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DECOHERENCE DUE TO SECOND ORDER CHROMATICITY IN THE NSLS-II STORAGE RING*

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INTRODUCTION

We study decoherence effects due to second order chromaticity for small amplitude kicks, in order to estimate the energy spread from TbT data. The measurements are taken for the bare lattice (no Damping Wigglers and Insertion devices) of the NSLS-II storage ring, since the long transverse radiation damping time $\tau_x = 54$ allows the study of decoherence effects on shorter time scales. To minimize the chromatic damping/antidamping from the slow-head tail effect, we used a short train of 50 bunches distributed over consecutive rf-buckets with an average current $I_0 = 1$mA, high enough to obtain a good BPM signal. The vertical and horizontal betatron motion have been excited independently with pinger magnets. In this contribution we limit the discussion to the horizontal case. A more detailed analysis discussing the vertical case, together with the transition to kicks of larger amplitudes to characterize decoherence with amplitude effects will be addressed in our future work. Decoherence studies have been done by independently exciting horizontal and vertical betatron oscillations with pingers, from small values of the pinger voltage for good BPM signal, to larger values in order to excite betatron oscillations of amplitude (peak to peak) 2mm. Larger betatron oscillations have been induced to clearly show the transition from decoherence due to second order chromaticity to decoherence with amplitude. Decoherence due to chromaticity and nonlinearities and beam energy spread measurements have been discussed in [1, 2].

DECOHERENCE DUE T0 LINEAR CHROMATICITY

Although in this contribution we discuss measurements for negligible linear chromaticity, in a small kick regime dominated by second order chromaticity, the decoherence due to nonzero linear chromaticity can be used as well to accurately estimate important beam parameters such as the synchrotron tune and the energy spread, based on the direct analysis of TbT data or on the analysis of their spectra. Of course, in second order chromaticity can not be made negligible, both effects (linear chromaticity and second order chromaticity) must be studied simultaneously. This case introduces little complications, for example formulae cannot eventually be put in closed form, however, as shown in the next Section, it does not prevent to estimate beam parameters. A formula for the decoherence of TbT data due to linear chromaticity is well known [3–5]. A derivation of the formula is given in Appendix. Here we discuss its use to estimate the synchrotron tune and energy spread from

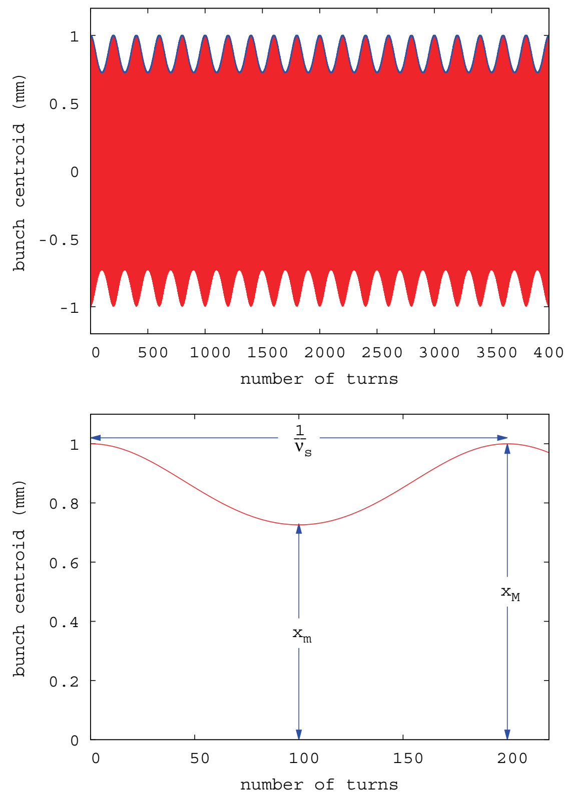

Figure 1: Top frame: evolution of the bunch centroid $\langle x(t)\rangle$. Bottom frame: enlargement of the top frame illustrating how the synchrotron tune and energy spread can be extracted from the signal of TbT data.

TbT data. Assuming for the betatron frequency ω a linear dependence on the relative energy deviation δ

$$\omega(t) = \omega_\beta + \xi\omega_0\delta(t), \tag{1}$$

where ξ is the linear chromaticity, the time evolution of the kicked (at t=0) bunch centroid $\langle x(t)\rangle$ reads (see Appendix)

$$\langle x(t)\rangle = \langle x_0\rangle e^{-2\frac{\xi^2\sigma_\delta^2}{\nu_s^2}\sin^2\frac{\omega_s t}{2}} \cos\omega_\beta t, \tag{2}$$

where η is the slippage factor, ω_s the synchrotron frequency, $\nu_s = \omega_s/\omega_0$ the synchrotron tune, σ_δ the energy spread and $\sigma_{z0} = \eta c/(\omega_s\sigma_\delta)$ the bunch length, where c is the speed of light. The top frame of Fig. 1 shows the evolution of the bunch centroid $\langle x(t)\rangle$ where its envelope shows a modulation with a characteristic wavelength and amplitude as given by Eq. 2. For illustration purposes, the linear chromaticity ξ has been chosen equal to 4. The bottom frame of Fig. 1 shows an enlargement of the top frame, and illustrates how the synchrotron tune and energy spread can be extracted from the signal of TbT data. The synchrotron tune

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can be estimated from the knowledge of the period of the signal, while from the maximum x_M and minimum x_m amplitude of the signal we can estimate the energy spread (or equivalently the bunch length) according to

$$\sigma_\delta = \frac{\nu_s}{\xi}\sqrt{\ln\sqrt{\frac{x_M}{x_m}}},\ \text{equivalently}\ \sigma_{z0} = \frac{\eta c}{\xi\omega_0}\sqrt{\ln\sqrt{\frac{x_M}{x_m}}}$$

To include radiation damping in the analysis, Eq.(2) is multiplied by the factor $\exp(-T_0 t/\tau_x)$, where T_0 is the revolution period.

DECOHERENCE DUE T0 SECOND ORDER CHROMATICITY

To study second order chromaticity effects at zero linear chromaticity, we assume for the betatron frequency ω a quadratic dependence on δ,

$$\omega(t) = \omega_\beta + \xi_2\omega_0\delta^2(t), \tag{3}$$

where ξ_2 is the second order chromaticity. From Eq.(7) (see Appendix), the transverse single particle dynamics reads

$$x(t) = x_0\cos\phi(t) + \frac{p_{x0}}{\omega_\beta}\sin\phi(t),$$
$$p_x(t) = p_{x0}\cos\phi(t) - \omega_\beta x_0\sin\phi(t), \tag{4}$$

where the betatron phase is

$$\phi(t,z_0,\delta_0) = \int_0^t d\tau\,\omega(\tau) = \frac{\xi_2\omega_0}{\eta c}\delta_0 z_0\sin^2\omega_s t + \tag{5}$$
$$\frac{t\omega_s^2\omega_0\xi_2}{2}\Big(\frac{z_0^2}{\eta^2c^2} + \frac{\delta_0^2}{\omega_s^2}\Big) - \frac{\omega_s\omega_0\xi_2}{4}\sin 2\omega_s t\Big(\frac{z_0^2}{\eta^2c^2} - \frac{\delta_0^2}{\omega_s^2}\Big).$$

The evolution of the bunch centroid can then be calculated by averaging over the initial conditions $z_0, \delta_0, x_0, p_{x0}$ (or initial particle positions) distributed according to the initial phase space density Ψ_0

$$\langle x(t)\rangle = \int dz_0 d\delta_0 dx_0 dp_{x0}\Big(x_0\cos\phi(t,z_0,\delta_0)$$
$$+\frac{p_{x0}}{\omega_\beta}\sin\phi(t,z_0,\delta_0)\Big)\Psi_0(z_0,\delta_0,x_0,p_{x0}). \tag{6}$$

Eq.(6) can be conveniently integrated with a Montecarlo method or by quadrature. The inclusion of both linear and second order chromaticity, together with amplitude dependent terms, is straightforward.

ENERGY SPREAD ESTIMATE FROM ANALYSIS OF TBT BPM DATA

The second order chromaticity ξ_2, at negligible linear chromaticity ξ, is used to estimate the energy spread in the NSLS-II storage ring with the bare lattice. The measured chromaticity, as shown in Fig. 2, is $\xi_x = 0.03, \xi_y = 0.01$ (linear chromaticity), and $\xi_{2x} = -135.8, \xi_{2y} = 52.7$ (second order chromaticity). The decoherence of the average

Figure 2: Measurement of chromaticity with the NSLS-II TbT tune measurement system: $\xi_x = 0.03, \xi_y = 0.01$, $\xi_{2x} = -135.8, \xi_{2y} = 52.7$.

horizontal BPM signal from TbT data at zero linear chromaticity as measured for the bare lattice of the NSLS-II storage ring is shown in Fig. 3 (red line). The theoretical fit, from the decoherence due to second order chromaticity, of the envelope of the BPM signal is shown by the blue line for three different kick amplitudes. The green line shows the decay of the BPM signal if only radiation damping were present. The fit uses the known radiation damping time, $\tau_x = 54$ms, the measured second order chromaticity $\xi_{2x} = -135.8$ and the nominal energy spread $\sigma_\delta = 0.0005$ of the bare lattice. The top frame of Fig. 3 shows a very good agreement between the theoretical fit and the average BPM signal for an initial kick of amplitude 80μm, giving therefore an indirect good estimate of nominal energy spread. In this small kick amplitude regime, we can therefore state that decoherence with amplitude is negligible. For kicks of larger amplitudes, the decoherence with amplitudes becomes significant. The middle frame of Fig. 3 shows the onset of the decoherence with amplitude for a kick of 100μm, while the bottom frame of Fig. 3 shows the transition to a regime with significant decoherence with amplitude for kicks of 800μm. We plan to analyzed this regime in our future work, in conjunction with the study done in [6].

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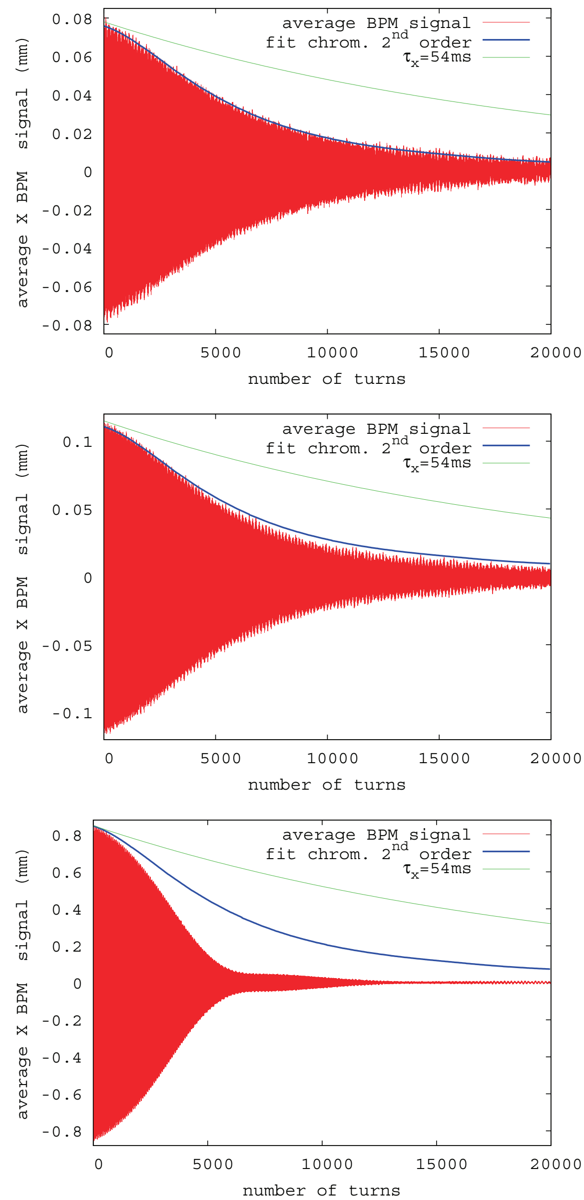

Figure 3: Decoherence of the average horizontal BPM signal from TbT data at zero linear chromaticity as measured (red line) for the bare lattice of the NSLS-II storage ring.

APPENDIX: DECOHERENCE FORMULA DUE TO LINEAR CHROMATICITY

From the longitudinal single particle dynamics

$$z(t) = z_0 \cos\omega_s t - \frac{\eta c}{\omega_s}\delta_0 \sin\omega_s t,$$
$$\delta(t) = \delta_0 \cos\omega_s t + \frac{\omega_s}{\eta c} z_0 \sin\omega_s t, \qquad (7)$$

assuming the betatron frequency ω a linear in δ

$$\omega(t) = \omega_\beta + \xi\omega_0\delta(t), \qquad (8)$$

the transverse single particle dynamics reads

$$x(t) = x_0 \cos\phi(t) + \frac{p_{x0}}{\omega_\beta}\sin\phi(t),$$
$$p_x(t) = p_{x0}\cos\phi(t) - \omega_\beta x_0 \sin\phi(t), \qquad (9)$$

where the betatron phase is

$$\phi(t, z_0, \delta_0) = \int_0^t d\tau\, \omega(\tau) = \omega_\beta t - \chi(z(t) - z_0). \qquad (10)$$

The time evolution of the bunch centroid $\langle x\rangle$ is obtained by averaging over the initial conditions

$$\langle x(t)\rangle = \int dz_0 d\delta_0 dx_0 dp_{x0}\Big(x_0\cos\phi(t, z_0, \delta_0) + \frac{p_{x0}}{\omega_\beta}\sin\phi(t, z_0, \delta_0)\Big)\Psi_0(z_0, \delta_0, x_0, p_{x0}). \qquad (11)$$

Assuming $\Psi_0(z_0, \delta_0, x_0, p_{x0}) = f(z_0, \delta_0)g(x_0, p_{x0})$ and $\langle p_{x0}\rangle = \int d_{x0}dp_{x0}p_{x0}g(x_0, p_{x0}) = 0$, it follows

$$\langle x(t)\rangle = \langle x_0\rangle \int dz_0\, d\delta_0 \qquad (12)$$
$$\times\Big(\cos(\omega_\beta t + z_0\chi(1-\cos\omega_s t))\cos(\delta_0\kappa\sin\omega_s t) - \sin(\omega_\beta t + z_0\chi(1-\cos\omega_s t))\sin(\delta_0\kappa\sin\omega_s t)\Big)g(z_0, \delta_0).$$

For a Gaussian distribution

$$g(z_0, \delta_0) = \frac{1}{2\pi\sigma_{z0}\sigma_{\delta_0}} e^{-\frac{1}{2}\left(\frac{z_0^2}{2\sigma_{z0}^2} + \frac{\delta_0^2}{\sigma_{\delta_0}^2}\right)}, \qquad (13)$$

the decoherence formula due to linear chromaticity reads

$$\langle x(t)\rangle = \langle x_0\rangle e^{-2\frac{\xi^2\sigma_{\delta_0}^2}{\nu_s^2}\sin^2\frac{\omega_s t}{2}} \cos\omega_\beta t, \qquad (14)$$

where we used $1 - \cos\omega_s t = 2\sin^2\frac{\omega_s t}{2}$, $\sigma_{z0} = \frac{\eta c}{\omega_s,}\sigma_{\delta_0}$ and $\frac{1}{\sqrt{2\pi}\sigma}\int_{-\infty}^{+\infty} dx\, \cos ax\, e^{-\frac{x^2}{2\sigma^2}} = e^{-\frac{1}{2}\sigma^2 a^2}$.

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