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LORENTZ BOOSTED FRAME SIMULATION OF LASER WAKEFIELD ACCELERATION USING HYBRID YEE-FFT SOLVER IN QUASI-3D GEOMETRY

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Abstract

We present results from a preliminary study on modeling Laser wakefield acceleration (LWFA) with OSIRIS in a Lorentz boosted frame using a quasi-3D algorithm. In the quasi-3D algorithm, the fields and currents are expanded into azimuthal harmonics and only a limited number of harmonics are kept. Field equations in (r, z) space are solved for a desired number of harmonics in ϕ. To suppress the numerical Cerenkov instability (NCI) that inevitably arises due to the relativistic plasma drift in the simulation, we use a hybrid Yee-FFT solver in which the field equations are solved in (k_z, r) space, where $\hat{z}$ is the drifting direction. Preliminary results show that high fidelity LWFA boosted frame simulations can be carried out with no evidence of the NCI. Good agreement is found when comparing LWFA boosted frame simulations in the full 3D geometry against those in the quasi-3D geometry. In addition, we discuss how the moving window can be combined with the hybrid Yee-FFT solver to further speed up the simulation. The results indicate that unprecedented speed ups for LWFA simulations can be achieved when combining the Lorentz boosted frame technique, the quasi-3D algorithm, and a moving window.

INTRODUCTION

Laser wakefield acceleration (LWFA) [1] offers the potential to construct compact accelerators that have numerous potential applications including the building blocks for a next generation linear collider and for compact light sources. Due to the strong nonlinear effects that are present in LWFA [2], particle-in-cell (PIC) simulations play a very important role in LWFA research. The PIC algorithm follows the self-consistent interactions of particles through the forces directly calculated from solving the full set of Maxwell equations with the currents and charge densities calculated from the particle trajectories. However, using a standard PIC code to study LWFA in a nonlinear regime can be very CPU-time consuming, e.g. a 10GeV stage run takes approximately 30 million CPU hours. While computing resources now exist to do a few of such simulations, it is not possible to do parameter scans in full three-dimensions (3D).

Recently, there has been much research focused on performing simulations in a Lorentz boosted frame with a plasma drifting towards the laser with a Lorentz factor γ_b [3, 4]. So long as there is no reflected light, then the effective time and space scales to be resolved in a numerical simulation may be minimized. The increase in time step and decrease in the plasma length in this frame lead to savings of factors of γ_b^2 as compared to a lab frame simulation using the so-called moving window [5]. Another reduced model that has been recently proposed is to expand the fields in azimuthal mode numbers m and truncate the expansion [6]. This can reduce modeling a 3D problem with low azimuthal asymmetry into the similar computational cost as using a 2D $r - z$ code. The quasi-3D algorithm was implemented into OSIRIS [7] including a new charge conserving current deposit for improved accuracy [8].

It was pointed out in [9, 10] that it would be intriguing to combine the two methods. Similarly to full PIC simulations in cartesian geometry, it was found [9] that in cylindrical geometry, one of the obstacles that needs to be overcome the numerical Cerenkov instability (NCI) [11], that arises due to the inevitable numerical coupling between the Langmuir modes (main and aliasing) and electromagnetic (EM) modes in relativistically drifting plasma frame [12–14]. This makes clear that Lorentz invariance is not strictly true in a PIC code [15, 16]. However, while the multi-dimensional NCI theory in 2D/3D Cartesian coordinates has been well studied, there is currently no dispersion relation for the NCI in the quasi-3D geometry.

In cartesian geometry several ideas have been proposed to minimize and in some cases essentially eliminate the NCI [10, 13–15, 17]. For example, theory and simulations show how to eliminate the NCI through the use of FFT (spectral) solvers and additional strategies including filters [15]. Recently, it was proposed and demonstrated that a hybrid Yee-FFT solver could also be used to greatly minimize the NCI and that this scheme can be used to suppress the NCI in quasi-3D geometry [10] . In this solver, the Maxwell equation is solved in (k_z, r) space, where $\hat{z}$ is the drifting direction of the plasma for each azimuthal mode number. In this way, the fastest growing modes of the NCI at $(\mu, \nu_1) = (0, \pm 1)$ can be well suppressed by filtering the current in k_z space. Here μ and ν_1 refer the time and space aliased modes [14]. Furthermore, the highly localized $(\mu, \nu_1) = (0, 0)$ NCI modes have similar patterns to that of a 2D spectral solver in Cartesian coordinate, and can be suppressed by reducing the simulation time step, or using a local dispersion modification that accurately eliminate the $(\mu, \nu_1) = (0, 0)$ NCI modes [15].

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We have implemented the hybrid Yee-FFT solver into the quasi-3D OSIRIS code. Here, we present results from a preliminary attempt to combine the Lorentz boosted frame technique, the quasi-3D algorithm, and the moving window technique to model LWFA in a nonlinear regime. We briefly discuss the simulation setups, and show that the NCI can be very well suppressed in the quasi-3D geometry with the use of the hybrid solver. These results demonstrate that high fidelity LWFA Lorentz boosted frame simulation in the quasi-3D geometry can be conducted with the potential for unprecedented speed up.

SIMULATION SETUP

Quasi-3D Hybrid Yee-FFT solver

The hybrid Yee-FFT solver for the quasi-3D solver is a natural extension of its counterpart in Cartesian coordinates. We solve the fields in (k_z, r) space for each azimuthal mode m, and in the $\hat{r}$ direction we use second order finite difference operators [10]. The fields are transformed back to (z, r) space to push the particles and deposit the current. In quasi-3D OSIRIS, a charge conserving current deposition is used for the quasi-3D geometry [8]. Similarly as in Cartesian geometry, this current is corrected in k_z, r space so that charge is still conserved [10], i.e., $\tilde{j}_z^{n+\frac{1}{2}} = \frac{\sin k_z \Delta z/2}{k_z \Delta z/2} j_z^{n+\frac{1}{2}}$, where j_z is defined at half-integer time with n the time step index, Δz is the grid size in $\hat{z}$ direction. Since the solver uses finite difference operators in the directions transverse to the drifting direction, boundary conditions that are designed for FDTD solvers can be readily applied in the hybrid solver.

Setup for the LWFA boosted frame simulation

We present a layout of the simulation setup in Fig. 1. In the Lorentz boosted frame LWFA simulation with the Lorentz factor γ_b, a laser is launched using a moving antenna from right to left into a drifting plasma drifting from right to left. Compared to their counterparts in the lab frame, the laser wavelength and pulse length stretch by $\gamma_b(1 + \beta_b)$, while its Rayleigh length is γ_b times shorter. The plasma column is length contracted by γ_b, and it is drifting at β_b.

This setup is used for cases with and without moving window. When the moving window is applied, we place a damping section at the rear end of the moving window to absorb any radiation that propagates back from the plasma. Note that even though there are EM waves that are reflected by the damping section, this reflected light will not be able to catch up with the moving window unless it moves superluminally with a speed significantly larger than c.

SAMPLE SIMULATION

In this section, we present the preliminary simulation results for a 1.3 GeV LWFA case using the parameters in Ref. [2]. The Lorentz factor of the boosted frame is $\gamma_b = 12.0$, and other parameters are listed in Table 1. Periodic boundary conditions are used in $\hat{z}$ direction. As for

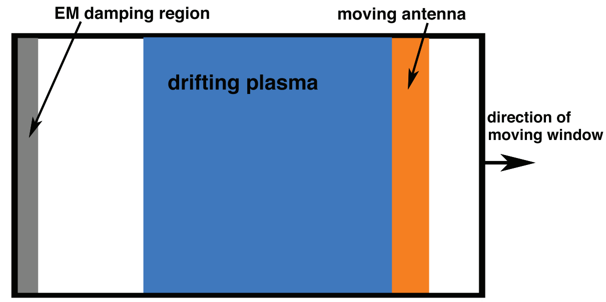

Figure 1: Cartoon of simulation setups for the LWFA simulation in the Lorentz boosted frame. The plasma, and moving antenna are drifting from left to right, in the same direction as the moving window.

the upper $\hat{r}$ boundary, we are currently using the 2D Cartesian PML algorithm. Note at $r_{\max}$ the field equations begin to approximate those in 2D cartesian geometry. While the measured refection coefficient at the r_max boundary in the quasi-3D geometry is larger than in the 2D Cartesian geometry, we found it to be sufficient for our preliminary attempts. For the quasi-3D simulation, we kept the $m = 0, -1, 1$ modes.

In Fig. 2 we present the comparison between LWFA boosted frame simulation in 3D Cartesian geometry and quasi-3D geometry. We plot the plasma electron density and the E_z field on the upper half of the (x_1, x_2) cross-section in 3D [corresponding to (r, z) with $\theta = 0$ in quasi-3D]. Cases with and without moving window are shown. From Fig. 2 (b), (f), (d), and (h), we can see there is no evidence of NCI in the quasi-3D simulations. By comparing the first and second rows of Fig. 2, it can be seen that the quasi-3D results agree very well with the full 3D results, which indicates that the physics we are interested in is well preserved when only the $m = 0, -1, 1$ azimuthal harmonics are kept in the quasi-3D simulations.

Note in this paper, we only present plots at fixed times in the boosted frame. A detailed comparison of the laser evolution, as well as injected particle information between the boosted frame simulations and a lab frame simulation will be included in a future work.

SUMMARIES AND DISCUSSIONS

We presented preliminary OSIRIS results on combining the Lorentz boosted frame technique with quasi-3D algorithm for the modeling of Laser wakefield Acceleration. To eliminate the NCI that arises due to the relativistically drifting plasma, we applied a quasi-3D hybrid Yee-FFT solver together with a low-pass filter on current in k_z space. Good agreement between the LWFA boosted frame in 3D Cartesian geometry and quasi-3D geometry shows that the NCI is well suppressed for the quasi-3D algorithm.

In addition to combining Lorentz boosted frame technique with the quasi-3D algorithm, we also attempted to apply the moving window technique. Since the hybrid solver requires periodic boundary conditions along the drifting direction, we applied a damping region at the back of the moving window

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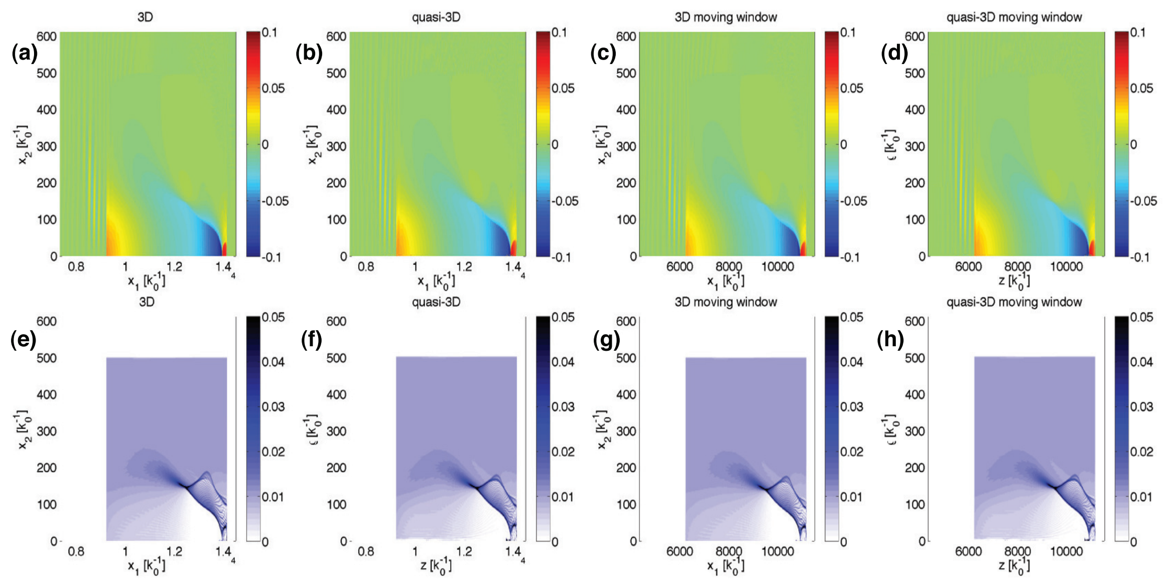

Figure 2: First row shows the E_z fields for the 3D and quasi-3D cases, both with and without the moving window. The second row shows the corresponding plasma electron density. The plasma is drifting from left to right in all these cases.

Table 1: Parameters for the LWFA simulation in the Lorentz boosted frame with $\gamma_b = 12$, and $\beta_b = (1 - \gamma_b^{-2})^{-1/2}$.

Plasma	
density n_0	$8.62 \times 10^{-4} n_0 \gamma_b$
length L	$5.89 \times 10^4 k_0^{-1}/\gamma_b$
Laser	
pulse length τ	$86.9 k_0^{-1} \gamma_b (1 + \beta_b)$
pulse waist W	$153.0 k_0^{-1}$
normalized vector potential a_0	4.0
Simulation parameters	
grid size $\Delta x_{1,2}$	$0.1 k_0^{-1} \gamma_b (1 + \beta_b)$
time step $\Delta t / \Delta x_1$	0.25
number of grid	
without moving window	8192×256
with moving window	3000×256
particle shape	quadratic

to prevent backward moving radiation from re-entering the box on the other side. Our preliminary results indicate that this combination of techniques has much potential unprecedented speedup. Future work will also include integrating this with the GPU and Intel Phi enabled versions of OSIRIS for even more dramatic speedups.

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