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SPACE CHARGE STUDIES IN FFAG USING THE TRACKING CODE ZGOUBI

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Abstract

A method is implemented in Zgoubi that allows the computation of space charge effects in 2D distributions and with some restrictions in 3D distributions. It relies on decomposing field maps or analytical elements into slices and applying a space charge kick to the particles. The aim of this study is to investigate the accuracy of this technique, its limitations/advantages by comparisons with other linear/nonlinear computation methods and codes, and to apply it to high power fixed field ring design studies.

INTRODUCTION

Accelerator Driven Systems are still in the early development stage. One of the main challenges is the average beam current required in order to achieve high transmutation rates. By choosing the effective multiplication factor as to accommodate any possible positive reactivity insertion during the operation of the reactor, it can be shown that the minimum average beam current is $\sim$ 10 mA. So, in order to investigate the possible benefits of FFAGs for Accelerator Driven Systems, one has to develop techniques to understand and master the space charge effects.

IMPLEMENTATION

Zgoubi [1] is a ray-tracing code which can track particles through electric and magnetic fields introduced as field maps or as analytic elements. We carry out a self consistent multi-particle simulation that is based on the space charge KV model [2] : the beam is supposed to have a uniform distribution with elliptical cross section, and a constant linear charge density. In that case, the free-space self-field solution within the beam is:

$$E_x = -\frac{\partial\phi}{\partial x} = \frac{\lambda}{\pi\epsilon_0}\frac{x}{(r_x+r_y)r_x} \tag{1}$$

$$E_y = -\frac{\partial\phi}{\partial y} = \frac{\lambda}{\pi\epsilon_0}\frac{y}{(r_x+r_y)r_y} \tag{2}$$

The kick received transversly by the particles after each integration step Δs is thus given by:

$$\Delta x' = \frac{2Q}{(r_x+r_y)r_x}x\Delta s \tag{3}$$

$$\Delta y' = \frac{2Q}{(r_x+r_y)r_y}y\Delta s \tag{4}$$

where Q is the generalized pervance term defined by, $Q = \frac{q\lambda}{2\pi\epsilon_0 m_0 c^2\beta^2\gamma^3}$. So, in order to evaluate the space charge force, one has to evaluate the beam radii and vice-versa.

Slicing

If we cut the magnet into thin slices, we may assume that the beam radii do not change much within each slice. Thus, each particle will experience a transverse space charge kick given by the formula (3) and (4) above. The main assumption here is that each particle in the beam does not see its immediate neighbors, but the smooth potential which is derived from the bunch (with uniform distribution) as a whole. Taking the statistical averages (rms quantities) is the natural way to derive analytically the evolution of such quantities as the beam emittance or the beam edge radius. Here we restrict ourselves to the KV model for which we define:

$$r_x = 2 < x^2 >^{1/2} \tag{5}$$

$$\epsilon_x = 4[< x^2 >< x'^2 > - < xx' >^2]^{1/2} \tag{6}$$

Given that the space charge forces are linear, the rms edge emittance in this model is constant and represents the maximum Courant-Snyder invariants. This was successfully checked. Also, the transverse particle equation of motion (when dispersion is neglected and only valid within the beam) is given by [3]:

$$x''(s) + \left(\kappa_x(s) - \frac{2Q}{[r_x(s)+r_y(s)]r_x(s)}\right)x(s) = 0 \tag{7}$$

If we consider a KV beam composed of N particles and with zero rms emittance, such as x_i(s=0)=x_i and x_i'(s=0)=0 $\forall i \in [\![1\,;\,N]\!]$. Then the envelope edge radius in a drift is given by:

$$r_x(s) = x_i^{max}\cosh\left[\left(\frac{2Q}{r_{x0}(r_{x0}+r_{y0})}\right)^{1/2}\times s\right] \tag{8}$$

where r_{x0} and r_{y0} are the envelope edge radii at the entrance of the drift. Fig. 1 shows a comparison of the axial beam envelope in a drift obtained from tracking with the analyti-cal formula (8). Similar tests were performed in quadrupole element which gave agreement as well. A natural test of the convergence of this method is to vary the number of slices until the beam envelope stabilizes: Fig. 2 below illustrates the convergence in a drift element (which is easy to picture). This result can be interpreted in the following way: it shows the speed at which the space charge force evolves in a KV beam (in a drift).

SPACE CHARGE EFFECTS IN SCALING FFAG

Analytical Model of the 150 MeV KURRI FFAG

The current work is a first step to understand the space charge effects in the 150 MeV scaling FFAG at KURRI in

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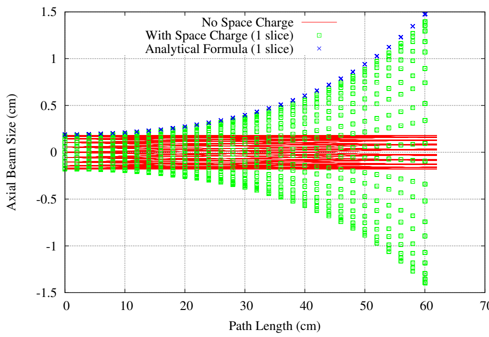

Figure 1: Axial beam envelope in a drift. The analytical formula (8) in blue shows that the space charge kick is applied correctly.

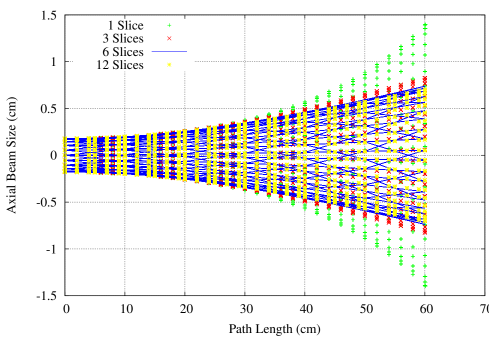

Figure 2: Convergence of the slicing method in a drift.

Japan. The FFAG ring consists of 12 DFD triplets and accelerates a proton beam up to 150 MeV for subcritical reactor application. 3D field map (see Fig.3) was first used to perform low intensity benchmarking using different codes. The results of tracking showed excellent agreement between the different codes. More details about this work and comparisons with results of recent experiments can be found in the recent paper [6]. On the basis of the successful comparison of the different codes the FFAG analytical model in Zgoubi [5] was coupled with the KV model in order to examine the key ingredients in the space charge effects.

Tune Shift

We investigate the change in betatron oscillation frequency due to space charge forces. When dispersion is neglected, the linear Laslett tune shift is given by:

$$\begin{aligned} \Delta Q_x &= \frac{1}{4\pi}\int_0^C \beta_x(s) K_{x,SC} ds \\ &= \frac{1}{4\pi}\int_0^C \beta_x(s) \frac{2Q}{r_x(r_x+r_y)} ds \end{aligned} \quad (9)$$

The Laslett tune shift is proportional to the orbit radius (contained in the integral) and inversely proportional to $\beta^2\gamma^3$. In order to check the validity of our simulations, the tune shift is computed as a function of the energy: We launch several beams of particles that have a uniform distribution, at different energies. The emittance is the same. This allowed us to

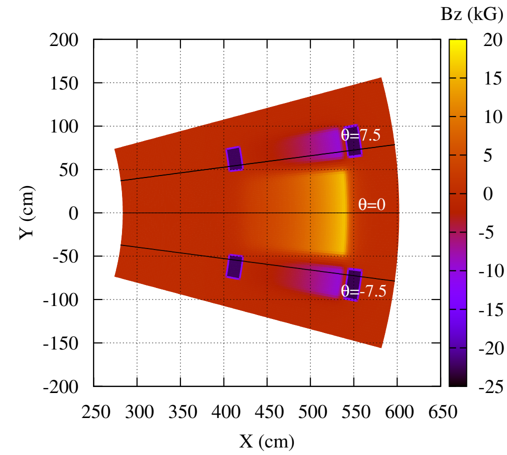

Figure 3: Median plane field map of the KURRI 150 MeV scaling FFAG (only one sector is represented): the straight lines show an example of the slicing into 4 elements.

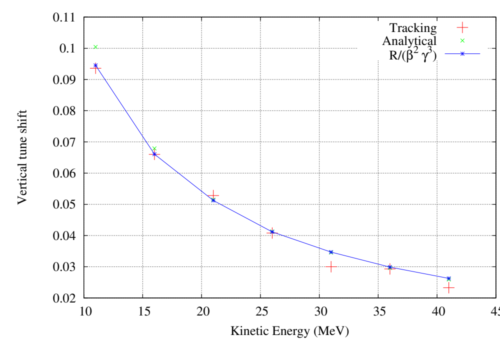

Figure 4: Tune shift as a function of the energy in the KURRI scaling FFAG. Good agreement between the analytical formula 9 and the tracking results is shown. (The perveance taken here is $Q = 6.4 \times 10^{-8}$).

have the same rms edge radii for different energies. Since the betatron functions do not change much with the energy, the Laslett tune shift should scale as $R/(\beta^2\gamma^3)$ (the linear charge density is kept the same). The results of tracking are compared to the analytical formula 9 which showed a good agreement with the expected scaling law (see Fig.4).

Damping Law in Scaling FFAG

In reality, the beam size reduces with acceleration, the normalized emittance of the beam is conserved. To show this, we first write the linearized Hill's equations of motion with acceleration:

$$\begin{aligned} \frac{d^2x}{ds^2} + \frac{(\gamma\beta)'}{\gamma\beta}\frac{dx}{ds} + \frac{1-n}{\rho^2}x = 0 \\ \frac{d^2y}{ds^2} + \frac{(\gamma\beta)'}{\gamma\beta}\frac{dy}{ds} + \frac{n}{\rho^2}y = 0 \end{aligned} \quad (10)$$

where n is the field index. In a scaling FFAG, if we neglect the scalloping of the orbits, n can be expressed in terms of the scaling factor:

$$n = -\frac{\rho}{B}\frac{dB}{dx} \approx -\frac{\rho}{B}\frac{dB}{dR} = -\frac{\rho}{R}k(R) \quad (11)$$

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where k is the average scaling factor that is allowed to change radially in order to account for any design imperfections. Substituting the above expression of n into Eq (10), and assuming that the amplitude and phase of the solution vary slowly (the energy increase is much slower than the betatron oscillations), one can use the WKB approximation [4] in order to solve the above differential equation with variable coefficients. An approximate solution is given by:

$$x(s) = x_0 A(s) \exp\left[\int i h_x(s) ds\right] \tag{12}$$

Using the hard edge model, one can show that $\frac{\rho}{R}$ does not change much with energy, which was confirmed by tracking. Thus, solving for the vertical y-component, one obtains:

$$h_y^2(s) = -\frac{k(R)}{\rho R} = -\frac{k(R)}{\alpha R^2} \tag{13}$$

$$A(s) = \frac{1}{\sqrt{\beta\gamma}} \times \frac{1}{\sqrt{|h_y(s)|}} = \frac{\sqrt{R}}{\sqrt{\beta\gamma}} \times \left(\frac{|\alpha|}{k(R)}\right)^{1/4} \tag{14}$$

In conclusion, the damping law for a scaling FFAG is given by:

$$\epsilon_{norm} \propto \frac{\beta\gamma}{R} \left(\frac{k(R)}{|\alpha|}\right)^{1/2} \times \epsilon \tag{15}$$

where ϵ is the geometrical beam emittance. Thus, for a KV beam, the y-space charge kick is given by:

$$K_{y,SC} = \frac{2Q}{(r_x + r_y) r_y} \propto \frac{q\lambda}{m_0 c^2 \beta\gamma^2 R} \left(\frac{k(R)}{|\alpha|}\right)^{1/2} \tag{16}$$

This shows that, in the KV model, if the scaling factor increases with R, the space charge effects would be amplified. In other words, if the applied magnetic field average variation increases faster with increasing energy, then the envelope amplitude damps faster, which implies a more important space charge kick . The radial dependence of the linear space charge kick shows that, for a scaling FFAG, the Laslett tune shift is independent of the orbit radius. In other words, due to the fact that the damping amplitude of the beam is proportional to the orbit radius (damps less fast), this counterbalances the fact that the tune shift increases with the path length.

NB: In reality, one should make the distinction in this analysis between the F and D-magnet, but this will be presented more in detail in later publications. For the moment, note that α alternates its sign for the F and D magnet.

FUTURE PLANS

Much of the high intensity issues for proton drivers centers on preserving the beam quality by understanding and controlling emittance growth and halo formation due to nonlinear forces. In general, beam dynamics in FFAG are intricate due to the non-linearities of the field. However, nonuniform beam distributions may exhibit even a more complicated behavior: consider a Gaussian charge distribution for instance: $\rho(x,y) = \frac{Ne}{2\pi\sigma_x\sigma_y}\exp(-\frac{x^2}{2{\sigma_x}^2} - \frac{y^2}{2{\sigma_y}^2})$. The transverse force due to the self fields of such a distribution is a non-linear function of x and y, and to the second order expansion in round beam geometry, the space charge kick is given by [7]:

$$\Delta x' = \frac{Q}{(\sigma_x + \sigma_y)\sigma_x} \exp[-\frac{x^2 + y^2}{(\sigma_x + \sigma_y)^2}] x \Delta s \tag{17}$$

$$\Delta y' = \frac{Q}{(\sigma_x + \sigma_y)\sigma_y} \exp[-\frac{x^2 + y^2}{(\sigma_x + \sigma_y)^2}] y \Delta s \tag{18}$$

Thus, the betatron tune shift of particles near the center of the beam is larger than in the equivalent uniform beam, and large amplitude particles will experience a zero tune shift. Also, such a distribution, evolving in time, may become non-Gaussian. Another key question that one needs to tackle is whether a particular distribution is stable or unstable against perturbations. From the thermodynamic point of view, a collisional kinetic system will tend to converge to an equilibrium distribution according to Boltzmann's H theorem. However, on a short time scale, the Vlasov Poisson equation is the correct equation to describe the space charge effects. Thus, simulations using the OPAL code [8] are foreseen in the near future.

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