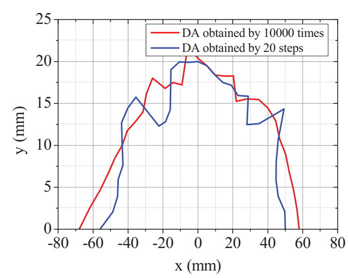

Figure 6: DAs obtained by the step-by-step chromaticity compensation method (blue line) and Method 1 (red line).

We applied Method 1 to the optimization of DA for the studied lattice, and the AI algorithm that we used is the particle swarm optimization (PSO) algorithm. The quantitative values of DAs obtained at N = 20, 50 and 10,000 times are shown in Fig. 5 (red column). For a comparison, the quantitative values of DAs obtained at N = 20, 50 steps using the step-by-step chromaticity compensation method are also shown in Fig. 5 (blue column). We can see that the DA quantitative value obtained by Method 1 becomes larger when N becomes larger. This is because the values of the chromaticities contributed by each chromatic sextupole pair become more precise. We also can see that Method 1 can obtain larger DA quantitative value, i.e., better DA than using the step-by-step chromaticity compensation method. The DA obtained at N = 10,000 times using Method 1 (red line) is better than that obtained at N = 20 steps using the step-by-step chromaticity compensation method (blue line), which can be seen in Fig. 6.

Method 2: Chromatic Sextupole Pair Optimization without Chromaticity Compensation Path

In Method 1, the ratio of N_m to N determines the values of the chromaticities contributed by m^{th} chromatic sextupole pair. In fact, in the CSPO method we can directly use the chromaticities contributed by each chromatic sextupole pair as the decision variables instead of indirectly using N_m and N. Besides, in Method 1 for each chromatic sextupole pair the ratio of the contributed horizontal chromaticity to the vertical is a constant equal to the reciprocal of the slope of the path in Fig. 1. In the CSPO method we can take away this constraint of the constant ratio.

In the improved CSPO method, the number of decision variables is $2M$. The decision variables can be written as ($\xi_{1x}, \xi_{2x}, \cdots, \xi_{mx}, \cdots, \xi_{Mx}; \xi_{1y}, \xi_{2y}, \cdots, \xi_{my}, \cdots, \xi_{My}$), where ξ_{mx} and ξ_{my} are the horizontal and vertical chromaticities contributed by m^{th} chromatic sextupole pair, respectively, and ξ_{mx} and ξ_{my} are nonnegative values. The two constraints for these variables are expressed as follows:

$$\xi_{1x} + \xi_{2x} + \cdots + \xi_{mx} + \cdots + \xi_{Mx} = \xi_{x1} - \xi_{x0}, \quad (2)$$

$$\xi_{1y} + \xi_{2y} + \cdots + \xi_{my} + \cdots + \xi_{My} = \xi_{y1} - \xi_{y0}. \quad (3)$$

We call this CSPO method the CSPO without chromaticity compensation path (Method 2). In the case that, for each chromatic sextupole pair, the following constraint is satisfied:

$$\xi_{mx} / \xi_{my} = (\xi_{x1} - \xi_{x0}) / \xi_{y1} - \xi_{y0}, \quad (4)$$

Method 2 is equivalent to the case of Method 1 with infinite large value of N. Because N is set to be a very large value when using Method 1, we can say that Method 1 is a special case of Method 2.

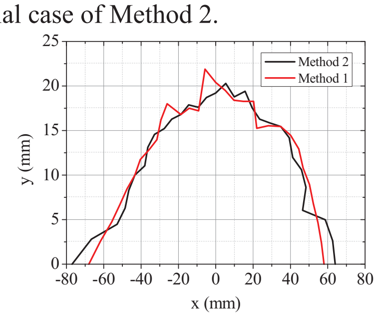

Figure 7: DAs obtained by Method 2 (black line) and Method 1 (red line).

We applied Method 2 to the DA optimization for the studied lattice, and the optimized DA is shown in Fig. 7 (black line). For a comparison, the DA obtained at N = 10,000 times using Method 1 is also shown in Fig. 7 (red line). In fact, these two DAs and the corresponding values of decision variables are almost the same.

Method 3: Chromatic Sextupole Pair Optimization with Fewest Variables

We are familiar with the equations for chromaticity compensation by sextupoles:

$$\xi_{x1} - \xi_{x0} = \frac{1}{4\pi} \oint \lambda(s) \eta \beta_x ds, \quad (5)$$

$$\xi_{y1} - \xi_{y0} = \frac{1}{4\pi} \oint \lambda(s) \eta \beta_y ds, \quad (6)$$

where $\lambda(s)$ is the sextupole strength, β_x, β_y and η are beta functions and dispersion function at the sextupole location, respectively.

In fact, the recent application of AI algorithms is related to the right part of Equations 5 and 6 above, in which the decision variables are sextupole strengths $\lambda(s)$. While the CSPO methods proposed here are related to the left part of the two equations, in which the decision variables are chromaticities. In other words, the two equations above connect the two kinds of numerical methods of optimizing chromatic sextupole pair related quantities and chromatic sextupole strengths, respectively, using AI algorithms. In the following, we will demonstrate the equivalence of the two kinds of methods.

In the AI algorithm based numerical method in which the decision variables are sextupole strengths, the number of variables to be optimized is $F = N_{SF} + N_{SD} - 2$, since the other two variables are used to compensate the horizontal and vertical chromaticities to the desired values according to Equations 5 and 6. In the CSPO methods, the minimum number of variables to be optimized is also equal to F. In the Method 1 and Method 2, the decision variables are set to be nonnegative values. This constraint can be taken away, i.e., the decision variables can be negative. If so, the minimum number of chromatic sextupole pairs used in Method 1 is equal to $F+1$ (since there is an equality constraint, i.e., Equation 1), and in Method 2 it is equal to $\lceil (F+2)/2 \rceil$ (two equality constraints, i.e., Equation 2 and 3) , where $\lceil\ \rceil$ denotes rounding up to the nearest integer. We call the CSPO method in such cases the CSPO with fewest variables (Method 3).

We used Method 3 to optimize the DA for the studied lattice (with F = 2). We chose two chromatic sextupole pairs ($\lceil (F+2)/2 \rceil = 2$): (S1, S2) and (S3, S4). The decision variables are four chromaticities, and the number of variables to be optimized is 2 equal to F (the other two are determined by Equations 2 and 3). The DA obtained by Method 3 is shown in Fig. 8 (black line). The DA obtained by optimizing the sextupole strengths using the PSO algorithm is also shown in Fig. 8 (red line). We can see that two DAs obtained by optimizing chromatic sextupole pair related quantities (i.e. chromaticities) and chromatic sextupole strengths are almost the same, which demonstrates the equivalence of these two kinds of AI algorithm based numerical methods.

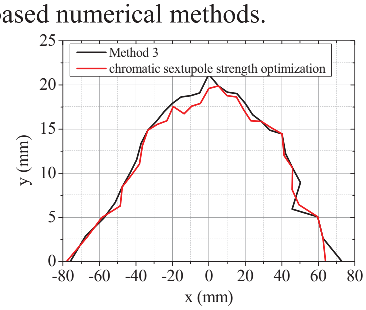

Figure 8: DAs obtained by Method 3 (black line) and optimizing sextupole strengths using PSO (red line).

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[1] E. Levichev, P. Piminov, "Algorithms for Chromatic Sextupole Optimization and Dynamic Aperture Increase," Proceedings of EPAC 2006, WEPCH085, p. 2116.