SPACE CHARGE AND CAVITY MODELING FOR THE ESS LINAC SIMULATOR

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Abstract

The proton linac of the European Spallation Source will operate at unprecedented beam power of 5 MW. Such power requires a precise modeling of the beam dynamics in order to protect its components from losses. The high peak current of 62.5 mA produces a space charge force that dominates the dynamics at low energy, while the high gradient required to accelerate up to 2 GeV in the 500 m of linac length is challenging for the dynamics in the RF cavities. This paper presents modelings of the space charge force and RF cavities used in the ESS Linac Simulator. The simulator is under development as part of the XAL on-line model, and it will be adopted for the ESS linac operations.

INTRODUCTION

To reach the 5 MW beam power required by the ESS linac there are two key non-linear components that have to be designed carefully: the space charge and the radio frequency cavities. The space charge dominates the dynamics of the proton beam at low energies: in the Drift Tube Linac the force of the space charge competes with the strength of the quadrupoles and in some cases can reach 80% of the force of the magnets [1]. Such a dominant force cannot be treated with the linear approximation but must be evaluated with a numerical integrator that calculates the field with high accuracy. Moreover, the usual approximation for long bunches, where the traversal σ is much smaller than the longitudinal σ is not valid for ESS. The emittances of ESS are almost the same in the three planes as reported in Table 1 of the beam parameters.

In this paper the model for a Gaussian distribution is presented. Such a model was selected, because it represents well the beam without affecting too much the performances of the simulator. A Particle In Cell (PIC) model is under consideration for a more advanced description of the space charge in the ELS code.

Table 1: Beam Parameters

Parameter	Value
Power on Target	5 MW
Peak Power	125 MW
Top Energy	2 GeV
Beam Peak Current	62.5 mA
Particles per bunch	1.1×10^9
Duty Cycle	4%
Freq. before the Medium β sect.	352.21 MHz
Freq. after the Medium β sect.	704.42 MHz
Hor. & Ver. Normalized Emit.	0.25×10^{-6} m rad
Longitudinal Normalized Emit.	0.33×10^{-6} m rad

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The second key component is the RF cavity model: those elements will push the beam with a peak surface field of 45 MV/m. This gradient influences the longitudinal dynamics producing a strong non-linear behavior. In order to maximize the longitudinal acceptance of the beam it is important to evaluate the strength that each particle feels with a model of the cavity based on a field map.

In this paper the field map along the z axis is generated from the first and third harmonics of the field.

SPACE CHARGE MODEL

The space charge is the electromagnetic force that a particle experiences when it is in a bunch of particles with the same charge. In the rest frame, the space charge depends on the beam intensity (number of particles), and the relative position of the particles (what is in general called the beam distribution or the geometry of the bunch). The factor that depends by the current is given by the product of the elementary charge e and the number of particles in the bunch N. This factor can be scaled according to the constant $\frac{1}{4\pi\epsilon_0}$ from the force of Coulomb. The space charge potential is:

$$U_{sc} = \frac{eN}{4\pi\epsilon_0} G \quad (1)$$

This is in general converted in parameters more familiar for the accelerator such as:

$$U_{sc} = \frac{R_0 m_p c^2}{e^2} \frac{I}{f} G \quad (2)$$

with $R_0 = \frac{e^2}{4\pi\epsilon_0 m_p c^2}$ is the classical radius of the proton and the number of particles $N = \frac{I}{ef}$ is expressed as the ratio between the beam current and the frequency of the RF multiplied by the elementary charge. In the lab-frame, the magnetic field contributes to the potential reducing it with a Lorentz factor γ^2:

$$U_{sc} = \frac{R_0 m_p c^2}{e^2 \gamma^2} \frac{I}{f} G \quad (3)$$

The geometrical factor G depends on the assumptions used to model the bunch shape. In literature there are three main ways to evaluate G:

- the bunch is assumed as an ellipsoid that contains all the charge uniformly distributed [2];
- the bunch is assumed as a Gaussian distribution in the three spatial coordinates;

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ISBN 978-3-95450-122-9

- the distance between particles is evaluated using a mesh of the space and computing the charge of the particles in a grid (the so-called Particle In Cell);

the ELS uses the Gaussian option, and consequently the geometrical factor is expressed as

$$G = \frac{1}{\sqrt{\pi}} \int_0^\infty \frac{e^{-\frac{x^2}{2\sigma_x^2+t} - \frac{y^2}{2\sigma_y^2+t} - \frac{z^2}{2\sigma_z^2+t}} - 1}{\sqrt{(2\sigma_x^2+t)(2\sigma_y^2+t)(2\sigma_z^2+t)}} dt. \quad (4)$$

The factor $\frac{1}{\sqrt{\pi}}$ normalizes the distribution to 1. Integration in the spatial coordinates is done by introducing the dummy variable t. The resulting potential is:

$$U_{sc} = \frac{IR_0 m_p c^2}{fe^2\sqrt{\pi}\gamma^2} \int_0^\infty \frac{e^{-\frac{x^2}{2\sigma_x^2+t} - \frac{y^2}{2\sigma_y^2+t} - \frac{z^2}{2\sigma_z^2+t}} - 1}{\sqrt{(2\sigma_x^2+t)(2\sigma_y^2+t)(2\sigma_z^2+t)}} dt \quad (5)$$

The kick to apply arise from the force:

$$\mathbf{F} = -e\nabla U_{sc} \quad (6)$$

$$\gamma m_p \mathbf{a} = -\frac{IR_0 m_p c^2}{fe\gamma^2}\nabla G \quad (7)$$

$$\mathbf{a} = -\frac{IR_0 c^2}{fe\gamma^3}\nabla G \quad (8)$$

The kick is the acceleration divided by the modulus of the momentum:

$$\Delta x' = -\alpha x \int_0^\infty \frac{e^{-\frac{x^2}{2\sigma_x^2+t} - \frac{y^2}{2\sigma_y^2+t} - \frac{z^2}{2\sigma_z^2+t}}}{\sqrt{(2\sigma_x^2+t)^3(2\sigma_y^2+t)(2\sigma_z^2+t)}} dt \quad (9)$$

$$\Delta y' = -\alpha y \int_0^\infty \frac{e^{-\frac{x^2}{2\sigma_x^2+t} - \frac{y^2}{2\sigma_y^2+t} - \frac{z^2}{2\sigma_z^2+t}}}{\sqrt{(2\sigma_x^2+t)(2\sigma_y^2+t)^3(2\sigma_z^2+t)}} dt \quad (10)$$

$$\Delta z' = -\alpha z \int_0^\infty \frac{e^{-\frac{x^2}{2\sigma_x^2+t} - \frac{y^2}{2\sigma_y^2+t} - \frac{z^2}{2\sigma_z^2+t}}}{\sqrt{(2\sigma_x^2+t)(2\sigma_y^2+t)(2\sigma_z^2+t)^3}} dt, \quad (11)$$

with $\alpha = \frac{IR_0}{fe\sqrt{\pi}\beta^2\gamma^3}$. It is important to recall that the kick is evaluated in the frame of the bunch, this means that the z position is the position that the particles have in the bunch frame and not in the laboratory frame. For the x and y coordinates the transformation of coordinates between the laboratory and the bunch is trivial.

The integral cannot be evaluated analytically so it is calculated using an adaptive algorithm for the Gaussian quadrature as implemented in the GNU Scientific Libraries [3–5]. An example of the field calculated with this algorithm is shown in Fig. 1: this is a section of the beam that shows the distribution of the space charge force. The maximum of the force is normalized to 1.

This model of the space charge is already implemented in the ELS and it was benchmarked against the TraceWin simulator. The results are presented in this conference in the paper [6].

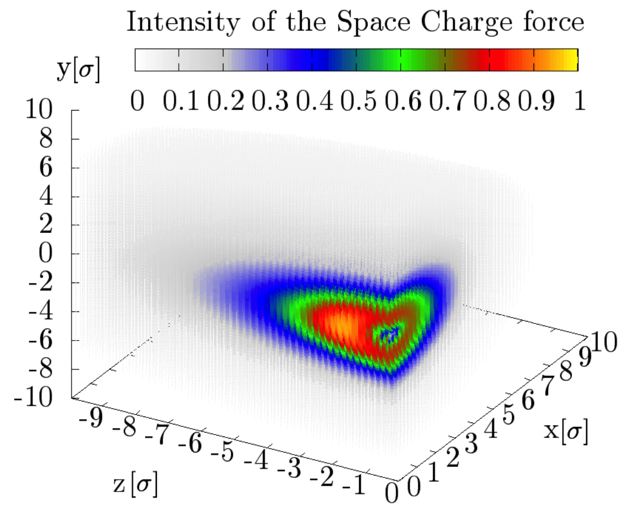

Figure 1: Force due to the space charge represented in a section of the bunch.

CAVITY MODEL

The ELS is using the kick-drift-kick cavity model at the time being. Such a model requires pre-calculated transit time factors, TTF, as one of the parameters. For the multi-cell cavities the phase of the cavity is also pre-calculated based on the distance between the gaps and particle energy. For the case of a linac at its nominal settings, these values are exact and the results are in good agreement with the integration of fields [6]. However, when the particle energy differs significantly from that of the synchronous particle the energy gain and phase advances are systematically wrong. To avoid this problem, a new cavity model was developed which using few parameters regenerates the original field [7].

The field is divided in three main regions: the input cell, the inner cells and the output cell. The input and output cells are each divided into two sections, the inner-side sinusoidal field and the outer-side exponential decay field as shown in Fig. 2. To include the field asymmetry due to power coupler, High Order Modes (HOM) coupler or different beam tube apertures, the input and output cells are treated independently.

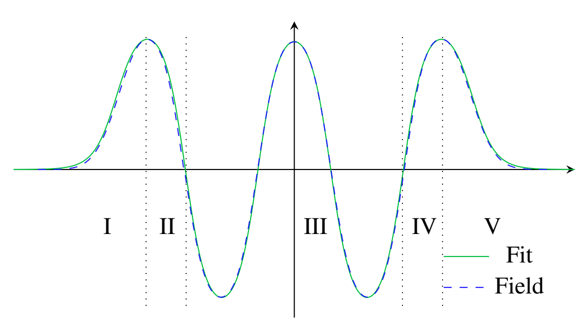

Figure 2: Simulated field, matched field and the sections of the matched field. The input/output exponential decay (I/V), input/output sinusoidal (II/IV), and inner cells (III).

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The inner cells are modeled as the sum of first and third harmonic oscillations; the inner-side end-cells are modeled using the same method, but with a different wave number and amplitude, while the exponential decay cells are modeled with a exponential function which takes a Gaussian form when the power of the exponent is exactly 2.

The matching for the sinusoidal cells is done using a Fourier transform, and the matching parameters for the exponential decay cells are based on analytic calculations. To regenerate the field, 15 parameters are requires: geometric β of the cavity (β_g), number of cells, frequency (f_{rf}), amplitude of the first and third harmonic for the inner cells, input cells, and output cells, the gap shift for the input and output cells, and exponent power and σ of the input and output cells. Then to use this field in the simulations one must add the synchronous phase and amplitude to the parameters. Another parameter which could be useful, especially for the online modeling, is the TTF for the synchronous particle.

The field in the five regions could be generated by the following equation in regions I and V (considering the different parameters).

$$F(z) = A_{\mathrm{exp}} \cdot e^{-\left(\frac{|z-L_{\mathrm{exp}}|}{2\sigma_{\mathrm{exp}}}\right)^{p_{\mathrm{exp}}}}, \tag{12}$$

where A_{exp} is the coefficient of the exponential, L_{exp} is the position of transition from region I to II (IV to V) for the input (output) cells, σ_{exp} represents the width of the decay field, and p_{exp} is the power of the exponential. Both σ and p could be found analytically. The governing equation for regions II and IV is:

$$F(z) = B_1 \cdot \cos\left(2\pi\frac{z-\phi_1}{L'}\right) + B_3 \cdot CS\left(6\pi\frac{z-\phi_3}{L'}\right), \tag{13}$$

where the function $CS(x)$ is a sine for even number of cells and a cosine for odd number of cells, B_1 and B_3 are the coefficients for the first and third harmonic, ϕ_1 and ϕ_3 are phases to make the function continues at the transition between region II and III (III and IV) for the input (output) cells, and L' is the adjusted cell length for the end cells taking into account the gap shift. The field in the inner cells is generated using:

$$F(z) = A_1 \cdot \cos\left(2\pi\frac{z}{L}\right) + A_3 \cdot \cos\left(2\pi\frac{3z}{L}\right), \tag{14}$$

where A_1 and A_3 are the coefficients of the first and third harmonic for the inner cells, and L is the period length equal to $\beta_g\lambda$ ($\lambda = c/f_{\mathrm{rf}}$).

One way to measure the accuracy of the fitted field to the original is to look the fitting parameters such as χ^2 or R^2. It is also possible to look at the TTF calculated using the fitted field with the TTF of the original field and compare the χ^2 or R^2. For the example presented here the R^2 is 0.999 for both the field and the TTF. The curves do overlap in the functional area of the cavity, *i. e.* from a beta of 0.65 onward as shown in Fig. 3.

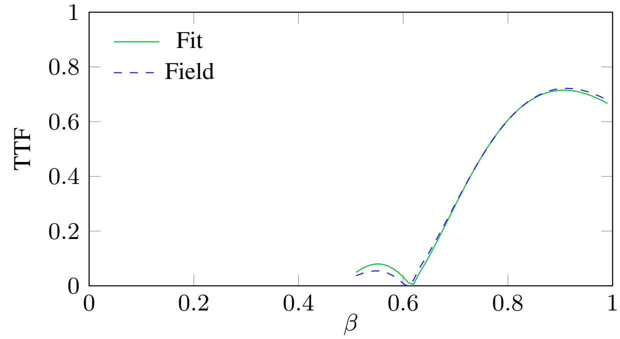

Figure 3: TTF calculated for a range of βs using the field from RF simulations and the fitted field.

Further Development

The new cavity model will be implemented to the ELS in near future and the results will be benchmarked against the kick-drift-kick model. The transverse field generation must further developed from the current Bessel expansion around the beam axis. The model at its current state could not generate fields with complete asymmetry in each single cell which is the case for some cavities, *e. g.* Spoke cavities.

CONCLUSION

The space charge model for a 3D Gaussian beam was here presented. This model is benchmarked against TraceWin in [6] showing that the dynamics predicted is in good agreement with a well known simulator.

The RF model here proposed is capable to generate the field map of a multy-gap cavity based on 15 parameters with a very small difference with respect to the classic field map. This new model for the cavities is under implementation in the ESS Linac Simulator and will be included in the next version.

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ISBN 978-3-95450-122-9