

MEASUREMENT OF MOMENTUM COMPACTION FACTOR VIA DEPOLARIZING RESONANCES AT ELSA*

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Abstract

Measuring beam depolarization at energies in close proximity to a depolarizing integer resonance is an established method to determine the beam energy of a circular accelerator. This technique offers high accuracy due to the small resonance widths. Thus, also other accelerator parameters related to beam energy can be measured based on this method. This contribution presents a measurement of the momentum compaction factor with a high precision of 10^{-4} . It was performed at the 164 m stretcher ring of the Electron Stretcher Facility ELSA at Bonn University, which provides a polarized electron beam of up to 3.2 GeV.

MEASUREMENT PRINCIPLE

Precise knowledge of key accelerator parameters is important for the analysis of numerous beam dynamics measurements and user experiments as well as to adjust the accelerator model.

One essential parameter is the beam energy. An established method to precisely determine the beam energy in a storage ring is based on spin polarization measurements. If both a polarized beam and polarimetry are available, a decrease of polarization at certain, theoretically well known beam energies can be observed. The characteristic energy of such a depolarizing resonance can be determined experimentally by measuring polarization at various storage energies around the expected resonance. The small energy width of these resonances allows for an energy measurement with a precision of $\Delta E/E \lesssim 10^{-4}$.

Building on this procedure, other accelerator parameters can be measured with likewise precision, if they are related to beam energy. One example is the momentum compaction factor. Recently, it was measured at the ELSA stretcher ring at Bonn University using a polarized electron beam.

DEPOLARIZING RESONANCES

In a flat circular accelerator (without solenoids) the stable spin axis, also known as invariant spin axis, usually points in vertical direction due to the strong vertical guiding fields of the bending magnets. The spins of revolving electrons precess around this axis γa times per turn according to the Thomas-BMT equation [1]. The spin tune $Q_{\text{spin}} = \gamma a$ is given by the gyromagnetic anomaly $a = (g_s - 2)/2$ of the electron and the Lorentz factor γ and thus depends linearly on beam energy. Only the projection of the polarization on the invariant spin axis can be preserved, since the finite energy width of the beam implies a spin tune spread, that leads to a spin spreading in the precession plane.

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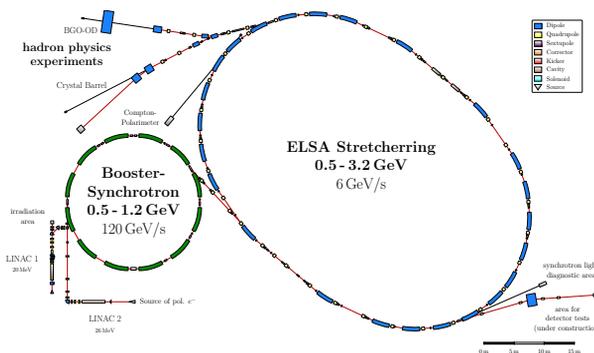


Figure 1: The Electron Stretcher Facility ELSA in Bonn.

The electrons experience horizontal magnetic fields along their trajectories occurring with specific spatial frequencies, which are related to the accelerator lattice and beam dynamics. On the one hand, magnet misalignments and vertical closed orbit distortions cause revolution harmonic contributions, which are not present in an ideal accelerator. On the other hand, betatron motion causes contributions of the quadrupole magnets depending on the betatron tunes.

Usually, spin precession around these horizontal magnetic fields averages out due to the changing phase relationship between these fields and the precession around the vertical axis. However, for certain beam energies the spin tune is in phase with one of the horizontal field contributions and the spin motion results in an additional precession around the horizontal fields. On such a depolarizing resonance, the stable spin axis is aligned completely within the horizontal plane and a beam stored at this energy has zero vertical polarization.

Though, the resonance energy can be determined experimentally by measuring vertical polarization at several energies in close proximity to the resonance. The measurements presented below use one of the so called integer resonances, which occur every 440 MeV at spin tunes

$$\gamma a = n \in \mathbb{N}. \quad (1)$$

In this case, the theoretically expected resonance energy is known precisely, because it only depends on the gyromagnetic anomaly a of the electron. For that reason, the corresponding beam energy can be precisely determined by this method. Integer resonances are driven by revolution harmonic horizontal field distributions. They are mostly generated by vertical closed orbit displacements in the quadrupole magnets and rotations of the bending dipole magnets around the beam axis.

When crossing a depolarizing resonance on an energy ramp, the preserved vertical polarization decreases and increases with the strength of the resonance driving fields and increases

with the crossing speed as described by the Froissart-Stora formula [2].

POLARIZED ELECTRON BEAM AT ELSA

For more than a decade, a polarized electron beam with typically 2.35 GeV has been provided by the Electron Stretcher Facility ELSA at Bonn University (Fig. 1). The polarized electrons produced by an inverted source are accumulated in the 164 m stretcher ring via an 1.2 GeV booster synchrotron, then accelerated utilizing a fast energy ramp with up to 6 GeV/s. Thereby, a polarized beam at any storage energy up to 3.2 GeV is available within 1 s. Then, the stored beam can be extracted slowly via resonance extraction. Afterwards, the magnets of the stretcher ring are ramped back to injection energy within some 100 ms and a new cycle is started. In this way, a duty factor of more than 80 % is achieved for the hadron physics experiments.

Each depolarizing resonance which is crossed during the energy ramp can reduce the polarization significantly. Though, every single resonance has to be compensated applying dedicated countermeasures [3]. For example, at ELSA the influence of each integer resonance is reduced with an empirically adjusted harmonic field distribution [4]. Including all countermeasures a polarization of up to 70 % can be achieved at 2.35 GeV.

The internal Compton polarimeter in the stretcher ring is still under construction. So the polarization of the extracted beam is measured with a Møller polarimeter at the Crystal Barrel experiment. During the measurements presented below, about 20 mA were accumulated in the stretcher ring and ramped up close to the integer resonance $\gamma a = 5$ slightly above 2.2 GeV. A beam current of about 1 nA was continuously extracted to the polarimeter for 5 s. Each polarization measurement comprised about 3 min (30 accelerator cycles) to reach a statistical error below 2.5 %.

ENERGY CALIBRATION

Figure 2 shows the vertical beam polarization, measured by the Møller polarimeter, in the vicinity of the integer resonance $\gamma a = 5$. It is plotted against the extraction energy set in the control system.

The polarization is quite low, because the machine was not optimized for high polarization during these measurements. Nevertheless, the resonance can be clearly identified from the measurement: If the beam is stored at an energy below the resonance, it obviously has no influence. Exactly on the resonance, there is zero vertical polarization, as explained above. When crossing the resonance on the energy ramp, its effect depends on the resonance strength. In case of high resonance strength, the spins are flipped to the opposite direction as in the measurement shown in red. Here, the resonance was deliberately excited by a vertical closed orbit distortion. In case of low resonance strength, polarization is not affected as in the measurement without additional excitation shown in blue.

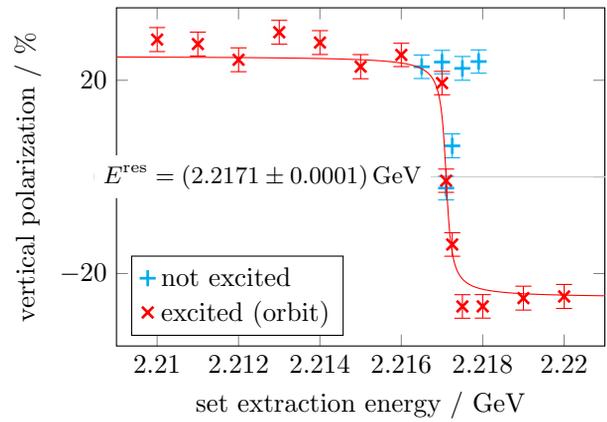


Figure 2: Møller polarimeter measurement for energy calibration using the integer resonance $\gamma a = 5$. The resonance was deliberately excited during the measurements shown in red.

So it is much easier to identify the resonance energy when additionally exciting the resonance and fitting the resonance energy E^{res} via

$$P(E) = P_0 \cdot \arctan(\text{const} \cdot (E - E^{\text{res}})) . \quad (2)$$

E^{res} is the extraction energy which has to be set in the control system to achieve an actual beam energy corresponding to the resonance ($\gamma a = 5$). In this case it was determined to

$$E^{\text{res}} = (2.2171 \pm 0.0001) \text{ GeV} ,$$

achieving a relative accuracy of $\Delta E/E \sim 4.5 \times 10^{-5}$. Applied for several resonances, this procedure principally can be used to calibrate the beam energy. Here, the same resonance has been used to additionally measure the momentum compaction factor.

MOMENTUM COMPACTION FACTOR

The momentum compaction factor α_c numbers the relation between the orbit circumference L and the energy E of an ultrarelativistic particle:

$$\frac{\Delta L}{L_0} = \alpha_c \cdot \frac{\Delta E}{E_0} . \quad (3)$$

The orbit circumference can be varied by changing the frequency f of the field in the accelerating RF cavities. The higher the RF frequency is, the shorter is the bucket length and the orbit L . Thus, increasing the frequency reduces the beam energy, which can be measured with the previously described method. It should be noted that a lower actual beam energy means that the resonance (energy corresponding to $\gamma a = 5$) is reached at a higher set extraction energy E^{res} . So α_c can be determined from the relation

$$\frac{\Delta f}{f_0} = \alpha_c \cdot \frac{\Delta E^{\text{res}}}{E_0^{\text{res}}} . \quad (4)$$

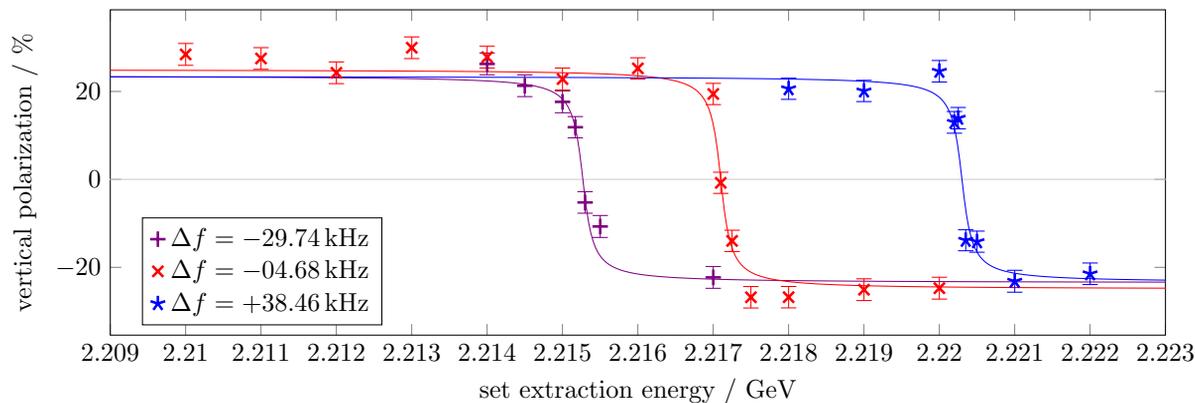


Figure 3: Møller polarimeter measurements for energy calibration using the integer resonance $\gamma a = 5$. The three measurements are taken with slightly different frequencies of the field in the accelerating RF cavities. The resonance was deliberately excited.

Figure 3 shows measurements analog to Figure 2, but for three slightly different RF frequencies. The resonance was excited during all measurements as described above. The data clearly demonstrates the energy shift due to the changed RF frequency. Each resonance energy is determined with a fit to the function from Equation 2. Then, the RF frequency offset is plotted against the energy shift.

The result, displayed in Figure 4, follows the linear dependence of Equation 4. A linear one-parameter fit yields

$$\alpha_c = 0.0601 \pm 0.0002 .$$

The choice of f_0 and the corresponding E_0^{res} within the data range does not affect the result significantly. The procedure benefits from the precision of the energy measurement. Therefore, the momentum compaction factor can be determined with four decimal places. The measurement is robust against relatively large polarization measurement errors. Table 1 gives a comparison of the measured value with simulation results from MAD-X and ELEGANT. They confirm the good quality of the measurement and underline that accurate parameter measurements can be very helpful to check and improve models for simulations.

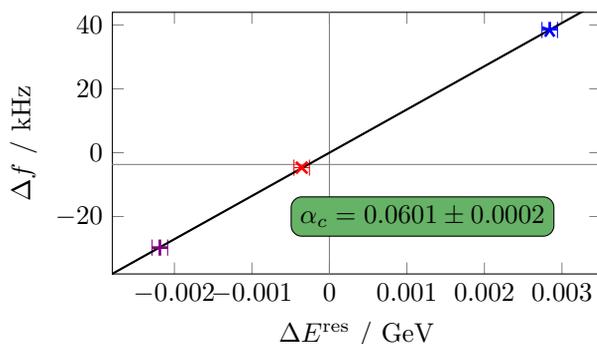


Figure 4: Linear relation of RF frequency shift and energy shift of integer resonance $\gamma a = 5$. Data was calculated from the measurement shown in Figure 3.

Table 1: Numerical Simulation of the Momentum Compaction Factor of the ELSA Stretcher Ring

MAD-X	0.0617
ELEGANT	0.0605

CONCLUSION

The established energy calibration method for storage rings based on resonant spin depolarization was used to precisely measure the momentum compaction factor at ELSA. Using an integer resonance, it does not depend on any other accelerator parameters. This method can be used for any storage ring if both a polarized beam and polarimetry are available. Neither a high polarization nor very precise polarimetry are essential, as long as the influence of the resonance is significantly larger than the polarization measurement error and as long as the beam energy can be varied sensitively enough to resolve the resonance width. The duration of the measurement is heavily influenced by the time needed to provide a polarized beam at every new energy. The infrastructure available at ELSA allowed to complete the whole measurement in about 4 h.

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