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SIMULATIONS AND MEASUREMENTS OF LONGITUDINAL COUPLED-BUNCH INSTABILITIES IN THE CERN PS

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Abstract

Among various and challenging objectives of the LHC Injectors Upgrade project (LIU), one aim is to double the beam intensity of the CERN Proton Synchrotron (PS) in order to achieve the integrated luminosity target of the High-Luminosity LHC project (HL-LHC). A known limitation to reach the required high intensity is caused by the longitudinal coupled-bunch oscillations developing above the transition energy. The unwanted oscillations induce large bunch-to-bunch intensity variations not compatible with the specifications of the future LHC-type beams. A wide-band longitudinal damper has been installed in the PS to suppress these instabilities and is going to be commissioned. A measurement campaign of coupled-bunch oscillations has been launched to substantiate the extrapolations and predictions for the future High Luminosity LHC beam with the final aim to determine the maximum intensity that could be provided to the LHC. In parallel a Simulink© model of the PS is going to be implemented to predict the machine behavior in the parameter space of LIU and to be used during the beam commissioning and optimization of the feedback system.

INTRODUCTION

The Proton Synchrotron (PS) [1] is the LHC injector where the longitudinal structure of the beam train serving LHC is established. RF systems at multiple harmonics between $h = 7$ and $h = 458$ provide the flexibility required for the longitudinal bunch splitting [2]. After the beam is accelerated above transition energy [3], dipolar longitudinal coupled-bunch (CB) instabilities are observed [4, 5]. The machine longitudinal impedance changes according to the dynamic tuning of the main accelerating cavities along the magnetic cycle. During the ramp all cavities are tuned at $h = 21$ and the longitudinal impedance is maximized, thus causing a crosstalk among bunches. Presently this instability is cured by a controlled longitudinal emittance blow-up at injection energy and dedicated feedback at 15 f_{rev} and 20 f_{rev}. From the present bunch population of $1.3 \cdot 10^{11}$ p/b (72 bunches in $h = 84$), the LIU project [6] aims to reach $2.6 \cdot 10^{11}$ p/b [7]. This makes the present counter measurement (blow-up) inadequate to control the instability, producing, during the splitting at top energy, a bunch-by-bunch intensity variation not compatible with the required LHC luminosity performance.

In the following we assume as working hypothesis that the train of equidistant bunches is regularly distributed along the machine azimuth (to compare with the nominal filling pattern with 18 bunches in $h = 21$). Under this condition, the bunches are not sortable from the physical point of view (circulant symmetry). Assuming in addition that the CB oscillations can be described by a linear approximation, the circulant matrix formalism [8] becomes the ideal mathematical tool to analyze the dipolar coupled-bunch oscillations. The purpose of the study is to develop an algorithm to analyze the longitudinal profiles of the bunch train and, using the circulant matrices formalism, to perform the mode analysis of the system to study its stability. In addition a Simulink© [9] model of the PS and the new damper cavity is being implemented to take into account the non idealities of the system, i.e. the limited voltage of the damper cavity, the noise level of the longitudinal pickup (PU), the errors introduced by the quantization and sampling of the low level electronics of the feedback.

THE CIRCULANT MATRIX FORMALISM

A $2n_b \times 2n_b$ circulant matrix, $\mathbf{F}$, has the general form

$$\mathbf{F} = \begin{pmatrix} f_1 & \dots & f_2 \\ \dots & \dots & \dots \\ f_{2n_b} & \dots & f_1 \end{pmatrix}. \quad (1)$$

The evolution in the normalised longitudinal phase space of n_b bunches from turn n to turn $n+1$ can be linearly approximated by the following

$$\begin{pmatrix} x_1 \\ \Delta p_1/p \\ \dots \\ \dots \\ x_{n_b} \\ \Delta p_{n_b}/p \end{pmatrix}_{n+1} = \mathbf{M} \times \begin{pmatrix} x_1 \\ \Delta p_1/p \\ \dots \\ \dots \\ x_{n_b} \\ \Delta p_{n_b}/p \end{pmatrix}_{n}, \quad (2)$$

where $\mathbf{M}$ is a stationary block circulant matrix where each block represents a rotation matrix. This paper will discuss how to derive the $\mathbf{M}$ matrix starting from the measurement data from the longitudinal PU. Once $\mathbf{M}$ is known, the stability of the system described in Eq. 2 can be investigated by its eigenvalues if the matrix can be put in diagonal form. Since we consider a perturbative approach, it is possible to assume that all the bunches in the system have the same synchrotron tune Q_s. Therefore each bunch position x_i and each bunch momentum deviation $\Delta p_i/p$ can be written as

$$x_i = \Re\left\{a_i e^{j\phi_i} \cdot e^{j2\pi Q_s \times n}\right\} \quad (3)$$

$$\Delta p_i/p = \Im\left\{a_i e^{j\phi_i} \cdot e^{j2\pi Q_s \times n}\right\} \quad (4)$$

where $i \in \{1, \dots, n_b\}$, a_i and ϕ_i are the amplitude and the phase of the longitudinal bunch oscillation. The dynamic

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in Eq. 2 together with the assumption in Eq. 3 can be reformulated in a complex amplitude space and described by the new variable $\mathbf{X_n}$

$$\begin{pmatrix} X_1 \\ \cdot \\ \cdot \\ X_{n_b} \end{pmatrix}_{n+1} = \mathbf{X_{n+1}} = \mathbf{C} \times \begin{pmatrix} X_1 \\ \cdot \\ \cdot \\ X_{n_b} \end{pmatrix}_{n} = \mathbf{C} \times \mathbf{X_n} \quad (5)$$

where $X_i = a_i \cdot e^{j\phi_i}$ is the phasor representing the complex amplitude of the i^{th} bunch and $\mathbf{C}$ is a $n_b \times n_b$ circulant matrix.

The eigenvectors of a circulant matrix can be written as

$$v_j = \frac{1}{\sqrt{n_b}}(1, \omega_j, \omega_j^2, \ldots, \omega_j^{n_b-1})^T \quad (6)$$

with $j = 0, 1, \ldots, n_b - 1$ and $\omega_j = \exp\left(\frac{2\pi i j}{n_b}\right)$. Thus $\mathbf{C}$ can be *always* written in the form

$$\mathbf{C} = \mathbf{P} \times \mathbf{D} \times \mathbf{P}^{-1} \quad (7)$$

where $\mathbf{D}$ is a diagonal matrix of eigenvalues and $\mathbf{P}$ is the base of the eigenvectors of the system. From Eq. 6, one obtain

$$\mathbf{P} = \mathrm{DFT}\,(\mathbb{I}) \quad (8)$$

where $\mathbb{I}$ is the $n_b \times n_b$ identity matrix and DFT represents the Discrete Fourier Transform [10]. It is possible to rewrite the evolution of Eq. 5 from the bunch space $\mathbf{X_n}$ in the modes space $\mathbf{W_n}$, yielding

$$\mathbf{W_{n+1}} = \mathbf{D} \times \mathbf{W_n}. \quad (9)$$

From previous equations the expression of the modes evolution can be achieved

$$\mathbf{W_n} = \mathbf{P}^{-1}\mathbf{X_n} \quad (10)$$

which leads, in combination with the definition of eigenvectors for circulant matrices in Eq. 6, to the next relation

$$\mathbf{W_n} = \mathrm{IDFT}(\mathbf{X_n}). \quad (11)$$

Eq. 11 allows to compute amplitude, A_i, and phase, Φ_i, for each oscillation mode

$$W_i = A_i \cdot e^{j\Phi_i} \quad (12)$$

from the a_i and ϕ_i of each bunch. We will show how the formalism discussed above can be applied to the measured data.

MEASUREMENTS CAMPAIGN

During the 2014 run, a measurements campaign of the CB instabilities was performed. In the following the setup, the analysis algorithms and the results are discussed. The aim is to derive the matrix $\mathbf{M}$ of Eq. 2 starting from the measured data and use it as an input for the Simulink© model of the machine. A large memory digitiser has been used to acquire 160 ms of the beam data from a longitudinal PU at a fixed sampling frequency of 400 MHz starting the acquisition just after the transition crossing. To obtain, from measured data, the $\mathbf{X_n}$, one needs to gate the bunches. During the acquisition the beam revolution period, T_{rev}, varies making non trivial the bunch gating. To make the gating possible during the post processing phase, the T_{rev} signal was recorded on the same digitiser together with the PU signal (Fig. 1). The

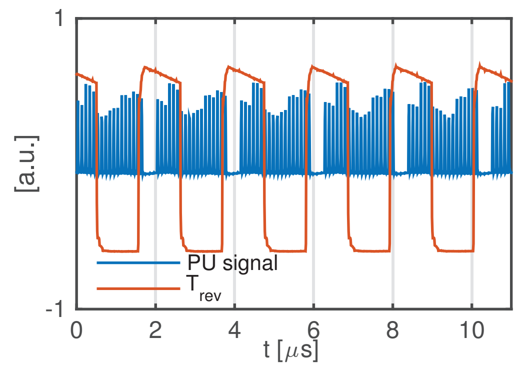

Figure 1: Longitudinal PU signal (WCM95) and beam synchronous T_{rev} signal (it is a signal proportional to the harmonic of the machine) acquired during measurements.

center of mass oscillation x_i in Eq. 3 is computed using the data represented in Fig. 1. An example of the result is plotted in Fig. 2 where it is shown that sub-ns sensitivity for the synchrotron oscillation can be attained. From the data in Fig. 2, a_i and ϕ_i are computed using a non linear sinusoidal fit

$$x_i = \alpha_i + a_i \cdot \sin(2\pi Q_s n + \phi_i) \quad (13)$$

on a time moving window. In this case, with $N \approx 80\ 10^3$ turns, the window has been selected to cover about two synchrotron oscillation (~2000 turns) with a Δ span of 100 turns between two consecutive windows (~700 windows in total). An alternative method is to compute from the machine parameters the expected ω_s and to find the bunch oscillation phasor directly from a sliding interpolated FFT of the signal [11]. From a_i and ϕ_i one obtains $\mathbf{X_n}$ and finally, using Eq. 11, the $\mathbf{W_n}$ (see Fig. 3).

The equation describing the evolution of $\mathbf{W_n}$ in the mode space (Eq. 9) is valid between turns n and $n+1$. It is possible anyway to rewrite this vector equation in a matricial form by substituting the vectors $\mathbf{W_{n+1}}$ and $\mathbf{W_n}$ with two matrices, $\mathbf{A}$ and $\mathbf{B}$, defined as

- $\mathbf{A} = [W_2, W_3, \ldots, W_N]$;
- $\mathbf{B} = [W_1, W_2, \ldots, W_{N-1}]$.

Using this approach one can write

$$\mathbf{A} = \mathbf{D} \times \mathbf{B} \quad (14)$$

and

$$\bar{\mathbf{D}} = \mathbf{A} \times \mathbf{B}^{+} \quad (15)$$

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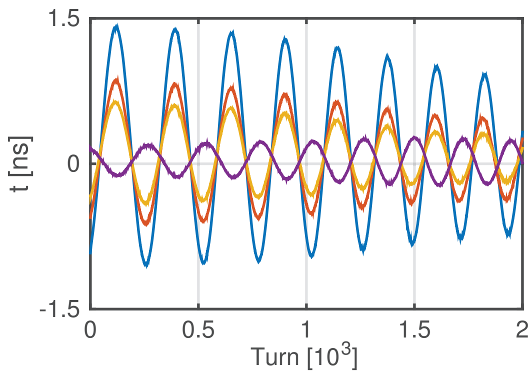

Figure 2: Plot of center of mass oscillation of 4 bunches in the case of $n_b = 18$ in $h = 21$.

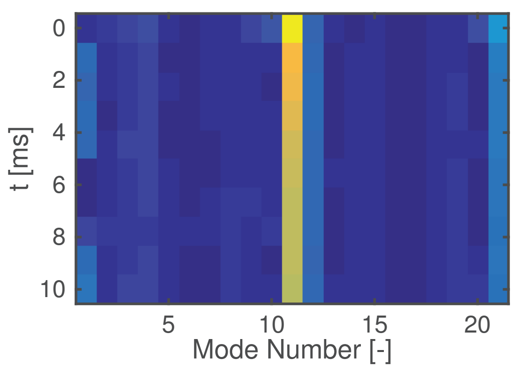

Figure 3: Mode amplitude evolution in $h = 21$ and 21 bunches. One unstable mode (i=11, μ=10) is visible.

where $\mathbf{B}^+$ is the pseudo-inverse [12] of $\mathbf{B}$ and $\bar{\mathbf{D}}$ is the least square approximation, in Eq. 15, of $\mathbf{D}$. Once the matrix $\bar{\mathbf{D}}$ is computed, the $\mathbf{C}$ is derived from Eq. 7 and the $\mathbf{M}$ can be determined from Eq. 3, 4 and 5, by transforming each element of $\mathbf{C}$, as an example c_{11}, in a pure rotation block of $\mathbf{M}$

$$\begin{aligned} m_{11} &= \Re(c_{11}\ e^{j2\pi Q_s \times n}) \\ m_{12} &= -\Im(c_{11}\ e^{j2\pi Q_s \times n}) \\ m_{21} &= \Im(c_{11}\ e^{j2\pi Q_s \times n}) \\ m_{22} &= \Re(c_{11}\ e^{j2\pi Q_s \times n}). \end{aligned}$$

LONGITUDINAL FEEDBACK AND SIMULINK MODEL

The PS can presently work with a narrowband FB system [13] by using the spare 10 MHz RF cavity as a longitudinal kicker. It has a voltage availability up to 20 kV and it is tunable from 2.8 MHz to 10.1 MHz ($h = 7, \ldots, 21$). Due to its limited bandwidth, the FB can address only two adjacent modes. To relax this limitation, during the long shutdown (LS1) in 2013-14, a new dedicated larger bandwidth cavity has been installed in the straight section 2 of the PS: the Finemet© cavity [14]. With a voltage up to 5 kV and bandwidth from 0.4-5.5 MHz, the cavity can provide the correcting RF voltage to address all the oscillation modes in $h = 21$. In fact for each dipolar mode $\mu = i - 1$, the frequency spectrum, $\Omega_\mu(f)$, can be expressed for $f > 0$ by the Dirac comb [15]

$$\Omega_\mu(f) = \sum_{l=-\infty}^{+\infty} a_l\ \delta(f - |(\mu + l\ n_b)f_0 - f_s|) \qquad (16)$$

where $a_l \in \mathbb{C}$. Therefore all modes $\mu \in \{1, \ldots, 20\}$ have at least one frequency component in the Finemet© frequency range and can be in principle cured by the feedback. The $\mu = 0$ mode instead is damped by the beam phase loop. The digital card of the low level RF is available [16] but the firmware for the signal processing has still to be finalized. In order to have an optimization tool in the design phase of the firmware and an operational tool during the commissioning of the FB, a Simulink© model of the machine loop and of the FB loop is being prepared. The machine loop will describe the linear system of Eq. 2 (linear loop) whilst the FB loop will consider also the non linear aspects

- the limited maximum voltage (5 kV or lower depending on requirements of the active loop reducing the effective impedance of the cavity), the error induced by the 14-bit quantization of the ADCs and DACs, the noise of the wall current monitor, the noise in the mode detection due to the limited clock frequency, f_{CLK}, of the digital card ($f_{CLK} = 256 f_{rev}$)
- and design parameters: optimization of the gains and the digital filters to isolate the synchrotron bandwidths of Eq. 16.

CONCLUSION

In this work a CB mode analysis technique has been presented by using the circulant matrices formalism. The mathematical model has been explained and applied to the measured data showing under which hypotheses the evolution matrix $\mathbf{M}$ of the system can be obtained. The development of a Simulink© model of the machine and the feedback is ongoing as an auxiliary tool in view of the 2015 commissioning of longitudinal feedback in the PS.
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