

# TRANSVERSE EMITTANCE MEASUREMENT AT REGAE

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## Abstract

The linear accelerator REGAE at DESY produces short and low charged electron bunches, on the one hand to resolve the excitation transitions of atoms temporally by pump probe electron diffraction experiments and on the other hand to investigate principal mechanisms of laser plasma acceleration. For both cases a high quality electron beam is required which can be identified with a small beam emittance. A standard magnet scan is used for the emittance measurement which is in case of a low charged bunch most sensitive to the beam size determination (2<sup>nd</sup> central moment of a distribution). Therefore the diagnostic and a routine to calculate proper central moments of an arbitrary distribution will be introduced and discussed.

## INTRODUCTION

The Relativistic Electron Gun for Atomic Exploration (REGAE, Fig. 1) at DESY is a small 5 MeV linear accelerator with a bunch charge range of a few to some hundreds fC. The beam energy is delivered by an S-band photo-injector cavity. In addition to the gun a 4-cell buncher cavity is installed. It is designed for velocity bunching down to 10 fs. Due to the low energy a beam optics consisting of solenoids is sufficient. They are compact and focusing in both directions simultaneously.

The machine is built for two types of experiments: first a time-resolved electron diffraction experiment in order to make atomic transitions 'visible' [1] and secondly investigations of new plasma-wakefield acceleration schemes [2]. Both experiments require a low transverse beam emittance down to 10 nm (normalized emittance). Hence, there are two challenges: generate such a high quality electron bunch and measure this quality for a low-charge bunch with high precision which is discussed below.

Due to the small energy spread as well as in first approximation negligible space charge effects a phase advance method can be utilized for the emittance measurement. For this purpose a charge sensitive detector system was developed which has the required spatial resolution to measure the beam profile despite the unavoidable noise and background signals.

## EMITTANCE MEASUREMENT VIA A SOLENOID SCAN

### Phase Advance Method

A commonly used method to determine the transverse emittance of an electron bunch is a magnet scan. Here the phase advance between a magnet and a downstream screen is changed by varying the magnet current. Analyzing the RMS beam size as function of the focusing strength yields the emittance. Alternatively, it is possible to measure the beam

size at different positions without any additional change of the optics. At REGAE the first method is used.

Assuming a small energy spread the *trace space emittance* is determined as

$$\epsilon_x = \sqrt{\langle x^2 \rangle \langle x'^2 \rangle - \langle x x' \rangle^2}$$

and is related in the following way to the *normalized emittance*:  $\epsilon_x = \frac{1}{\beta\gamma} \epsilon_{n,x}$  with  $\beta = v/c$ , the Lorentz factor  $\gamma$  and the velocity  $v$ . Furthermore,  $\langle \rangle$  denotes the central moment of a distribution, in this case the 2<sup>nd</sup> central moment where the square root is normally called RMS:  $\langle x^2 \rangle = x_{rms}^2$ .

In order to measure the transverse beam emittance the envelope equation can be used [3]:

$$x_{rms}^2 = \begin{pmatrix} R_{11}^2 & 2 R_{11} R_{12} & R_{12}^2 \end{pmatrix}^T \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \quad (1)$$

$$\begin{aligned} \text{with } a_1 &= x_{0,rms}^2 \\ a_2 &= x_{0,rms} (x_{0,rms})' \\ a_3 &= \frac{\epsilon_x^2}{x_{0,rms}^2} + (x_{0,rms})'^2. \end{aligned}$$

$x_{rms}$  denotes the RMS beam size at the screen,  $x_{0,rms}$  the beam size and  $(x_{0,rms})'$  the envelope slope at the position of the solenoid. In order to calculate the RMS beam size a Gaussian fit is often used. But the width of the normal distribution is only equal to the RMS if the beam profile is really normal distributed. In any other case this assumption doesn't hold which causes false results. It is important to emphasize that Eq. 1 only holds for RMS quantities. But the calculation of the RMS is difficult because the whole signal has to be taken into account which means the beam signal as well as background and noise. A post-processing routine for images will be introduced in the next section.

Taking Eq. 1 as a model describing the beam size development at a certain position in dependence of varying phase advances, the emittance can be found with the method of least squares.

### Detector System at REGAE

For the diffraction experiment at REGAE a highly sensitive detector system is installed which is able to detect single electrons. This detector combined with the solenoids Sol45 or Sol67 (Fig. 1) can be used for the emittance measurements. The detector contains a CsI-crystal-screen which is evaporated onto light guides, called FOS (Fiber Optics Scintillator), and a charge sensitive Electron Multiplying CCD (EMCCD). The overall spatial resolution of this detector system is  $\sim 20 \mu\text{m}/\text{pixel}$ . A schematic layout is shown in Fig. 2. The FOS is orientated perpendicular to the beam propagation. To avoid high energy photons or electrons hitting the EMCCD camera a mirror reflects the visible light

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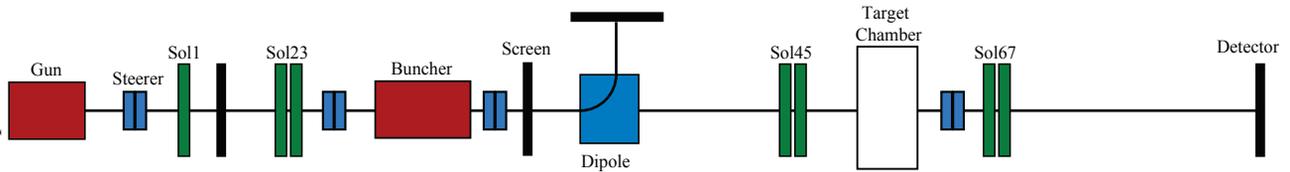


Figure 1: Schematic REGAE layout.

emitted by the FOS under 90° in direction of the camera. An Aluminum cover, directly layered onto the FOS reflects the emitted light of the FOS back in direction of the mirror. The whole setup is light-tight and the EMCCD is cooled down to -70 °C in order to reduce the noise from stray light and electronics of the EMCCD, respectively. All these factors increase the charge sensitivity by a factor of ~ 10<sup>3</sup> compared to a 'standard scintillator-CCD-setup' at REGAE ('Screen' in Fig. 1). Conditioned by the low charge and the high noise reduction the dominating noise is shot noise.

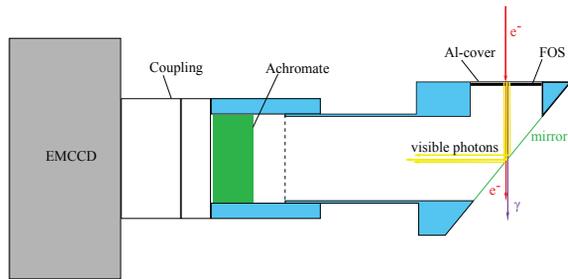


Figure 2: REGAE detector system layout.

### IMAGE POST-PROCESSING

For any kind of image-based beam diagnostic the noise and background signals are an issue which can have tremendous impact on the analysis of the beam signal. The relevant factor is the signal-to-noise ratio. Therefore the low bunch charge down to a few fC at REGAE is a challenge. In the following we want to describe a post-processing routine for

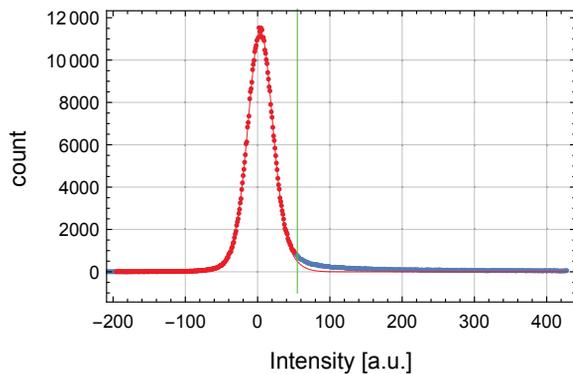


Figure 3: Histogram of pixel intensity (blue dots). Red dots were used for double Gaussian fit. The green line marks the 99 %-cut.

camera images and the calculation of central moments of an arbitrary distribution.

As already mentioned shot noise is the dominating source of noise at REGAE. It can be described by a Poisson-distribution

$$f(k; \lambda) = \frac{\lambda^k e^{-\lambda}}{k!}$$

$k \in \mathbb{N}$  and  $\lambda$  is the expected value and the variance of the distribution.

In order to get rid of the background signals like dark current an image of the background is subtracted from a beam signal image. In other words, a Poisson-distributed image is subtracted from another Poisson-distributed image. The only difference is the beam signal which normally covers just a fraction of an image. The difference of two Poisson distributions is called Skellam distribution [4,5] and is defined as

$$f(k; \lambda_1, \lambda_2) = e^{-(\lambda_1 + \lambda_2)} \left( \frac{\lambda_1}{\lambda_2} \right)^{k/2} I_k \left( 2\sqrt{\lambda_1 \lambda_2} \right)$$

where  $\lambda_1$  and  $\lambda_2$  are related to the two Poisson distributions and  $I_k$  is the modified Bessel function of the first kind. In case that  $\lambda_1 = \lambda_2 = \lambda$  and for a large  $\lambda$  the distribution tends to a normal distribution:

$$f(k; \lambda) \sim \frac{e^{-k^2/4\lambda}}{\sqrt{4\pi\lambda}} \tag{2}$$

For a 14-bit image, taken with our detector camera, this assumption is reasonable but Eq. 2 holds just for each single pixel. To get a reasonable sample number hundreds of images would be necessary. Assuming that all  $\lambda_i$  ( $i = 1 \dots N$ ;  $N$ : Number of pixels) just slightly differ we take a single image to analyze the noise. Fig. 3 shows a histogram of a beam image minus a background image. The remaining noise is centered around zero. Negative values are taken into account in order to fit a normal distribution. Due to the slightly differing  $\lambda$ s the noise is not a 'pure' normal distribution. It is rather a overlap of normal distributions which can be approximated by the following double normal distribution

$$f(k) = A_1 \sqrt{2\pi} \sigma_1 e^{-(x-\mu_1)^2/(2\sigma_1^2)} + A_2 \sqrt{2\pi} \sigma_2 e^{-(x-\mu_2)^2/(2\sigma_2^2)} \tag{3}$$

Fitting the noise distribution offers the possibility to define a signal threshold. For every signal below the threshold

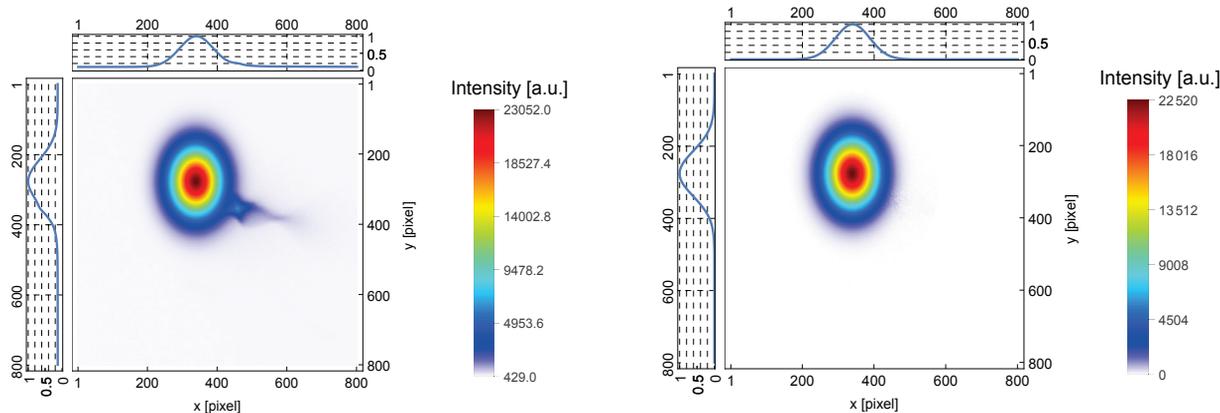


Figure 4: **Left:** Gaussian beam profile plus real REGAE background/noise image. **Right:** Same beam profile after applying post-processing routine.

it is not possible to distinguish between noise and signal. The threshold we are using suppresses 99 % of the integrated noise intensity. In Fig. 4 an example of an artificial Gaussian beam profile overlaid with a real background/noise image taken at REGAE is shown. The post-processing reduces the noise and improves the calculations of the central moments (Tab. 1).

Table 1: Theoretical and Post-Processed Barycenter (1<sup>st</sup> Central Moment) and RMS (2<sup>nd</sup> Central Moment) of a Gaussian Beam Profile (Fig. 4)

Mode	$x_{rms}$	$y_{rms}$	$x_{mean}$	$y_{mean}$
Theoretical	48	63	340	278
Post-pro	48.0(2)	62.7(1)	340.1(1)	278.11(6)

## MEASUREMENTS

Due to a repaired vacuum leak of the gun it is running only at moderate field gradients. This limits the mean beam energy. For the emittance measurement shown in Fig. 5 the measured mean beam energy was  $E_{kin} = 2.43$  MeV. The bunch charge was determined as  $Q_{bunch} = 245(2)$  fC - the number in brackets denotes the uncertainty of the last digit. The least-square fit parameters are shown in Tab. 2. The goodness of the fit is estimated by the *reduced Chi Square*  $\tilde{\chi}^2$  which should tend to one.

Table 2: Least-Square fit Parameters of a Solenoid Scan at REGAE

	Horizontal	Vertical
$\epsilon_n$ [ $\pi$ mm mrad]	0.157(3)	0.209(2)
$\mathbf{x}_{0,rms}$ [mm]	0.192(4)	0.259(3)
$(\mathbf{x}_{0,rms})'$ [ $\mu$ rad]	-32(2)	-19(1)
$\tilde{\chi}^2$	1.6	1.3

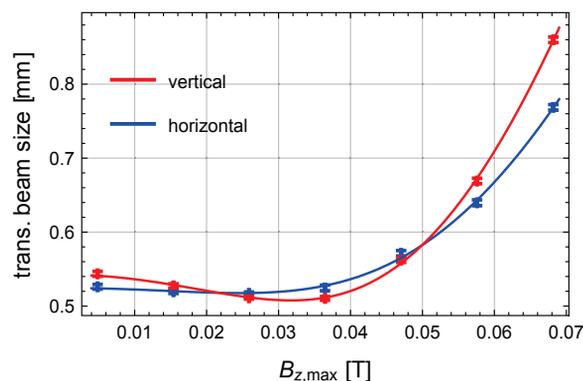


Figure 5: Measured beam size at *detector* in dependence of the maximum magnetic field  $B_{z,max}$  of solenoid *Sol67*. Solid lines are least-square fits.

## CONCLUSION

A new post-processing routine could be successfully integrated at REGAE. It offers the ability to find a quantified criterion for a threshold from analyzing camera images in order to improve the calculation of central moments of an arbitrary electron distribution. Any routine will be limited by the signal-to-noise ratio. At the transition of signal to noise a loss of information is unavoidable which cannot be regained. This loss could be minimized by the introduced detector system.

Depending on the cut-off criterion the calculated moments can be over- as well as underestimated. Therefore, the determination of a threshold has to be done carefully.

## REFERENCES

- [1] J.R. Dwyer et al., "Femtosecond electron diffraction: 'making the molecular movie' ", Philosophical Transactions of the royal society, 364:741-778 (2006).
- [2] B. Zeitler et al., "Merging conventional and laser wakefield accelerators", Proceedings SPIE 8779 (2013).

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- [3] M. Hachmann, "Transverse emittance measurement at REGAE via a solenoid scan", *Diploma thesis*, Hamburg, Germany (2013).
  - [4] J. G. Skellam, "The frequency distribution of the difference between two Poisson variates belonging to different populations", *Journal of the Royal Statistical Society, Series A*, 109 (3), 296 (1946).
  - [5] Y. Hwang et al., "Sensor noise modeling using the Skellam distribution: Application to the color edge detection", *IEEE Conference on Computer Vision and Pattern Recognition* (2007).