ANALYSIS OF THE NON-LINEAR FRINGE EFFECTS OF LARGE APERTURE TRIPLETS FOR THE HL-LHC PROJECT*

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Abstract

The HL-LHC project relies on large aperture quadrupoles which are compatible with the very large beam sizes in the inner triplets resulting from the strong reduction of beta*. As a result the beam is much more sensitive to non-linear perturbations in this region, such as those induced by the fringe fields of the low-beta quadrupoles. The spatial extension of these fringe fields increases as well more or less linearly with the coil aperture, which is an additional motivation to analyse this aspect in detail in the framework of the High Luminosity LHC design study. This paper will quantify this effect by direct analytical estimates using first order Hamiltonian perturbation theory, applied to quadrupole and dipole fringe fields. Both detuning with amplitude and chromatic effects will be considered. A numerical estimate for the proposed triplet quadrupoles will be presented, and the implementation of special symplectic integrators in SixTrack for tracking simulations outlined.

INTRODUCTION

The proposed High-luminosity upgrade of the LHC (HL-LHC) is based, among other systems, on new large aperture superconducting magnets [1]. Indeed, the triplet quadrupoles are planned to have 150 mm coil diameter [2] to be compared with 70 mm for the current triplet magnets [3]. Similarly, the new D1 superconducting separation dipole would feature 160 mm coil diameter [2] against 80 mm now [3][1]. Almost a factor of two increase is applied to the new magnets. This fact calls for a careful analysis of the impact of stray fields on the particle's dynamics.

The impact of the linear part of the stray fields in the triplet's quadrupoles has been addressed in Ref. [4]. Nevertheless, also the non-linear effects may play an important role and should be evaluated. This topic is the focus of this paper. The starting point is the analytical evaluation of the detuning with amplitude and chromatic effects induced by the fringe fields of the large aperture quadrupoles and dipoles. Then, in a second step, the long-term beam dynamics should be analysed taking into account this additional source of potential harmful effects. Such a step requires numerical simulation tools to be implemented in the SixTrack code [5], which is the workhorse for beam dynamics simulations at CERN.

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[1] It is worth mentioning that in the nominal layout the D1 separation dipole in the high luminosity insertions IR1 and 5 is made of normal conducting magnets.

ANALYTICAL CONSIDERATIONS

Quadrupoles

According to the analysis presented in Refs. [6, 7, 8], the vector potential for a field with quadrupolar symmetry can be expressed as

$$
\begin{aligned}
A_x(s) &= \frac{1}{2}\sum_{i,k=0}^{\infty} \frac{y^{2i+2}x^{2k+1}a'_{ik}(s)}{(2i+1)(2i+2)}\,, \\
A_y(s) &= -\frac{1}{2}\sum_{i,k=0}^{\infty} \frac{y^{2i+1}x^{2k+2}a'_{ik}(s)}{(2i+1)(2k+2)}\,, \\
A_s(s) &= \sum_{i,k=0}^{\infty} \left[\frac{y^{2i}x^{2k+2}a_{ik}(s)}{2k+2} - \frac{y^{2i+2}x^{2k}a_{ki}(s)}{2i+2}\right].
\end{aligned}
$$

with coefficients a_{ik} satisfying the relations:

$$
a_{ik} = \frac{2i+1}{2k+1}a_{ki}\,, \qquad \text{and if }' \text{ stands for the } s\text{-derivative}
$$
$$
a''_{i-1,k-1} + 2i(2i-1)a_{i,k-1} + 2k(2k+1)a_{i-1,k} = 0\,.
$$

The expansion of the vector potential reads

$$
\begin{aligned}
A_x(s) &= \frac{G'xy^2}{4} - \frac{G'''}{12}\left(\frac{y^2x^3}{4} + \frac{xy^4}{8}\right) + O(7)\,, \\
A_y(s) &= -\frac{G'x^2y}{4} + \frac{G'''}{12}\left(\frac{yx^4}{8} + \frac{x^2y^3}{4}\right) + O(7)\,, \\
A_s(s) &= \frac{G(x^2-y^2)}{2} - \frac{G''}{12}\frac{(x^4-y^4)}{4} + O(6)\,,
\end{aligned}
$$

and the Hamiltonian governing the particle's motion is

$$
H = -\sqrt{1 - \left(P_x - \frac{e}{pc}A_x\right)^2 - \left(P_y - \frac{e}{pc}A_y\right)^2} - \frac{e}{pc}A_s\,,
$$

where $P_{x,y}$ are the generalised momenta, normalised with respect to $p = p_0(1+\delta)$, p_0 being the design particle's momentum. A series expansion in the co-ordinates and $\delta = \Delta p/p_0$ gives for $\bar{H} = H + 1$

$$
\begin{aligned}
\bar{H} &= \frac{P_x^2+P_y^2}{2} + \frac{(P_x^2+P_y^2)^2}{8} + \frac{P_x\,A_x}{B\rho} + \frac{P_y\,A_y}{B\rho} + \frac{A_s}{B\rho} \\
&= \frac{P_x^2+P_y^2}{2} + \frac{(P_x^2+P_y^2)^2}{8} + \frac{K_1}{1+\delta}\frac{(x^2-y^2)}{2} + \\
&+ \frac{K_1'}{1+\delta}\frac{(P_x\,x\,y^2 - P_y x^2\,y)}{4} - \frac{K_1''}{1+\delta}\frac{(x^4-y^4)}{48} + O(5)\,,
\end{aligned}
$$

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The Hamiltonian provides the required information about the detuning with amplitude, assuming that the original variables are transformed to action-angle J_z, ϕ_z, where z stands for x, y, so that the Hamiltonian averaged over the angles reads

$$\langle H\rangle_{\psi_x,\psi_y} = -1 + \frac{J_x}{\beta_x} + \frac{J_y}{\beta_y} + \frac{K_1'}{4} J_x J_y(\alpha_y\beta_x - \alpha_x\beta_y) + \\ + \frac{K_1''}{32}(J_y^2\beta_y^2 - J_x^2\beta_x^2) + \frac{3}{16}\frac{J_y^2}{\beta_y^2}(1+\alpha_y^2)^2 + \\ + \frac{3}{16}\frac{J_x^2}{\beta_x^2}(1+\alpha_x^2)^2 + \frac{1}{4}\frac{J_xJ_y}{\beta_x\beta_y}(1+\alpha_x^2)(1+\alpha_y^2)\,.$$

The tune shift is the computed as

$$\begin{aligned}\Delta\nu_x &= \frac{1}{2\pi}\oint \frac{\partial}{\partial J_x}\langle H\rangle_{\psi_x,\psi_y} ds = \alpha_{xx}J_x + \alpha_{xy}J_y\,,\\ \Delta\nu_y &= \frac{1}{2\pi}\oint \frac{\partial}{\partial J_y}\langle H\rangle_{\psi_x,\psi_y} ds = \alpha_{yy}J_y + \alpha_{yx}J_x\,,\end{aligned}$$

where

$$\begin{aligned}\alpha_{zz} &= \frac{3}{16\pi}\oint \frac{(1+\alpha_z^2)^2}{\beta_z^2} ds - \frac{1}{32\pi}\oint K_1''\beta_z^2 ds\,,\\ \alpha_{xy} &= \frac{1}{8\pi}\oint \frac{(1+\alpha_x^2)(1+\alpha_y^2)}{\beta_x\beta_y} ds + \\ &+ \frac{1}{8\pi}\oint K_1'(\alpha_y\beta_x - \alpha_x\beta_y) ds\,,\\ \alpha_{yx} &= \alpha_{xy}\,.\end{aligned}$$

Each term $\alpha_{z_1z_2}, z_1, z_2 = x, y$ consists of two parts, one depending on the optics, the so-called kinematic term, and one depending on s-derivatives of K_1, the latter being the direct effect of the fringe field. Using β_x^* for the value of the beta-function at the interaction point (IP), L for quadrupole length, and L^* for the length of the drift from the IP to quadrupole, then in such a drift $\beta_x(s) = \beta_x^* + s^2/\beta_x^*$, $\alpha_x(s) = -s/\beta_x^*$, $(1+\alpha_x(s)^2)/\beta_x(s) = 1/\beta_x^*$. Then the kinematic terms in the drift are given by

$$\alpha_{zz,kin} = \frac{3}{16\pi}\frac{L^*}{{\beta_z^*}^2}\,, \qquad \alpha_{xy,kin} = \frac{1}{8\pi}\frac{L^*}{\beta_x^*\,\beta_y^*}\,.$$

In a very similar way it is possible to derive the fringe field contribution to chromaticity starting from its very definition and assuming that dispersion can be neglected, obtaining

$$\begin{aligned}\frac{\partial\Delta\nu_x}{\partial\delta} &= \xi_x + \xi_{xx}J_x + \xi_{xy}J_y\,,\\ \frac{\partial\Delta\nu_y}{\partial\delta} &= \xi_y + \xi_{yy}J_y + \xi_{yx}J_x\,,\end{aligned}$$

where the coefficients $\xi_z, \xi_{z_1z_2}$ are given by

$$\begin{aligned}\xi_z &= -\frac{1}{4\pi}\oint K_1\beta_z ds,\\ \xi_{zz} &= \frac{1}{32\pi}\oint K_1''\beta_z^2 ds\,,\\ \xi_{xy} &= -\frac{1}{8\pi}\oint K_1'(\alpha_y\beta_x - \alpha_x\beta_y) ds\,,\\ \xi_{yx} &= \xi_{xy}\,.\end{aligned}$$

It is clear that the computation of the contribution of the fringe fields effect implies specifying a model for the s-dependence of K_1, and specific examples are given in Refs. [9, 10, 11, 12]. The results of numerical computations will be presented in a next section.

Dipoles

The computations carried out previously can be extended to the case of dipolar fringe fields. Assuming co-ordinates $u^i = \{x, s, y\}$, a basis $\vec{a}_1 = \vec{e}_x$, $\vec{a}_2 = (1+Kx)\vec{e}_s$, $\vec{a}_3 = \vec{e}_y$ and $K = -\dfrac{eB_0}{p_0c}$, then the vector potential reads as $A_1 = A_x$, $A_2 = (1+Kx)A_s$, $A_3 = A_y$ and its expansion as

$$\begin{aligned}A_1(s) &= B_0'\frac{y^2}{4} - KB_0'\frac{xy^2}{4} + K^2B_0'\frac{x^2y^2}{4} + \\ &- B_0'''\frac{y^4}{48} + O(5)\,,\\ A_2(s) &= B_0x + KB_0\frac{x^2}{2} - B_0''\frac{xy^2}{4} + K'B_0'\frac{x^2y^2}{8} + \\ &+ KB_0''\frac{x^2y^2}{8} - (2KB_0'' + K'B_0')\frac{y^4}{24} + O(5)\,,\\ A_3(s) &= -B_0'\frac{xy}{2} + KB_0'\frac{x^2y}{4} + \\ &- K^2B_0'\frac{x^3y}{6} + B_0'''\frac{xy^3}{12} + O(5)\,.\end{aligned}$$

In this case the Hamiltonian can be expanded as

$$\begin{aligned}H = &-1 + K^2\frac{x^2}{2} + \frac{P_x^2}{2} + \frac{P_y^2}{2} + \frac{(P_x^2+P_y^2)^2}{8} + \\ &+ Kx\frac{P_x^2+P_y^2}{2} + K'\frac{y^2\,P_x}{4} - K'\frac{x\,y\,P_y}{2} + \\ &- K''\frac{x\,y^2}{4} - (K'^2 + 8KK'')\frac{y^4}{96} - K'K\frac{x^2\,y\,P_y}{4} + \\ &+ (2K'^2 + KK'')\frac{x^2\,y^2}{8} + O(5)\,,\end{aligned}$$

if the dependence on δ is neglected. The tune shift can be derived also for this case and the result reads

$$\begin{aligned}\alpha_{xx} &= \frac{3}{16\pi}\oint \frac{(1+\alpha_x^2)^2}{\beta_x^2} ds\,,\\ \alpha_{xy} &= \frac{1}{8\pi}\oint \frac{(1+\alpha_x^2)(1+\alpha_y^2)}{\beta_x\beta_y} ds + \\ &+ \frac{1}{16\pi}\oint \beta_x(\alpha_y\,K\,K' + \beta_yK'^2 + \beta_yK\,K'') ds\,,\\ \alpha_{yx} &= \alpha_{xy}\,,\\ \alpha_{yy} &= \frac{3}{16\pi}\oint \frac{(1+\alpha_y^2)^2}{\beta_y^2} ds + \\ &- \frac{1}{64\pi}\oint (K'^2 + K\,K'')\beta_y^2 ds\,.\end{aligned}$$

The kinetic part of the coefficients $\alpha_{z_1z_2}$ is the same for dipoles and quadrupoles, while the proper fringe field terms are different. Furthermore, the symmetry between the H- and V-plane that is apparent in the fringe field contributions

Table 1: Contributions to the amplitude detuning and chromaticity by the triplet quadrupoles of IP5

Name	α_{xx}	$\alpha_{xy} = \alpha_{yx}$	α_{yy}	ξ_{xx}	$\xi_{xy} = \xi_{yx}$	ξ_{yy}
MQXC.3L5	$3.7 \cdot 10^3$	$8.8 \cdot 10^3$	$6.7 \cdot 10^3$	$-3.6 \cdot 10^3$	$-8.7 \cdot 10^3$	$-6.5 \cdot 10^3$
MQXC.B2L5	$6.3 \cdot 10^3$	$-10 \cdot 10^3$	$3.1 \cdot 10^3$	$-6.2 \cdot 10^3$	$10 \cdot 10^3$	$-2.8 \cdot 10^3$
MQXC.A2L5	$4.1 \cdot 10^3$	$7 \cdot 10^2$	$5.6 \cdot 10^2$	$-3.7 \cdot 10^3$	$-7 \cdot 10^2$	$-5.6 \cdot 10^2$
MQXC.1L5	$2.9 \cdot 10^3$	$-1.7 \cdot 10^3$	$4 \cdot 10^2$	$-2.5 \cdot 10^3$	$1.7 \cdot 10^3$	$-4 \cdot 10^2$
MQXC.1R5	$4 \cdot 10^2$	$1.7 \cdot 10^3$	$2.9 \cdot 10^3$	$-4 \cdot 10^2$	$-1.7 \cdot 10^3$	$-2.5 \cdot 10^3$
MQXC.A2R5	$5 \cdot 10^2$	$7 \cdot 10^2$	$4.1 \cdot 10^3$	$-5 \cdot 10^2$	$-7 \cdot 10^2$	$-3.8 \cdot 10^3$
MQXC.B2R5	$3.1 \cdot 10^3$	$10 \cdot 10^3$	$6.3 \cdot 10^3$	$-2.8 \cdot 10^3$	$-10 \cdot 10^3$	$-6.2 \cdot 10^3$
MQXC.3R5	$6.7 \cdot 10^3$	$-8.6 \cdot 10^3$	$3.7 \cdot 10^3$	$-6.5 \cdot 10^3$	$8.7 \cdot 10^3$	$-3.6 \cdot 10^3$
Total	$2.8 \cdot 10^4$	$1.6 \cdot 10^3$	$2.8 \cdot 10^4$	$-2.6 \cdot 10^4$	$-1.4 \cdot 10^3$	$-3.1 \cdot 10^4$

in the quadrupoles is broken for the dipole case. A number of considerations can be made. The underlying assumption for the computations presented here is that neither the beam trajectory, nor the beam optics is affected, at least to first order, by the stray fields as this allows using, e.g., the unperturbed optical parameters for the estimate of the stray fields effects for quadrupoles. As the extension of the fringe fields is proportional to the aperture it is justified to assume that larger aperture magnets can introduce a stronger perturbation of the beam dynamics. As the value of the beta-functions is considerably different at the location of the triplet quadrupoles and the separation dipole, the impact of the fringe fields effects is rather different in the two cases. Therefore, in the next section the focus will be on the numerical evaluation of the fringe fields effects for quadrupoles. Finally, integration by parts could be used to convert derivative of K, K_1 into derivatives of the optical parameters.

NUMERICAL CONSIDERATIONS

The formalism developed in the previous sections has been applied by considering magnet measurements of stray fields in models of triplet quadupoles [13] shown in Fig. 1. A special layout for the HL-LHC has been used, featuring

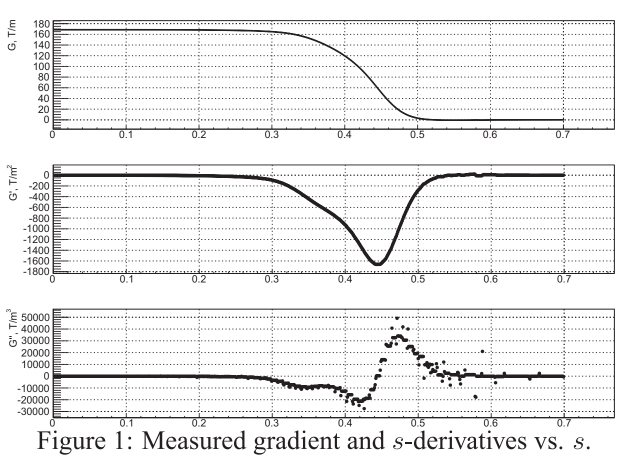

Figure 1: Measured gradient and s-derivatives vs. s.

round optics with $\beta^* = 10$ cm. By assuming $\epsilon_x = \epsilon_y = 5 \times 10^{-4}$ μm and action with amplitude of one sigma is $J_x = \varepsilon_x/2 = 2.5 \times 10^{-10}$ m. The contribution of the kinematic part of the $\alpha_{z_1 z_2}$ terms is negligible, ranging from about 1.4×10^2 m^{-1} to 4.6×10^1 m^{-1} for the drift and the quadrupole, respectively. The total effect is summarised in Table 1.

CONCLUSIONS AND OUTLOOK

Although the effect of the fringe fields is small, nevertheless it cannot be completely neglected. This means that the effect on the long-term beam dynamics should be evaluated via tracking simulations. Currently SixTrack does not provide the tools to perform symplectic integration of stray fields. Therefore, a new implementation is being considered, based on the approach presented in Refs. [14, 15] and a preliminary analysis has been made [16].

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