ISBN: 978-3-95450-168-7 doi:10.18429/JACoW-IPAC2015-MOPJE019

CATEGORIZATION AND ESTIMATION OF POSSIBLE DEFORMATION IN EMITTANCE EXCHANGE BASED CURRENT PROFILE SHAPING

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Abstract

Bunches with shaped current profiles can be used to increase the transformer ratio in the beam-driven collinear wakefield acceleration. Shaped current profiles can be generated with an emittance exchange (EEX) and controlled by the incoming beam parameters as well as the EEX beam line parameters. The ideal bunch shape, predicted from first order theory in the absence of collective effects, is deformed in the real case. In this paper, we categorize the sources of deformation with a deformation parameter and observe their deformation pattern on the current profile.

CATEGORIZATION OF DEFORMATION

Perturbation theory can be used to categorize the deformations of the bunch shape that arise with the emittance exchange (EEX) based bunch shaping. We define the ideal bunch shape, $N_0(\mathrm{z})$, as that which occurs under the ideal conditions of zero emittance, thin-lens deflecting cavity, first-order beam dynamics, and the absence of collective effects [1]. In this case, there is perfect linear relationship between the initial horizontal position (x_i) and the final longitudinal position ($z_f^0 = \{k\xi - s(\eta + k\xi(L+D))\}x_i$). The actual bunch shape, $N(\mathrm{z})$, is deformed from the ideal when realistic effects are included. In this paper, we characterize the deformation patterns in order to improve the beamline design. We analyse the deformation of an ideal single triangle current profile and determine how the deformation affects the transformer ratio (TR).

Single Particle Treatment

The final bunch position (z_f) can be treated as a perturbation to the ideal bunch position (z_f^0) by adding perturbation terms.

$$z_f = z_f^0 + \sum C_m \prod X_n, \tag{1}$$

where C_m are the perturbation coefficients of the beam parameters $X_n \in \{x_i, x_i', y_i, y_i', z_i, \delta_i\}$. Based on the polynomial form of Eq. (1), there are three categories. (1) random deformation where all X_n are not correlated to x_i This deformation spreads the particle from its ideal position without any preferred direction. The thick-lens effect from the deflecting cavity ($\frac{L_c}{6}\kappa^2\xi z_i + \frac{L_c}{6}\kappa^2\xi^2\delta_i$) belongs to this category since z_i and δ_i are not correlated to x_i [1]. (2) correlated-random deformation contains at least one X_n which is and is not correlated to x_i. This deformation makes a similar effect like the random perturbation, but it changes the shape of the current profile in the different way. Since a divergence after the deflecting cavity ($x_2' = \kappa z_i + \kappa\xi\delta_i$) is not correlated to x_i, and an energy spread after the deflecting cavity ($\delta_2 = \kappa x_i + \kappa(L + L_{bc})x_i{'}$) is correlated to x_i, the second order term $\mathrm{T}_{526}x_2'\delta_2$ is the one of the correlated random deformation terms [2]. (3) correlated random deformation only has X_n which is correlated to x_i. Since all parameters are correlated to x_i, particles move to the specific position based on its original position. This collective movement makes the strongest deformation to the current profile.

EEX BASED SHAPING DEFORMATION PATTERNS

Each major deformation categories can be further separated into several minor cases depending on the sign of the coefficients and the order of beam parameter terms. By considering up to the second order for the random and the correlated-random case, and the third order for the correlated case, there are total ten minor deformation cases.

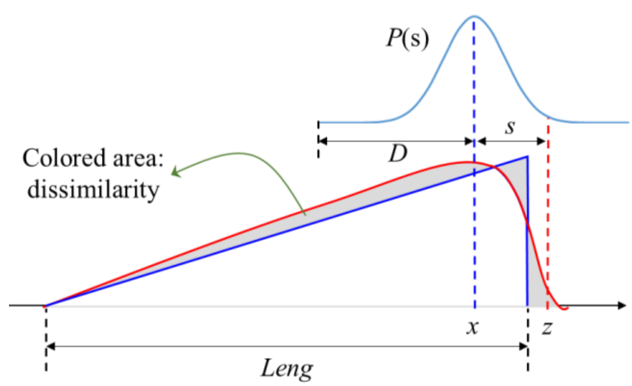

Figure 1: Definition of parameters

Deformed Density Profile

We define a perturbation function (which is similar to the probability density function) in order to analyse the cases. If s is the distance between two positions, $P(s \equiv z - x)ds$ indicates what portion of particles in position $[x, x + dx]$ moves to z (Fig. 1). This function satisfies $\int_{-\infty}^{\infty} P(s)ds = 1$. Also, the deformed profile related to the ideal profile, $N_0(x)$, can be expressed

$$N(z) = \int_{-\infty}^{\infty} N_0(x)P(z-x)dx. \tag{2}$$

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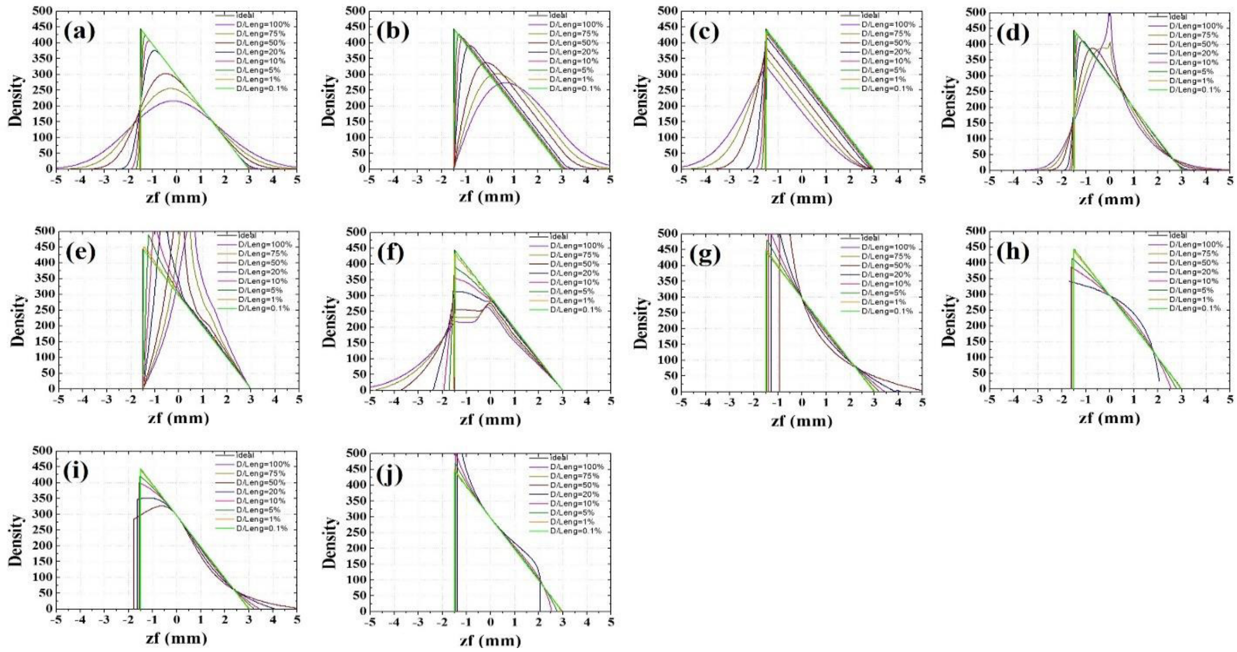

Figure 2: The deformation by the random deformation with a first order (a), a positive second order (b) and a negative second order (c). The deformation by the correlated-random deformation with a first order (d), a positive second order (e) and a negative second order (f). The deformation by the correlated deformation with a positive second order (g), a negative second order (h), a positive third order (i) and a negative third order (j). To consider the Gaussian beam, $P(s)$ has a Gaussian form,

$$P(s) = \frac{n}{\sqrt{2\pi}\sigma_s} exp[-s^2/(2\sigma_s^2)], \quad (3)$$

where $s \in [-D, D]$ and $\int_{-\infty}^{\infty} P(s)ds = 1$. Since the distance that particles can move is limited, the domain is chosen to D which is the maximum deformation level. Different domains are applied to each minor categories. Domains for each categories and the correlation for correlated-random case are described in Appendix.

Figure 2 shows the deformation of the triangle current profile for each minor cases depending on the deformation parameter which is defined by the maximum deformation level (D) divided by the ideal bunch length (Leng). For each cases, deformation parameters changes from 0.1% to 100%.

ESTIMATION OF DEFORMATION

It is difficult to compare the deformation categories because their domains are different. To quantify the difference between the deformed profile, $N(z)$, and the ideal profile, $N_0(z)$, we define a dissimilarity as the sum of the areas included in either the ideal or deformed profiles only; see Fig. 1. If we assume the total area is 1, the portion of the area that leaves the original profile area (indicated by the dissimilarity) moves to the deformation area.

Figure 3 (a) shows the dissimilarity as a function of the deformation parameter (D/Leng). Since the deformation parameter is the maximum movement of particles and the dissimilarity is the moved area, they have a linear relation for all categories. However, the dissimilarity itself is different for each category because of the domain. Fig. 3 (b) shows the variation of the TR in terms of the dissimilarity. This graph shows how serious the deformation is for the given change of current profiles.

The estimation was done using TR which is defined by the ratio of the maximum accelerating field behind the drive bunch to the maximum decelerating field inside the drive bunch. This number strongly depends on the shape of current profile. Also, this is the one of the key parameters for the collinear dielectric wakefield acceleration because the TR directly indicates the energy transfer efficiency. Thus, one can estimate the dissimilarity from the deformation parameter and compare all different categories together from the variation of the TR.

In the low dissimilarity region (<0.01), the reduction in TR is less than 10% which is tolerable. When the dissimilarity is in the intermediate region (<0.1), the random based deformations increase smoothly and cause a TR drop of about 30%. In this intermediate region, the current profile is asymmetric and preserves the triangle shape as can be seen in Fig. 1. The correlated random deformation, however, generates a significant change. Its TR drop starts from 10% at the dissimilarity of 0.01 and grows up to the maximum 60% at the dissimilarity of 0.1. In this level, TR is already less than the half of its original value. Thus, if the deformation parameter is in the intermediate region, it is very important to control the correlated deformation. In the high dissimilarity region (>0.1), the TR drop by the correlated deformation is saturated and the TR drop by other deformations increase

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rapidly and saturate too. In this region, the first order random deformation makes the largest TR drop because it generates an almost perfectly symmetric current profile by its random nature.

As shown in Fig. 3 (b), TR can change significantly by deformations. This deformation and the TR drop should be considered in the beam line design. If the dissimilarity is in a moderate range, it is good to ignore the other deformations, but the correlated random deformation should be carefully handled because it generates a large TR drop even in the intermediate region.

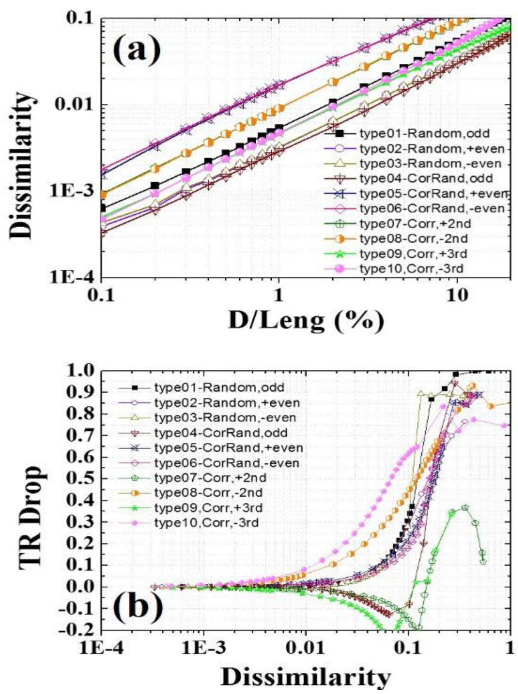

Figure 3: The relation between the deformation parameter and the dissimilarity (a). The relation between the dissimilarity and the TR drop (b). The TR drop is the ratio of the deviation of TR to the original value.

APPENDIX

Random Deformation

Since the random deformation does not have a preferred direction of movements, the domain is simply given by

$$s \in [-D, D] \text{ and } \int_{-D}^{D} P(s)ds = 1. \quad (4)$$

For second order cases, particles have a preferred direction depend on the sign of corresponding coefficients. Thus, its domain is only the half of previous case which is one of [0, D] or [-D, 0].

First Order Correlated -Random Deformation

For the correlated-random case, the particle movement is based on the random movement, but its maximum deformation level is changing depending on its original position because of the correlation term. Thus, the deformation pattern depends on what kind of relation they have. For the first order case, we consider the linear relation whose domain is

$$s \in [-\bar{x}D, \bar{x}D] \text{ and } \int_{-\bar{x}D}^{\bar{x}D} P(s)ds = 1, \quad (5)$$

where $\bar{x}$ is x/m to use it as amplification factor.

Second Order Correlated-Random Deformation

For the second order case, we introduce another special correlation. In the EEX based shaping process, the transverse beam shape is manipulated by the mask to achieve a specific horizontal profile. During this process, the initial horizontal position and the vertical position have a correlation depending on the mask shape. Fig. 4 shows a typical mask shape for the triangle current profile.

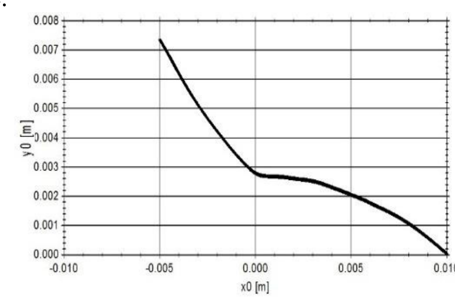

Figure 4: The typical mask shape to generate a triangle current profile. This shape shows positive y-side only. The shape is vertically symmetric.

If this relation is $f(x)$, the limit of the domain should be 0 to $f(x)D$ or $-f(x)D$ to 0. For the positive coefficients, the domain is

$$s \in [0, f(x)D] \text{ and } \int_{0}^{f(x)D} P(s)ds = 1. \quad (6)$$

Correlated Deformation

Previous deformations were based on the random movement, but the correlated deformation moves the particle to the specific position based on its original position. Thus, we can consider the delta function as $P(s)$ with an open domain $(-\infty, \infty)$.

$$P(s) = \delta(s \mp Dx^n). \quad (7)$$

where n is the order of deformation. Depending on the sign of coefficient, the sign in the equation changes.

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[2] G. Ha et al., "Initial EEX-based bunch shaping experiment results at the Argonne Wakefield Accelerator Facility", WEPWA035, these proceedings, IPAC'15, Richmond, USA (2015).