

# DOUBLE-TIP MAGNETIC FIELD ENHANCEMENT\*

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## Abstract

It is well known that sharp protrusions (tips) in microwave cavities enhance the local electric field. A pair of such tips can also increase the magnetic field. That is, our simulations show that when two tips are located along the path of rf current, there can be a significant magnetic field enhancement,  $\beta_H$ , between the tips, while the electric field enhancement can be fairly small. Moreover,  $\beta_H$  is determined purely by geometry and is independent of rf frequency and mode type. This double-tip magnetic field enhancement could play a role in triggering rf breakdown. In this paper, we present single and double tip simulation results and discuss their implications.

## INTRODUCTION

Although vacuum breakdown has been studied for more than a century, there are still many questions not fully answered, such as what triggers breakdown, and what is the physics behind the electric field enhancement factor (i.e.  $\beta_E$ ) deduced from the Fowler-Nordheim formulation of field emission. There have been many attempts to simulate the evolution of rf breakdown where a sharp protrusion and dense neutral gas [1-3] are assumed to exist initially. However, protrusions with large  $\beta_E$  are not generally seen in scanning electron microscope (SEM) images, and a mechanism that would generate a dense gas on a short time scale has not been clearly identified.

In this paper, we examine the field enhancement in an rf cavity from a double-tip structure that is oriented parallel to the surface current flow. We show the magnetic field enhancement factor,  $\beta_H$ , sharply increases as the gap between the two tips is reduced, even for small electric field enhancements. This is purely a geometrical effect, independent of rf frequency and mode type. With a large  $\beta_H$ , the resulting pulsed heating could be high enough to lead to surface cracking and melting, which may be a mechanism for generating a dense metal gas. This, together with an electric field enhancement from the altered surface, may then trigger rf breakdown.

## SINGLE TIP SIMULATION RESULTS

We first consider the electric and magnetic field enhancement for a single tip. A plane-wave normally incident on an infinite metal surface with a tangential magnetic field strength  $H_0$  will drive a surface current. When the current is blocked by a tip, it will be scattered and the charge density will increase at the end of the tip,

which enhances electric field there. Meanwhile, the surface current will be redistributed and enhance the magnetic field along the tip body. If the tip is small enough, much less than the rf wavelength, the magnetic field distribution around the tip is close to that for a perfect conductor in a uniform magnetic field, which is a sinusoidal distribution with a maximum value of  $2H_0$  [4].

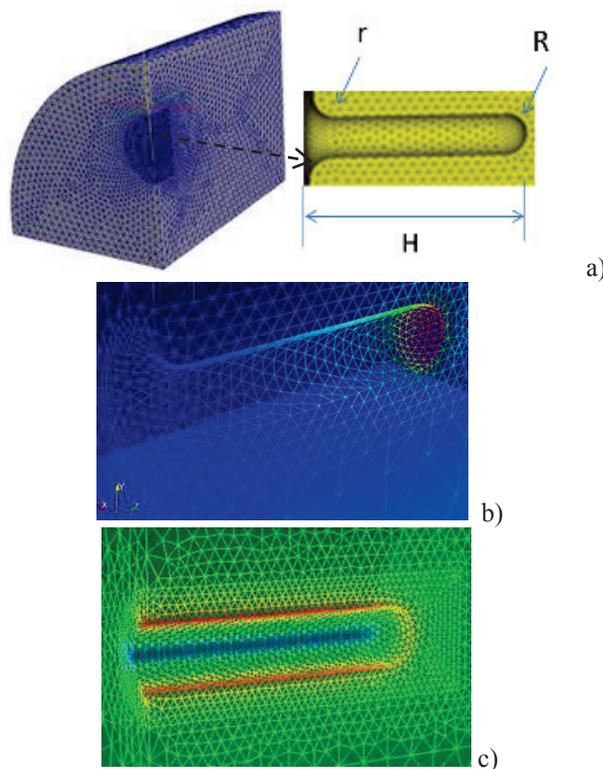


Figure 1: a) Cut-away view of single tip in a pill-box cavity with an 11.4 GHz TM<sub>010</sub> mode, b) and c) are, respectively, the electric and magnetic field distributions around the tip for this mode.

For our simulations, we assume the following: 1) the tips and rf cavity are perfect conductors, 2) the tip dimensions are of the order of microns and are much smaller than the rf wavelength, 3) the tips have a cylindrical shape with hemispherical end-caps, 4) the tip radii, heights and the gaps between them (measured from their centers) are denoted by  $R$ ,  $H$ , and  $d$ , respectively, and 5)  $\beta_E$  and  $\beta_H$  are the maximum surface electric and magnetic field enhancement factors due to the tips.

We modelled a single tip in a pill-box cavity with a TM<sub>010</sub> mode at 11.4 GHz, where the tip is located off center to avoid the current null (see Fig. 1a). To determine the surface field around the tip, the parallel finite element

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eigen-solver Omega3p was used that allows high-fidelity modelling of the geometry with large disparate scales [5].

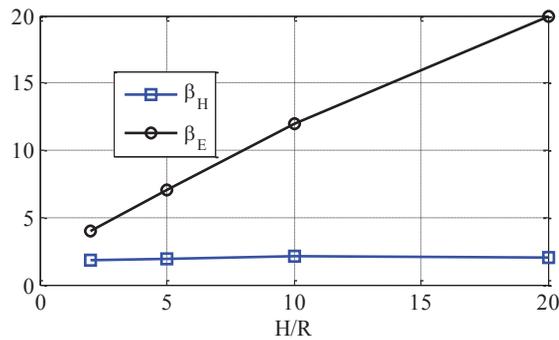


Figure 2: Maximum electric and magnetic field enhancement for a  $H = 50 \mu\text{m}$  single tip as a function of  $H/R$ .

The results show a magnetic field enhancement along the tip (see Fig. 1b) and an electric field enhancement at the end of the tip (see Fig. 1c). As expected, with a short ( $50 \mu\text{m}$ ) tip,  $\beta_H$  is almost constant and equal to 2, independent of tip radius. This is illustrated in Fig. 2, which also shows that  $\beta_E$  increases with smaller radii, i.e. roughly linearly as  $H/R$ .

### DOUBLE TIP SIMULATION RESULTS

We now add a second tip, in line with the radial rf current as shown in Fig. 3a. While the electric field enhancement is similar to that of a single tip (see Fig. 3b), the magnetic field between the tips is enhanced a lot (see Fig. 3c). The values of  $\beta_H$  between the tips is shown in Fig. 4 as function of  $d/R$  for various values of  $H/R$ . One sees that larger  $H/R$  and smaller  $d/R$  produce larger values of  $\beta_H$ .

We next simulated  $\beta_H$  at different frequencies and for different rf modes. The tips were lined radially and azimuthally on a pillbox cavity end plate for TM and TE modes, respectively. For  $H/R=2$ , the values of  $\beta_H$  between the tips for TM and TE modes are plotted in Fig. 5 as a function of  $d/R$ . One sees that  $\beta_H$  is independent of rf frequency and mode type, and that it sharply increases as the gap of the two tips is reduced. Thus, with our assumptions,  $\beta_H$  depends only on geometry, that is, on  $H$ ,  $R$  and  $d$ .

Note that the double-tip magnetic field enhancement is similar to a tip-on-tip electric field enhancement [6], where the second tip in each case needs to be at the position of the maximum magnetic or electric field to further enhance the field.

A capacitor model is being developed to quantify the magnetic field enhancement. It has the following form:

$$\beta_H = A \cdot g(H, R) \cdot f(H, R, d) \quad (1)$$

where

$$f(R, H, d) = 1 + \frac{k}{\left[ a + b \left( \frac{H}{R} \right)^{-1} \right] \ln \left[ \frac{d}{2R} + \sqrt{\left( \frac{d}{2R} \right)^2 - 1} \right]} \quad (2)$$

and  $A$ ,  $k$ ,  $a$  and  $b$  are constants related to the shape of the tip.

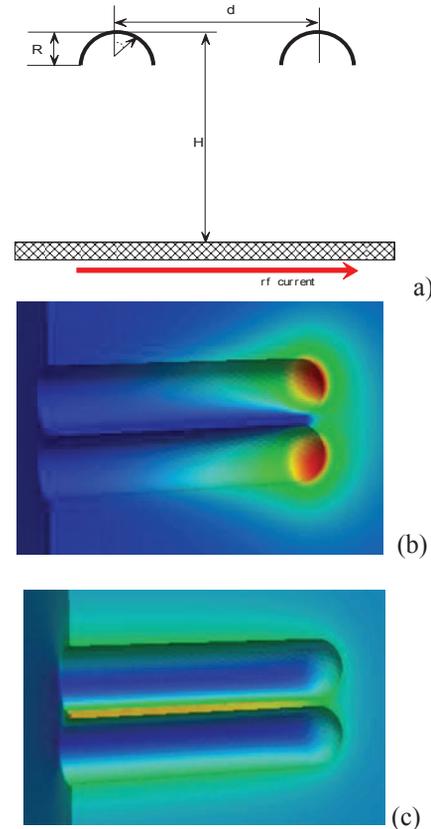


Figure 3: a) Double-tip layout, b) the electric field distribution around the two tips, which has the same peak value as that of a single tip, and c) the magnetic field distribution around the two tips, which is further enhanced compared to a single tip.

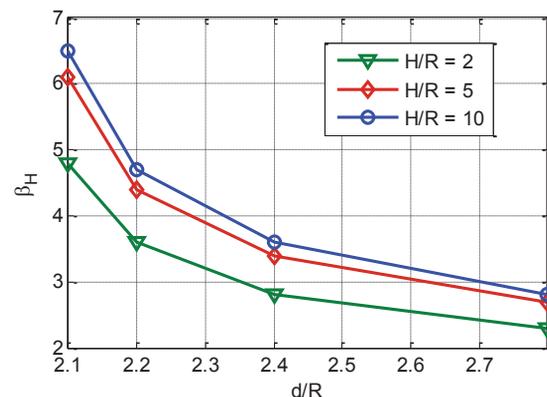


Figure 4: Magnetic field enhancement as a function of tip spacing (i.e.,  $d/R$ ) for different tip aspect ratios (i.e.,  $H/R$ ).

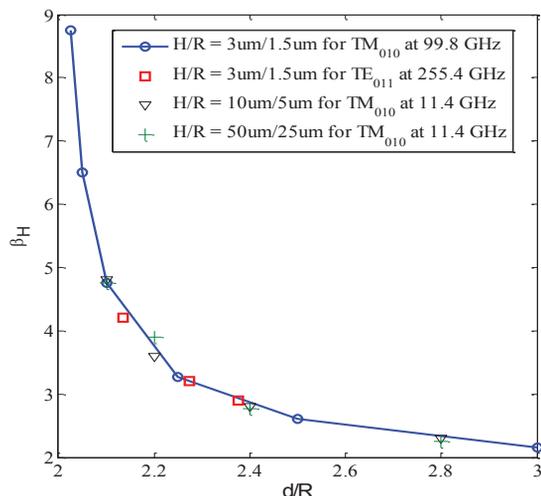


Figure 5: For  $H/R = 2$ ,  $\beta_H$  as a function of  $d/R$  for different rf frequencies, modes and values of  $H$  and  $R$ .

### SUMMARY AND DISCUSSION

Simulations of a double-tip structure oriented parallel to the rf current in a cavity show that a significant surface magnetic field enhancement can be produced. For small tips, this enhancement is a function of the tip geometry only. A capacitor model is being developed to reproduce the simulation results, and works reasonable well for the comparisons made so far.

In general, a magnetic field enhancement will occur for any tip shape (i.e., not just cylindrical) as long as such a double structure is oriented parallel to the surface current. Also, this enhancement can be large even when the electric field enhancement is small. Since pulsed heating scales as the square of  $\beta_H$ , and some structures already run with significant pulse heating, such enhancements could in principle cause local cracking and melting, and trigger a breakdown as noted above.

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