

Figure 2: A schematic drawing showing the process of normal forming the one turn map of a storage ring, where an action-angle dependant one turn map gets transformed into an action only dependance map.

However, it is believed that lowering the RDTs, together with reducing the tune shift with amplitudes and second order chromaticities, help to increase the stable region of particle revolution, i.e., enlarge the DA size.

The RDT coefficients h_{jklm} for N sextupoles, representing the lowest order RDTs, is usually calculated as

$$h_{jklm} = c \sum_{i=1}^N S_2 \beta_{xi}^{(j+k)/2} \beta_{yi}^{(l+m)/2} e^{i[(j-k)\mu_{xi} + (l-m)\mu_{yi}]}, \quad (4)$$

where S_2 is the integrated sextupole strength. To minimize the RDTs, we need to carefully pick the sextupole strengths and locations.

In eRHIC IR design, we modified the existing RHIC's anti-symmetric IR lattice to a symmetric lattice (Fig. 3), i.e., the dispersion functions on the two sides of IPs are symmetric. Furthermore, we design the lattice to have 90 degree phase advance from IP to the first sextupole. In such a way, the chromatic contribution from sextupoles in IR (to the lowest order) is cancelled and thus not contributed to the rest of the storage ring. The ARCs in eRHIC hadron lattice, which is composed of pure FODO cells, have 90 degree phase advance per cell to remove the coupling between sextupole families. We know the 2nd order chromaticities can be expressed as

$$C_x^{(2)} = -C_x^{(1)} - \frac{J_{p,x}^2}{4(\nu_x - p/2)\delta^2}, \quad (5)$$

where $J_{p,x}$ are the stopband integrals of resonances. The change in the stopband integrals thus reads

$$\Delta J_{p,x} = \frac{\delta}{2\pi} N [\beta_F(\Delta S)_F D_F + \beta_D(\Delta S)_D D_D e^{i\pi/4}]. \quad (6)$$

The stopband integral of resonances (and the 2nd order chromaticities) therefore can be easily minimized with careful tuning of the 24 families of sextupoles.

SIMULATION SETUP AND RESULTS

We employed multiple simulation tools to study this entire process: the linear lattice design is done in SYNCH [2] and MADX [3] for their fast convergence; the optimization of

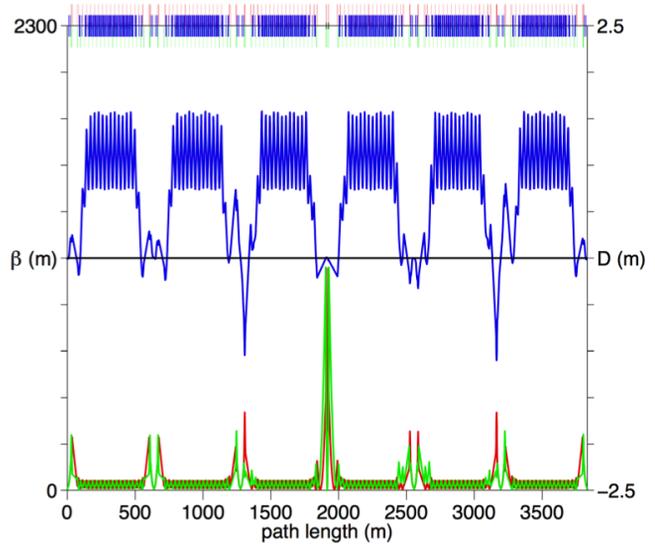


Figure 3: The interaction region (IR) of eRHIC is redesigned to have symmetric lattice, which helps to cancel the chromatic terms induced by non linear magnets.

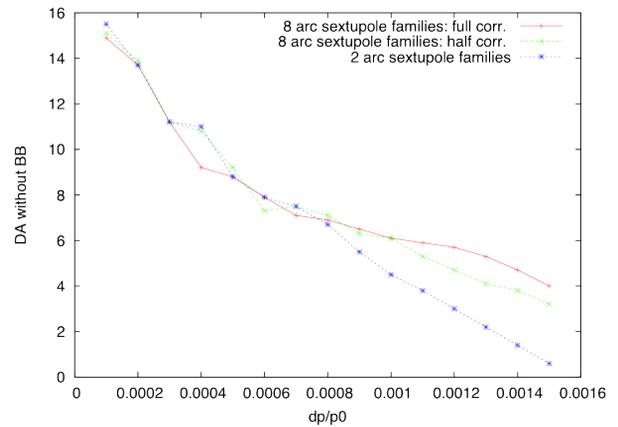


Figure 4: The plot of DA versus fractional energy deviation for a correction of chromaticities using 2 families of sextupoles. The DA size quickly reduces and diminishes for particles with large momentum offsets.

sextupole setups and locations is done in ELEGANT [4]; the tracking of DA over millions of beam revolution (due to the slow cooling rate for hadron rings) is done in SimTrack [5]. A self-developed Python script is developed for conversion and glueing between programs. The beam parameters we used for the self consistent simulation are listed in Table. 1.

Figure 4 shows the DA (in terms of rms beam size) versus momentum deviation using only two families of sextupoles. The result shows an acceptable (>10) size of stable region for on momentum particles however quickly diminishes for particles with large momentum offsets.

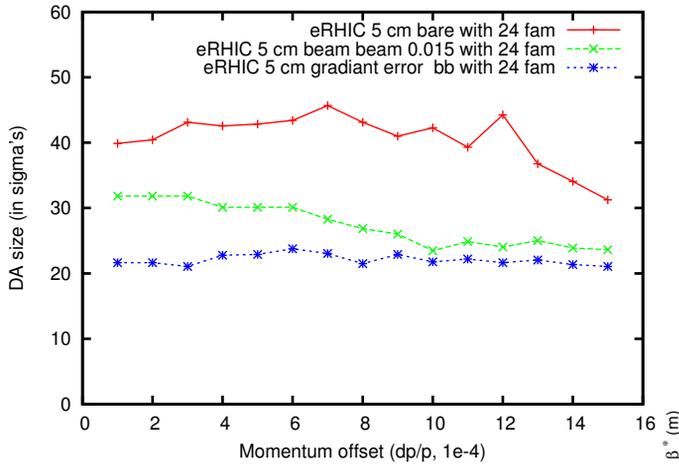


Figure 5: The DA with 24 families of sextupoles has been greatly improved. The DAs for bare lattice (red), lattice with beam beam (0.015) and misalignment (FW 100 μm , green) and lattice with beam beam and misalignment and magnet gradient errors ($\pm 0.1\%$, blue) are shown as a result of careful optimizing and tuning of the sextupoles.

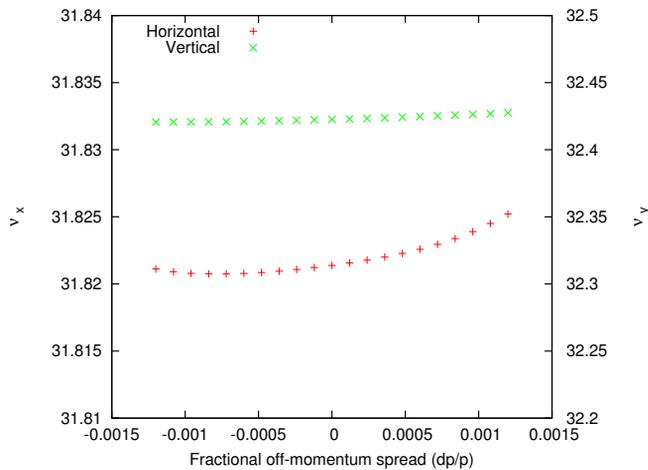


Figure 6: The chromaticities (1st and 2nd order) are well controlled after the optimization of sextupole families shown in the tune energy deviation plot.

With optimizing and tuning the 24 families of sextupoles as mentioned above, we achieved a much larger DA size shown as the red curve in Fig. 5. We studied the robustness of such scheme by assigning magnets misalignment (with FW 100 μm), magnets gradient errors ($\pm 0.1\%$ for quadrupoles and sextupoles) and 6D beam-beam interactions (with beam-

beam parameter at 0.015). The results are plotted as green and blue curves in Fig. 5. The DA size larger than 20 sigmas under the worst scenarios. Our study shows pretty good robustness for such sextupole working point.

The chromaticities (Fig. 6) and chromatic aberration on beta functions (Fig. 7) are well under controlled with such sextupole setup.

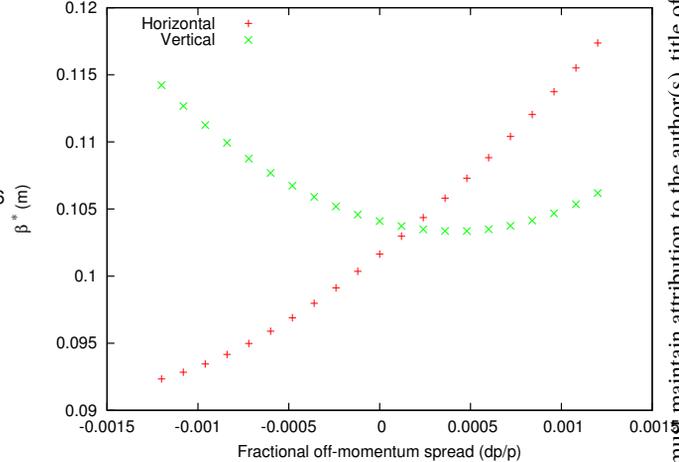


Figure 7: The chosen setup for sextupole families also minimize the beta function at collision dependence on energy deviation.

Table 1: Parameters for eRHIC DA Optimization

Hadron energy (GeV)	100
Hadron beam emittance (μm)	0.2
β^* at IP (cm)	5
ν_x, ν_y	0.82, 0.42
Maximum gradient sextupole ($S_2, 1/m^2$)	1.03

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