

# BEAM OPTICS MEASUREMENTS THROUGH TURN BY TURN BEAM POSITION DATA IN THE SLS

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## Abstract

Refined Fourier analysis of turn-by-turn (TBT) transverse position data measurements can be used for determining several beam properties of a ring, such as transverse tunes, optics functions, phases, chromatic properties and coupling. In particular, the Numerical Analysis of Fundamental Frequencies (NAFF) algorithm is used to analyze TBT data from the Swiss Light Source (SLS) storage ring in order to estimate on and off-momentum beam characteristics. Of particular interest is the potential of using the full position information within one turn in order to measure beam optics properties.

## INTRODUCTION

The SLS is a third generation light source which provides photon beams of high brilliance to 20 beam lines. The SLS a storage ring of 288 m with 2.4 GeV nominal energy and transverse tunes of (20.44,8.74), is equipped with beam position monitor (BPM) electronics capable of measuring the turn-by-turn position of the beam around the ring. In order to estimate the transverse tunes and other ring parameters, the bunches are kicked transversally by a kicker magnet to induce coherent betatron oscillations. The resulting data are analysed with refined Fourier algorithms. In this paper, the NAFF [1] method is employed in order to analyse a sample of TBT horizontal and vertical position data from the 73 BPMs of the SLS ring. Of particular interest is the comparison of tune evaluation with the traditional method of Fourier analysing single BPM data and for an alternate and ultra-fast method mixing all BPM signals for every turn [2]. Beta function estimations are also possible with two methods, using the Fourier amplitude of the main spectral line or by using the phase information between three consecutive BPMs [3] and its comparison with a perfect lattice model.

## BETATRON TUNES MEASUREMENTS

In the presence of chromaticity and amplitude dependent tune-shift, the BPM signal decoheres leaving a limited number of turns to be analysed, but also a tune modulation. On the other hand, the precision of the tune evaluation grows with the number of available turns. In this respect, and using the NAFF algorithm, the tunes for each BPM were estimated for different time-spans. In this first analysis, it was apparent that the signals from two BPMs (16 and 43) were noisy and they had to be ignored. In Fig. 1, the horizontal and vertical betatron frequencies are plotted

versus the BPM number for 50 (purple), 100 (blue) and 200 (red) turns. The fluctuation of the tune estimate for all the BPMs is large for 50 turns and reduced to minimum for 200 turns.

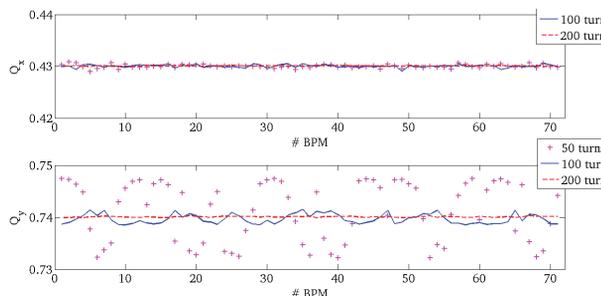


Figure 1: Measured horizontal (top) and vertical (bottom) betatron tunes as versus the BPM number for the analysis of 50 (purple), 100 (blue) and 200 (red) turns.

In Fig. 2, the standard deviation of the measured horizontal (blue) and vertical (red) tunes over all the BPMs is plotted as a function of the turn number. This provides an additional information about the accuracy with which the tunes are estimated for the different amount of analysed turns. The standard deviation for the horizontal plane is already less than  $10^{-3}$  for only 40 turns and drops even below  $10^{-4}$  towards 200 turns. In the vertical plane, though, at least 100 turns are needed for the standard deviation to reach the  $10^{-3}$  level. There is also a slight modulation apparent mostly in the vertical plane, which should be due to the fact the the vertical chromaticity is larger.

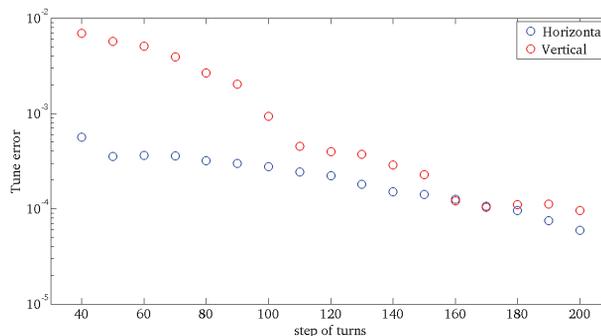


Figure 2: Standard deviation for the horizontal (blue) and vertical (red) tunes as measured for all BPMs as a function of the number of analysed turns.

The previous analysis has shown that more than 100 turns are needed for estimating with adequate precision the

tunes. An alternative method [2], which allows a very fast and accurate tune determination, consists of combining the data from all the BPMs and analysing them, ignoring the fact that the monitors are not symmetric neither in longitudinal position nor with respect to the machine optics. In Fig. 3, the horizontal (top right) and vertical (top left) tune estimates are plotted against the number of turns, combining the data from all the BPMs. The tunes converge to a value within 10 to 20 turns. This is also testified by the bottom plots, where the difference between consecutive tune values is presented as a function of the number of analysed turns. This difference becomes less than  $10^{-4}$  within 10 turns. It is worth noting that this method enables to evaluate accurately the integer part of the tune, provided that there are more BPMs than the tune integer units, as in the SLS case. In the previous analysis, all BPMs were used, including the "noisy" ones, without affecting in any significant way the final result.

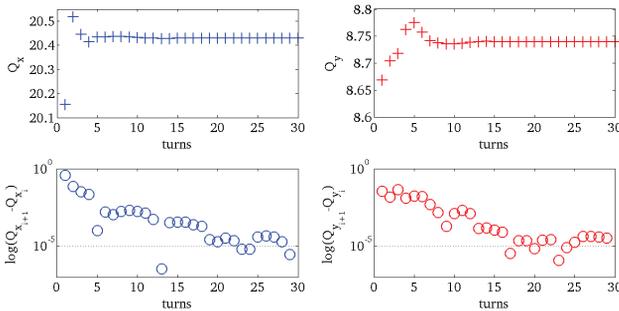


Figure 3: Horizontal (top left) and vertical (top right) tunes estimated by analysing all BPM data in consecutive turns, versus the number of turns, and consecutive difference between the tune estimates (bottom).

### BETATRON FUNCTION ESTIMATION

The Fourier component amplitude  $A_z^i$  of the spectral lines associated to the main tunes, for any plane  $z$ , is proportional to the square root of the beta functions  $\beta_z$ , at each BPM location  $i$ :

$$A_z^i = c_z \cdot S_i \sqrt{\beta_z^i} \quad (1)$$

where  $c_z$  is a constant which is related to a pseudo-invariant "emittance" and depends on the amplitude of the oscillation and  $S_i$  is a calibration factor which depends on the electronics of each BPM. Assuming that the monitors are well calibrated (i.e.  $S_i$  is close to 1), it is possible to estimate the beta function by a linear fit of the squared Fourier amplitudes to the linear machine model. Thereby the coefficient  $c_z$  can be determined. The estimate of this coefficient for the horizontal (blue curve) and vertical (red curve) plane is presented in the top plot of Fig. 4 as a function of the different time steps used in the analysis. The coefficient value drops as the beam decoheres, but at the same time the error bars corresponding to the standard deviation

of the least square fit are getting smaller, especially in the vertical plane, where at least 100 turns are needed for the standard deviation to become less than  $10^{-4}$ . In the horizontal plane, the fit works right from the beginning, even for 40 turns. This is also shown in the bottom plot where the correlation coefficient  $R^2$  is plotted for the horizontal and vertical data fit. In the horizontal plane the correlation coefficient is very close to 1 already for 40 turns.

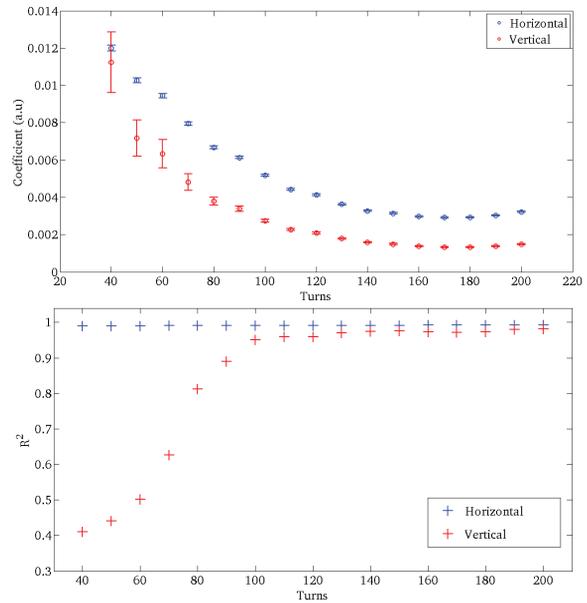


Figure 4: Constant  $c$  calculated from first order linear fit of  $A_z^2$  with SLS simulated  $\beta_{z_m}(s)$  for horizontal and vertical plane, along with the corresponding statistical offset from the fit. Increase of the step of turns converges to a single value of  $c$  with the least offset.

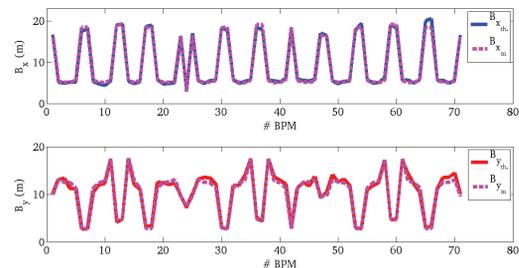


Figure 5: Horizontal (top solid blue) and vertical (bottom solid red) beta function given by the ideal SLS model, as compared to the measured beta functions (dashed magenta) for 200 turns.

The estimated beta function values for all BPM's around the SLS ring, after an analysis of 200 turns, are shown in Fig. 5 for the horizontal (top) and vertical (bottom) plane. For comparison, the ideal model beta functions are plotted. The agreement between the measured and ideal model values is excellent. This can be also observed in Fig. 6, where the rms value for all the BPMs of the relative beta difference between the model and the measurement is plot-

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ted as a function of the number of analysed turns. The rms difference is quite small in the horizontal plane right from the smallest number of turns, whereas for the vertical plane becomes around 10 % for 100 turns. For 200 turns, the horizontal and vertical rms differences are around 5% and 7%, testifying both the very good accuracy of the method and the fact that the SLS storage ring in operation is very close to the ideal machine model.

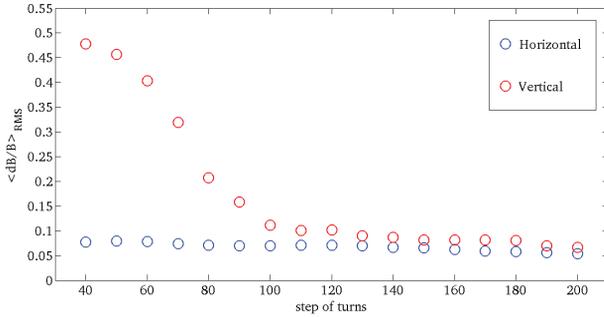


Figure 6: Horizontal (blue) and vertical (red) rms beta function relative difference between the ideal model and the measured data for all the BPMs as a function of the number of analysed turns.

The phases of the main spectral lines for each BPM can be also used for evaluating the difference of the beta function between the machine and the model [3]. Using the linear transfer matrices for 3 consecutive BPMs at locations  $s_1$ ,  $s_2$  and  $s_3$ , it can be shown that measured functions  $\tilde{\beta}$  should be proportional to the model  $\beta$ , with a proportionality factor depending on the measured phase advances  $\tilde{\phi}_{ij}$  and model ones  $\phi_{ij}$ , between the BPM at position  $i$  and  $j$ :

$$\begin{aligned} \tilde{\beta}_1 &= \beta_1 \frac{\cot \tilde{\phi}_{12} - \cot \tilde{\phi}_{13}}{\cot \phi_{12} - \cot \phi_{13}} \\ \tilde{\beta}_2 &= \beta_2 \frac{\cot \tilde{\phi}_{12} - \cot \tilde{\phi}_{23}}{\cot \phi_{12} - \cot \phi_{23}} \\ \tilde{\beta}_3 &= \beta_3 \frac{\cot \tilde{\phi}_{23} - \cot \tilde{\phi}_{13}}{\cot \phi_{23} - \cot \phi_{13}} \end{aligned} \quad (2)$$

As this method can be applied in a sliding window of three consecutive BPMs, three beta estimates per location can be obtained allowing for some statistics. The obtained average beta function as compared to the model is shown in Fig. 7 for an analysis of the first 200 turns. The agreement is quite good, although some discrepancies exist, especially in the vertical plane. This can be also observed in Fig. 8, where the relative beta difference between the model and measurement is plotted. The reason for this discrepancy is still under investigation.

### CONCLUSIONS

The NAFF algorithm was applied on beam position TBT data of the SLS storage ring. The measured fractional tune was measured quite accurately in around 100 turns with the

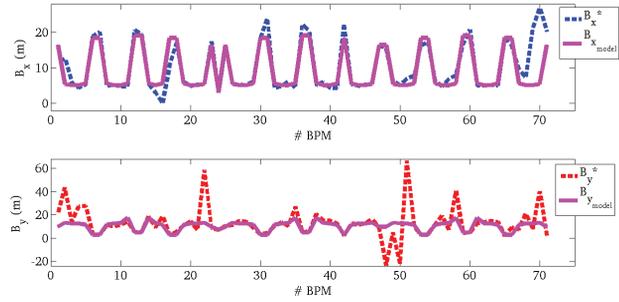


Figure 7: Horizontal (top solid blue) and vertical (bottom solid red) beta function given by the ideal SLS model, as compared to the measured beta functions (dashed magenta) for 200 turns.

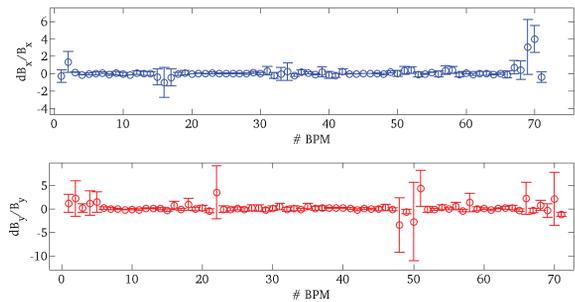


Figure 8: Horizontal (blue) and vertical (red) rms beta function relative difference between the ideal model and the measured data for all the BPMs as a function of the number of analysed turns. The error bars represent one standard deviation of the three estimated values for each BPM location.

traditional method of analysing single BPMs. The potential of the method of analysis by mixing BPM data in every turn and analysing them as a block, was once again demonstrated, enabling the measurement on the tune in around 10 turns. The amplitude and the phase of the principle spectral line for each BPM were finally used in order to measure the beta functions around the ring, which were found quite close to the ideal machine model.

### REFERENCES

- [1] J. Laskar, “Frequency analysis for multi-dimensional systems. global dynamics and diffusion”, Physica D 67, 257281, 1993.
- [2] Y. Papaphilippou et al., “Experimental Frequency Maps for the ESRF storage ring”, EPAC04, Lucerne, 2004.
- [3] P. Castro-Garcia, Doctoral Thesis, Luminosity and beta function measurement at the electron-positron collider ring LEP, CERN SL/96-70, p.51-57.